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Flow of a Dusty Gas with Suspended Particles in a Rotating Frame of Reference

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Abstract. The solution is given for the problem on the motion of a dusty gas under a rotating system of co-ordinates. The gas containing a uniform distribution of dust particles, occupies the infinite space above a rigid plane boundary. The motion of the dusty gas is induced by the motion of the plate moving with a velocity which decreases exponentially with time in a rotating frame of reference such that the plate and the gas are in a state of rigid body rotation about an axis normal to the plate. The velocity fields for the dusty gas, and the dust particles along and normal to the direction of motion of the plate are obtained in closed forms. Finally, some velocity distributions are calculated with particular reference to the effect of rotation at different times, and it is found that with the increase of the value of the rotation parameter ω, the velocity of the dusty gas along the direction of motion of the plate gradually increases while the velocity of the dust particle along the same direction gradually decreases.

Introduction

The theoretical study of two phase systems is an important prelude to understand many physical phenomena. For example, flows in rocket tubes where small carbon or metallic fuel particles are present, blood flow in capillaries, pneumatic conveyance of small grain-like particles.

The dynamics of a dusty gas was formulated by Saffman[1] in terms of a large number density $N(\vec{x}, t)$ of a cloud of undeformable spherical particles suspended in an incompressible fluid. In this formulation, the bulk concentration of the dust particles is assumed to be very small but the density of the dust material is taken to be large compared with the fluid density so that the mass concentration of the dust is an appreciable fraction of unity. A comprehensive review of the dynamics of dusty gases has been given by Marble[2]. Michael and Miller[3] have investigated the motion of dusty gas with uniform distribution of dust particles occupied in the semi-infinite space above a rigid plane boundary. Sone[4] has considered the steady flow past a body fixed in a uniform flow of dusty gas. Rukmangadachari and Arunachalam[5] have obtained the exact solutions for the flow of a dusty gas through a cylinder of triangular cross-section. Mitra[6] has investigated the flow of a viscous incompressible dusty gas between two concentric co-axial circular cylinders.

The study of motion of a dusty gas in a rotating system has some bearing on the pollution problems as well as on the motion of aerosol over the rotating earth. Gupta[7] and Pop considered the boundary layer growth in a fluid with suspended particles in a rotating frame of reference.

The present paper is concerned with the unsteady flow of a viscous incompressible dusty gas induced by the motion of an infinite flat plate moving with a velocity, which decreases exponentially with time in a rotating frame of reference. The dusty gas and the plate are rotating in unison with constant angular velocity Ω about an axis normal to the plate. The expressions for the velocity fields for the dusty gas, dust particles and the clean gas along and normal to the direction of motion of the plate are obtained in closed forms. Some velocity distributions for the gas, particle and the clean gas are drawn for the times $T = 0$, $T = 1$ in the region $0 \leqslant y \leqslant 1$ for different values of the rotation parameter ω.

Governing Equations

We consider an infinite plate lying along the plane $z = 0$, and situated in a viscous incompressible dusty gas suspended with a uniform distribution of dust particles with a small bulk concentration. The motion of the dusty gas occupying the space $z > 0$ is induced by the motion of the plate moving with a velocity $V \,.\, e^{-\lambda_1^2 t}$ in x-direction in a rotating frame of reference such that the plate and the gas are rotating in unison with constant angular velocity Ω about z-axis. The horizontal homogeneity of the problem demands that all the physical quantities will be functions of z and t only. Then using the formulation of Saffman[1], the equations of momentum for the dusty gas along the axes of x and y in a rotating frame of reference are respectively,

$$\frac{\partial u_1}{\partial t} = \nu \,.\, \frac{\partial^2 u_1}{\partial z^2} + 2\Omega u_2 + \frac{KN_0}{\rho}\,(u_1' - u_1) \tag{1}$$

$$\frac{\partial u_2}{\partial t} = \nu \,.\, \frac{\partial^2 u_2}{\partial z^2} - 2\Omega u_1 + \frac{KN_0}{\rho}\,(u_2' - u_2) \tag{2}$$

Similarly, the equations of momentum for the dust particle along x and y directions are respectively

$$\frac{\partial u_1'}{\partial t} = \frac{K}{m}\,(u_1 - u_1') + 2\Omega u_2' \tag{3}$$

$$\frac{\partial u_2'}{\partial t} = \frac{K}{m}\,(u_2 - u_2') - 2\Omega u_1' \tag{4}$$

In the above equations u_1, u_2, u_3 and u_1', u_2', u_3' are the components of velocity of the dusty gas and the dust particle at a point respectively, ν is the kinematic coefficient of viscosity of the gas, ρ is the gas density, m the mass of a dust particle, K is the Stokes' resistance co-efficient which is equal to $6\pi\mu a$, for spherical particles of radius a, μ being the gas viscosity and N_0 is the number density of the dust particles and is taken to be constant throughout the motion. From the equations of continuity of the gas and the dust particles, it follows that $u_3 = 0$ and $u_3' = 0$ everywhere in the flow field.

The boundary conditions are

$$\left.\begin{aligned} & u_1 = V \, . \, e^{-\lambda_1^2 t}, \; u_2 = 0 \text{ at } z = 0, \\ & (u_1, u_2) \to 0 \text{ as } z \to \infty. \end{aligned}\right\} \quad (5)$$

Solution of the Equations

We introduce the following non-dimensional quantities

$$u = \frac{u_1}{V}, \; v = \frac{u_2}{V}, \; u_p = \frac{u_1'}{V}, \; v_p = \frac{u_2'}{V}, \; T = \frac{t}{\tau},$$

$y = \dfrac{z}{\sqrt{\nu\tau}}$, where $\tau \left(= \dfrac{m}{K} \right)$ is the relaxation time of the dust particles. Then the Eqns. (1) to (4) respectively reduce to

$$\frac{\partial u}{\partial T} = \frac{\partial^2 u}{\partial y^2} + 2\omega v + f(u_p - u) \quad (6)$$

$$\frac{\partial v}{\partial T} = \frac{\partial^2 v}{\partial y^2} - 2\omega u + f(v_p - v) \quad (7)$$

$$\frac{\partial u_p}{\partial T} = (u - u_p) + 2\omega v_p \quad (8)$$

$$\frac{\partial v_p}{\partial T} = (v - v_p) - 2\omega u_p \quad (9)$$

where $f \left(= \dfrac{mN_0}{\rho} \right)$ is the mass concentration of the dust particles and $\omega \, (= \Omega\tau)$ is the rotation parameter. The boundary conditions (5) transform to

$$\left.\begin{aligned} & u = e^{-\lambda^2 T}, \; v = 0 \text{ at } y = 0 \\ & (u, v) \to 0 \text{ as } y \to \infty. \end{aligned}\right\} \quad (10)$$

We choose the solutions of (6) to (9) respectively as

$$\left.\begin{aligned} & u = F(y) \, . \, e^{-\lambda^2 T}, \; v = G(y) \, . \, e^{-\lambda^2 T} \\ & u_p = F_p(y) \, . \, e^{-\lambda^2 T}, \; v_p = G_p(y) \, . \, e^{-\lambda^2 T} \end{aligned}\right\} \quad (11)$$

Then the equations (6) to (9) respectively reduce to

$$\frac{d^2 F}{dy^2} + F(\lambda^2 - f) + 2\omega G + fF_p = 0 \quad (12)$$

$$\frac{d^2 G}{dy^2} + G(\lambda^2 - f) - 2\omega F + fG_p = 0 \quad (13)$$

$$(1 - \lambda^2) F_p - 2\omega G_p - F = 0 \quad (14)$$

$$(1 - \lambda^2)\ G_p + 2\omega F_p - G = 0 \tag{15}$$

The boundary conditions (10) transform to

$$\left.\begin{array}{l} F = 1,\ G = 0 \text{ at } y = 0 \\ (F, G) \to 0 \text{ as } y \to \infty. \end{array}\right\} \tag{16}$$

From equations (14) and (15) we get,

$$\left.\begin{array}{l} F_p = \dfrac{(1-\lambda^2)F + 2\omega G}{(1-\lambda^2)^2 + 4\omega^2} \\ G_p = \dfrac{(1-\lambda^2)G - 2\omega F}{(1-\lambda^2)^2 + 4\omega^2} \end{array}\right\} \tag{17}$$

Then the equations (12) and (13) respectively reduce to

$$\frac{d^2F}{dy^2} + \alpha F + \beta G = 0 \tag{18}$$

$$\frac{d^2G}{dy^2} + \alpha G - \beta F = 0 \tag{19}$$

where $$\alpha = (\lambda^2 - f) + \frac{f(1-\lambda^2)}{(1-\lambda^2)^2 + 4\omega^2} \tag{20}$$

and $$\beta = 2\omega\left[1 + \frac{f}{(1-\lambda^2)^2 + 4\omega^2}\right] \tag{21}$$

The solution of the simultaneous differential equations (18) and (19) under the boundary conditions (16) are

$$F = e^{-By}.\ \cos\ (Ay) \tag{22}$$

and $$G = -\ e^{-By}\ .\ \sin\ (Ay), \tag{23}$$

where

$$\left.\begin{array}{l} A = \left[\dfrac{\sqrt{\alpha^2 + \beta^2} + \alpha}{2}\right]^{1/2} \\ B = \left[\dfrac{\sqrt{\alpha^2 + \beta^2} - \alpha}{2}\right]^{1/2} \end{array}\right\} \tag{24}$$

Thus, from (11) the velocity components of the dusty gas and the dust particles are obtained as

$$u = e^{-By}\ .\ \cos\ (Ay)\ .\ e^{-\lambda^2 T} \tag{25}$$

$$v = -\ e^{-By}\ .\ \sin\ (Ay)\ .\ e^{-\lambda^2 T} \tag{26}$$

and

$$u_p = e^{-By}\ .\ \frac{(1-\lambda^2)\ .\ \cos Ay - 2\omega\ .\ \sin Ay}{(1-\lambda^2)^2 + 4\omega^2}\ .\ e^{-\lambda^2 T} \tag{27}$$

$$v_p = - e^{-By} \cdot \frac{(1 - \lambda^2) \sin Ay + 2\omega \cdot \cos Ay}{(1 - \lambda^2)^2 + 4\omega^2} \cdot e^{-\lambda^2 T} \tag{28}$$

For clean gas $f = 0$ and so from Eqns. (25) and (26), the velocity components of the clean gas are obtained as

$$u_c = e^{-B_c y} \cdot \cos (A_c y) \cdot e^{-\lambda^2 T} \tag{29}$$

and
$$v_c = - e^{-B_c y} \cdot \sin (A_c y) \cdot e^{-\lambda^2 T} \tag{30}$$

where A_c and B_c are respectively the values of A and B in Eqn. (24) for $\alpha = \lambda^2$ and $\beta = 2\omega$.

Numerical Results and Conclusions

The velocity profiles for the dusty gas, dust particle and the clean gas along and normal to the direction of motion of the plate are drawn in the region $0 \leqslant y \leqslant 1$ for times $T = 0, 1$, taking the parameters characterizing the flow as follows :

$\lambda^2 = 0.8, f = 0.5, \omega = 0.05, 0.10, 0.15$. From these figures the following points are observed :

(*i*) From Fig. 1, it is noted that with the increase of the rotation parameter ω, the velocity of the dusty gas along the direction of motion of the plate gradually increases. (*ii*) But from Fig. 2, it is observed that with the increase of the rotation parameter ω, the velocity of the dusty gas in a direction normal to the direction of

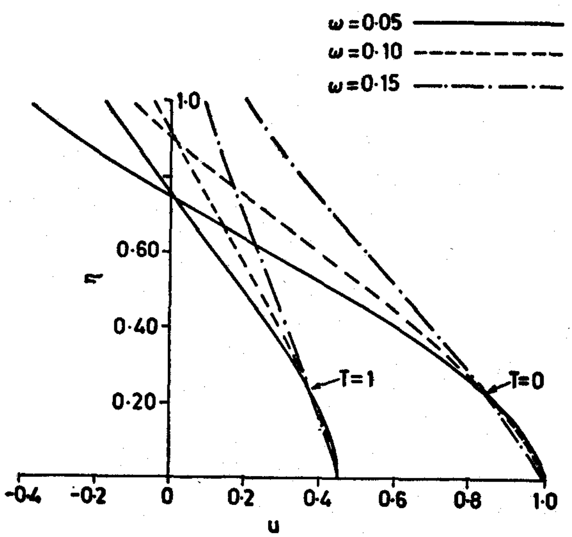

Figure 1. Velocity distributions of the dusty gas along with the direction of motion of the plate.

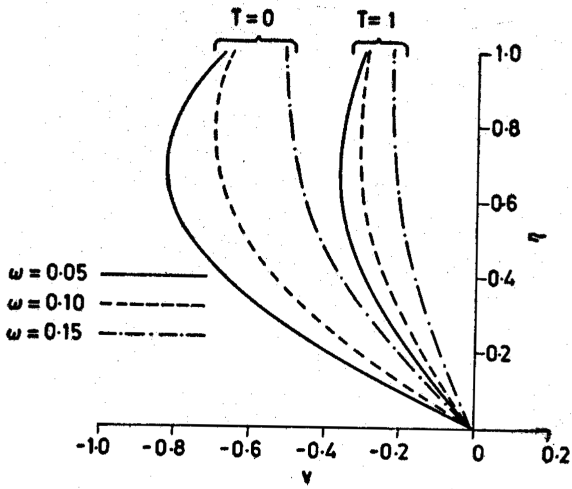

Figure 2. Velocity distributions of the dusty gas in a direction normal to the direction of motion of the plate.

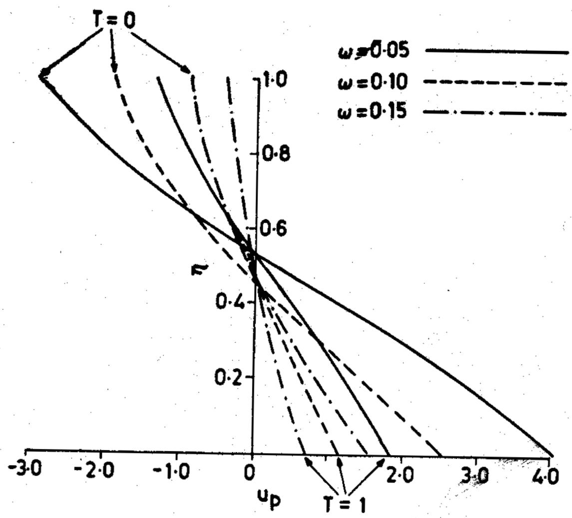

Figure 3. Velocity distributions of the dust particle along with the direction of motion of the plate.

motion of the plate decreases rapidly. (*iii*) From Figs. 3 and 4 we note that as the value of the rotation parameter ω increases, the velocities of the dust particle along and normal to the direction of motion of the plate gradually decrease.

(*iv*) For the clean gas, from Figs. 5 and 6, it is observed that with the increase of the rotation parameter ω, the velocities of the clean gas along and normal to the direction of motion of the plate gradually decrease.

(*v*) From Figs. 1 and 5, we note that the velocity of the clean gas along the direction of motion of the plate at each point of the flow field is greater than that of the dusty gas along the same direction. In otherwords, due to the presence of the dust particles, the velocity of the gas decreases along the direction of motion of the plate.
(*vi*) From Figs. 1, 3 and 2, 4 we observe that the velocities of the dust particle along and normal to the direction of motion of the plate are respectively greater than those of the dusty gas along the same direction.

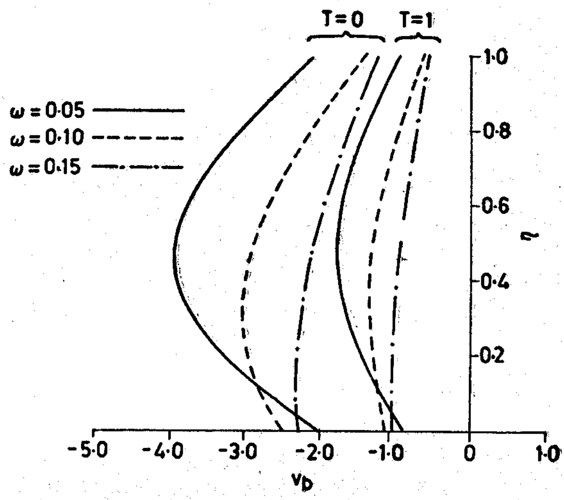

Figure 4. Velocity distributions of the dust particle in a direction normal to the direction of motion of the plate.

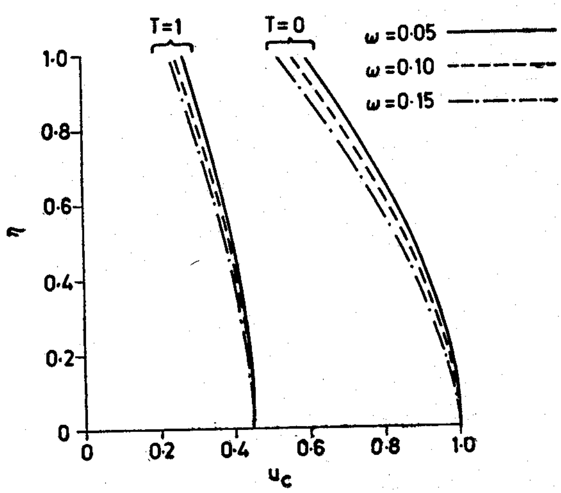

Figure 5. Velocity distributions of the clean gas along with the direction of motion of the plate.

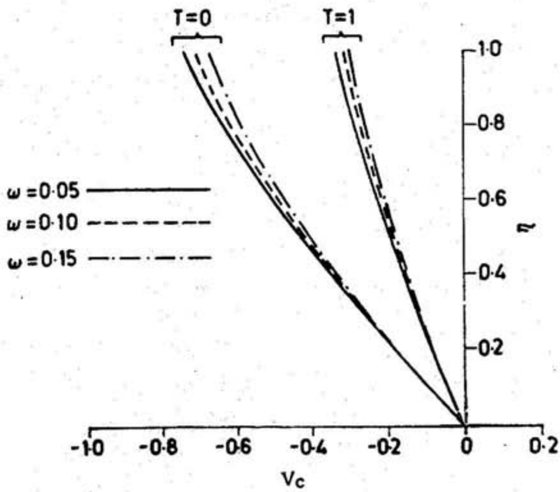

Figure 6. Velocity distributions of the clean gas in a direction normal to the direction of motion of the plate.

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