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Computer Method for Amplitude Spectrum Analysis

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Abstract. A computer programme based on Fortran IV language for doing Amplitude Spectrum Analysis (ASPEC) and the associated flow charts are explained. The importance of the programme in ionosphere and space research is briefly discussed.

1. Introduction

In view of the rapid growth of electronics, acoustics, vibrational mechanics and all the branches of engineering that are in some way related to the theory of oscillations, spectral representations found a broad application. The primary definition of a spectrum is based on the Fourier transform i.e., integration with respect to time carried out between infinite limits. Thus, when a time function is subjected to transformation as a whole, the result of the transformation is nothing but a spectrum that depends on the frequency as well as time.

In some data series like the temperature and pressure variations of a particular locality, the values generally fluctuate around a more or less constant value. In such series, the observed values are the result of the interaction of the various components, and as such, if from the original series the trend values are subtracted, the remaining values would represent the fluctuations. The trend values may be measured by separate methods. A computer programme based on Fortran IV language, has been developed for bringing out the various periodic components in a data series. Some details of the programme utilizing an IBM 1130 digital computer at the Department of Physics, Andhra University, Waltair, are communicated through this paper.

2. Theory

Every periodic function can be represented by a series of trignometric function as

$$f(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos\left(2\pi k \frac{x}{T} - \phi_k\right) \tag{1}$$

The periodic function $f(x)$ thus appears as a summation of components of the form

$$a_k \cos\left(2\pi k \frac{x}{T} - \phi_k\right),$$

each of which is a sinusoidal oscillation with an amplitude a_k and initial phase ϕ_k. T is a constant quantity called the period and a_0 is the dc component or constant component. The frequencies of the oscillations which make up the periodic function $f(x)$ form a harmonic sequence, where, the frequencies of all the components are multiplies of the fundamental frequency $1/T$. The individual components are called harmonics. Thus the oscillation with frequency $1/T$ is called the first harmonic ($k = 1$), that with frequency $2/T$ is called the second harmonic and so on.

The Eqn. (1) can also be written in a general form as

$$f(x) = a_0 + \sum_{k=1}^{\infty} \left(A_k \cos 2\pi k \frac{x}{T} + B_k \sin 2\pi k \frac{x}{T}\right) \tag{2}$$

where

$$A_k = C_k \cos \phi_k, \; B_k = C_k \sin \phi_k$$

so that

$$C_k = (A_k^2 + B_k^2)^{1/2} \text{ and } \tan \phi_k = \frac{B_k}{A_k}$$

The coefficients a_k and b_k are defined by the equations.

$$A_k = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \cos 2\pi k \frac{x}{T} \, dx \tag{3}$$

$$B_k = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \sin 2\pi k \frac{x}{T} \, dx \tag{4}$$

The constant component is given by as

$$a_0 = \frac{1}{T} \int_{-T/2}^{+T/2} f(x) \, dx \tag{5}$$

If the data refers to some physical phenomenon, say, an exponential curve, then naturally one should search for the coefficients of a good fit of an exponential, and if the data represents a periodic function, the only alternative is to go for Fourier analysis.

The Fourier series can also be written in a complex form as

$$f(x) = \sum_{k=-\infty}^{+\infty} A_k \, e^{j2\pi k} \frac{x}{T} \tag{6}$$

where

$$2A_k = C_k\, e^{-j\phi_k}$$

The quantity $2A_k$ is the complex amplitude and A_k is found from the equation

$$A_k = \frac{1}{T} \int_{-\pi/2}^{+\pi/2} f(x)\, e^{-j2\pi k \frac{x}{T}}\, dx \tag{7}$$

The summation is taken over both positive and negative integer values of k including the zero value.

The above Fourier series Eqn. (6) can also be written as

$$f(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos (k\, \omega_1\, x - \phi_k) \tag{8}$$

where

$\omega_1 = \frac{2\pi}{T}$ is the fundamental frequency. Thus the complex period function $f(x)$ can be fully defined by the set of quantities a_k and ϕ_k. The set of quantities a_k is called the Amplitude Spectrum and the set of quantities ϕ_k is correspondingly called as the Phase Spectrum.

3. Programme Discussion

This programme is developed in the course of upper atmospheric wind analysis of the Meteor Radar Observations at Waltair. Of course, there are some other methods for doing similar type of analysis like the one developed by Blackman & Tukey[1] and applied by Jones & Maude[2,3] for ionospheric drift measurements. According to their method, the autocovarience function and the critical frequency of the spectrum are respectively given by

$$C(\tau) = \underset{T\to\infty}{Lt}\ \frac{1}{T} \int_{-T/2}^{+T/2} f(t) \cdot f(t + \tau)\, dt \tag{9}$$

and

$$f_0 = \frac{\Delta t}{2\tau} \tag{10}$$

where τ and Δt are respectively the lag and time interval between any two successive observing points in the data. Thus the autocovariance function can not be estimated for arbitrarily long lags in continuous records of finite length and no estimate can be made for lags longer than the record. Further more, it is not desirable to use lags longer than a moderate fraction (5 to 10 per cent) of the length of the record. Their analysis gives the powers of various periodic components present in the data at discrete steps only which causes ultimately the possibility of missing the power of some interesting component.

In the case of discrete data points, the summations replace the integrations of the usual Fourier series presentation. The outstanding property of the Fourier series is the fact that if we take a finite number of terms of the series i.e., approximately the periodic function by a trignometric polynomial, then for any value of N, the least square deviation from the exact value is obtained when the coefficients of the polynomial are determined. As the number of terms N increases the approximation becomes better and as $N \to \infty$, the approximate equation goes over to an exact one. In the present case, the Fourier series is represented by some linear combination of the functions :

$$1, \cos\frac{\pi}{N}x, \cos\frac{2\pi}{N}x, \ldots, \cos\frac{\pi(N-1)}{N}x, \cos\frac{\pi}{N}Nx$$

$$\sin\frac{\pi}{N}x, \sin\frac{2\pi}{N}x, \ldots, \sin\frac{(N-1)\pi}{N}x,$$

and so the function $f(x)$ can be represented by a series of the form

$$f(x) = a_0/2 \sum_{k=1}^{N}\left(a_k \cos\frac{\pi}{N}kx + b_k \sin\frac{\pi}{N}kx\right) + \frac{a_n}{2}\cos\pi x \qquad (11)$$

For simplicity sake, let us consider an even number of sample points and to make the sum of the series exactly equal to the function values at $N = 2M$ sample points, the following orthogonality relations were used in finding out the coefficients a_k's and b_k's :

$$\sum_{x=0}^{2M-1} \sin\frac{\pi}{M}kx \sin\frac{\pi}{M}mx = \begin{cases} 0 \text{ if } k \neq m \\ N \text{ if } k = m \neq 0 \end{cases}$$

$$\sum_{x=0}^{2M-1} \sin\frac{\pi}{M}kx \cos\frac{\pi}{M}mx = 0$$

$$\sum_{x=0}^{2M-1} \cos\frac{\pi}{M}kx \cos\frac{\pi}{M}mx = \begin{cases} 0 \text{ if } k \neq m \\ M \text{ if } k = m \neq 0, M \\ 2M \text{ if } k = m = 0, M \end{cases}$$

On multiplying both sides of the assumed series by $\cos\left(\frac{\pi}{M}\right)mx$, $\sin\left(\frac{\pi}{M}\right)mx$ and summing over all x, finally we will get

$$a_0 = \frac{1}{M}\sum_{x=0}^{2M-1} f(x) \qquad (12)$$

$$a_n = \frac{1}{M}\sum_{x=0}^{2M-1} f(x)\cos\pi x \qquad (13)$$

$$a_k = \frac{1}{M} \sum_{x=0}^{2M-1} f(x) \cos \frac{\pi}{M} kx \qquad k = 0, 1, 2, ..., M \tag{14}$$

$$b_k = \frac{1}{M} \sum_{x=0}^{2M-1} f(x) \sin \frac{\pi}{M} kx \quad k = 0, 1, 2, ..., M \tag{15}$$

In this manner, the present programme can supply the continuous spectrum of the data points in the whole length of the time series. The flow charts for this computer programme are illustrated in Figs. 1 and 2. The first step (displayed in Fig. 1) will do the harmonic analysis for all the observing points by taking the first data point as FO (for notational consistency of all the data points in the arry F), followed by $(2M-1)$ additional points. The constant term a_0 can be computed by simple summation followed by a division by M. Excluding a_M, there are now $(M-1)$ sine coefficients and $(M-1)$ cosine coefficients to compute. The coefficients a's and b's are calculated in the programme by using a DO loop that runs from 1 to $(M-1)$ values. After that, the percentage of each harmonic is computed by taking fundamental frequency represented by a_1 and b_1 as 100 per cent. The relations involved

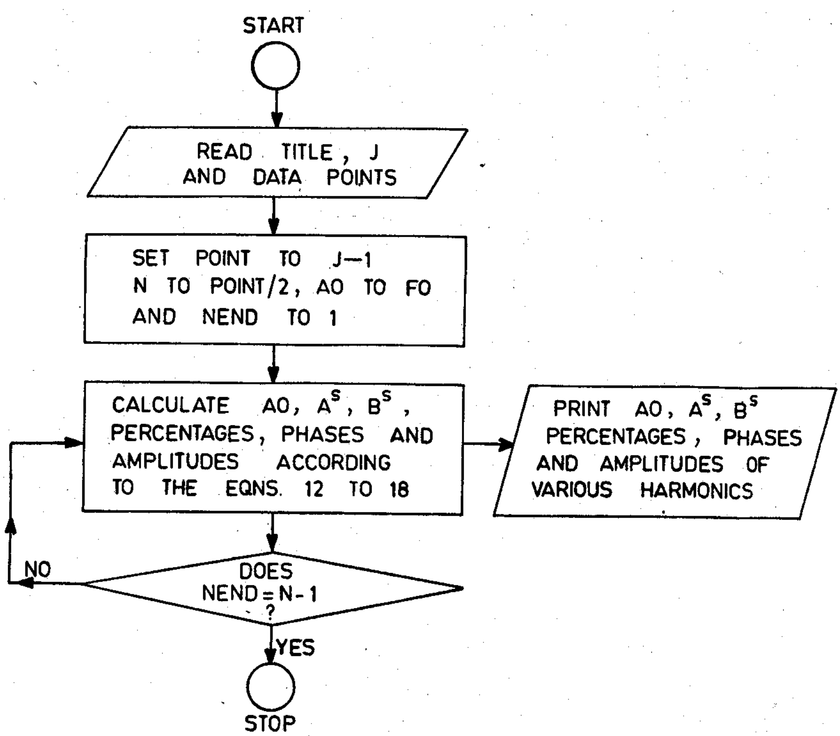

Figure 1. Flow chart showing calculation of phases and amplitudes of various harmonics.

in the calculation of percentage relative to fundamental, phases and amplitudes of various harmonics are given by

$$\mathrm{PER}\ (K) = \sqrt{\frac{a_k^2 + b_k^2}{a_1^2 + b_1^2}} \times 100.0 \tag{16}$$

$$\phi(K) = \tan^{-1}\left[\frac{b_k}{a_k}\right] \tag{17}$$

$$V(K) = \sqrt{a_k^2 + b_k^2} \tag{18}$$

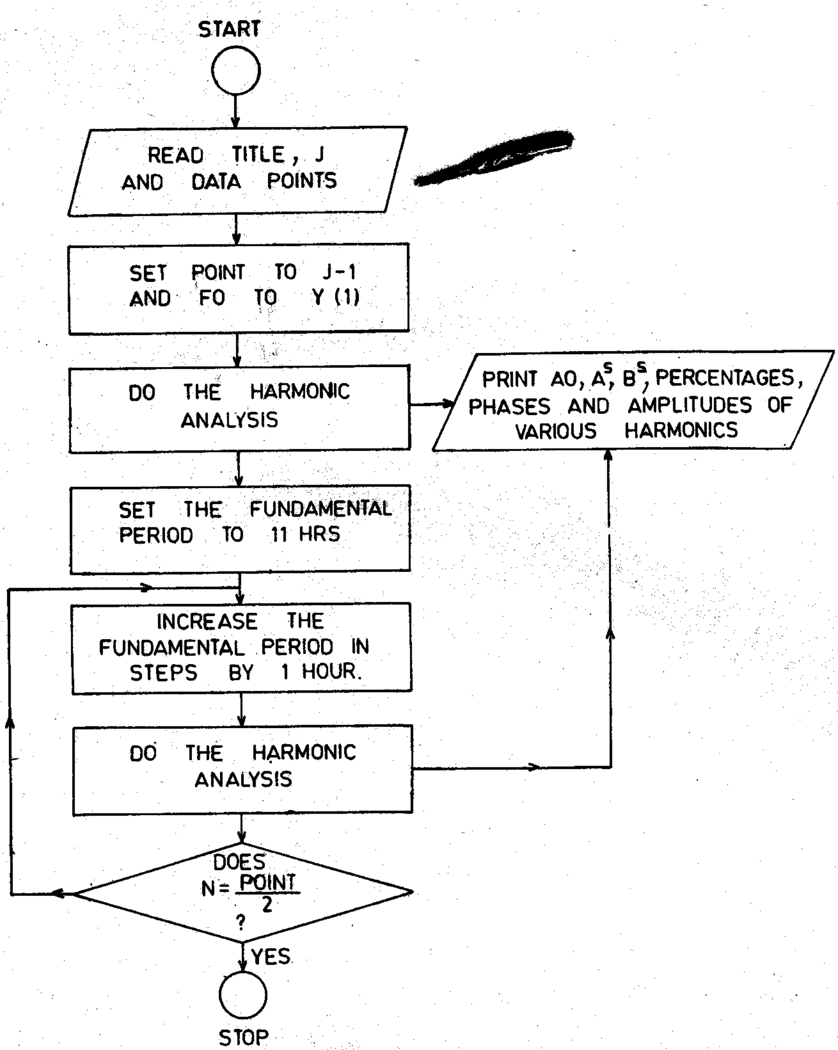

Figure 2. Flow chart showing the procedure for doing the amplitude spectrum analysis.

where $K = 1, 2, \ldots, M-1$.

The second step (displayed in Fig. 2) acts as source or main programme which actually constructs the spectrum by initially sampling a periodic wave form at different places (initial sampling was made in the present programme at 12 places i.e. $2\omega = 12$). After that, it will go on increasing the fundamental period in steps of one unit i.e., 1 minute or 1 hour or 1 day according to the availability of the data and at each step, the same harmonic analysis will be repeated at each fundamental period.

4. Application of the Programme to Space Research Problems

As mentioned earlier, the knowledge of the periodicities of various atmospheric waves which ultimately cause both regular and irregular changes of various atmospheric parameters is essential for the understanding of atmospheric dynamics. Since the programme is mainly developed for this purpose, it is necessary here to mention a few periodicities that generally or commonly appear at E and F regions and more recently found at D region also, of the ionosphere.

(a) Stationary planetary waves — Periodicities of the order of months.

(b) Travelling planetary waves or Rossby waves — Periodicities of the order of days.

(c) Atmospheric tides — Periodicities of the order of a few hours to days.

(d) Gravity waves and acoustic gravity waves — Periodicities of the order of a few minutes to hours.

The present programme has been tested and verified in various cases by applying it to the data involving both known and unknown periodicities at various occasions. Besides the amplitude information it gives for each and every periodic component present in the data, with the phase information, one can examine that whether the concerned wave phenomenon is propagating type or evanescent type.

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