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Dusty Viscous Flow Between Concentric Spheres Performing Longitudinal Oscillations

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Abstract. The primary flow of dusty viscous fluid due to longitudinal oscillations of two concentric spheres is considered. The velocity fields for the fluid and dust particles are determined. The pressure is evaluated and the drag on a sphere is calculated assuming that the frequency of oscillations is small. The oscillation of a single sphere is discussed. The presence of dust in the fluid increases the virtual mass of the sphere and the magnitude of the damping force acting on the sphere.

1. Introduction

Interest in problems of multi-phase flows has developed rapidly in recent years and several authors have already solved various problems[1–4] using Saffman's model[5] for a two-phase motion. In this paper, laminar motion of a dusty fluid between two concentric spheres performing longitudinal oscillations is discussed. The velocity of the fluid and dust have been found exactly and the pressure field obtained for small frequency oscillations. The drag on a sphere has been calculated. Results pertaining to the oscillation of a single sphere are deduced. Some numerical work has been done and the results have been presented graphically and in tables.

2. Formulation and Solution

The governing equations of Saffman[5], in the usual notation, are

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \frac{KN}{\rho}(\vec{v} - \vec{u}) \tag{1}$$

$$m\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right] = K(\vec{u} - \vec{v}) \tag{2}$$

$$\operatorname{div}\vec{u} = 0 \tag{3}$$

$$\frac{\partial N}{\partial t} + \text{div}\, N\vec{v} = 0 \tag{4}$$

When we assume the number density N to be constant and introduce stream functions ψ and ψ^* for fluid and dust respectively, Eqns. (1)–(4), under the usual assumptions of slow flow, reduce to

$$\frac{\partial^2}{\partial t^2} E^2\psi + \frac{1+f}{\tau}\frac{\partial}{\partial t} E^2\psi = \nu\left(\frac{\partial}{\partial t} E^4\psi + \frac{1}{\tau} E^4\psi\right) \tag{5}$$

$$\frac{\partial}{\partial t} E^2\psi^* = \frac{1}{\tau} E^2(\psi - \psi^*) \tag{6}$$

where

$\tau = \dfrac{m}{K}$ is the relaxation time of the dust

$f = \dfrac{mN}{\rho}$ is the mass concentration of dust

and

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) \tag{7}$$

Let two concentric spheres of radii a and b $(a < b)$ containing between them a dusty viscous incompressible fluid perform oscillations along their common diameter which is taken as z-axis $(\theta = 0)$. If $(u, v, 0)$, $(u^*, v^*\ 0)$ denote respectively the velocity vectors of fluid and dust in (r, θ, ϕ) directions and if $2\pi/\sigma$ be the period of oscillation then the boundary condions are

$$(u, v) = (\cos\theta, -\sin\theta)\, Ue^{i\sigma t} \quad (r = a, b) \tag{8}$$

We adopt the convention that whenever complex expressions are quoted for physical quantities only their real parts are understood.

Substituting into (5) and (6)

$$(\psi, \psi^*) = \{\chi(r, \theta), \chi^*(r, \theta)\}\, e^{i\sigma t} \tag{9}$$

we obtain

$$E^2(E^2 - k^2)\,\chi = 0$$

$$E^2\left(\chi^* - \frac{\chi}{1 + i\sigma\tau}\right)$$

where

$$k^2 = \frac{i\sigma}{\nu}\left(\frac{1 + f + i\sigma\tau}{1 + i\sigma\tau}\right)$$

When χ is found solving Eqn. (10), then Eqn. (11) yields χ^*. Clearly

$$\chi^* = \frac{\chi}{1 + i\sigma\tau}$$

Now, substituting $\chi = \chi_1 + \chi_2$ into Eqn. (10), we have

$$E^2 \chi_1 = 0, \tag{14}$$

$$(E^2 - k^2)\chi_2 = 0 \tag{15}$$

Solving these, we obtain

$$\chi = (\chi_1 + \chi_2) e^{i\sigma t} = e^{i\sigma t} \sin^2\theta \sum_{n=1}^{\infty} \Big[(A_n r^{-n} + B_n + C_n r^{1/2} I_{n+1/2}(kr) + D_n r^{1/2} K_{n+1/2}(kr) \Big] P_n^{(1)}(\cos\theta) \tag{16}$$

where $P_n^{(1)}(\cos\theta)$ is the Associated Legendre Polynomial of the first kind of order n and degree 1; $I_{n+1/2}(kr)$ and $K_{n+1/2}(kr)$ are the modified Bessel functions of the first and second kind of the indicated order.

Imposing the boundary conditions (8), we get an infinite set of Eqns. to determine the constants A_n, B_n, C_n and D_n. But, if we set $A_n = B_n = C_n = D_n = 0$ ($n \geqslant 2$) we will have the following four equations involving the four constants A_1, B_1, C_1 and D_1, which we henceforth denote dropping suffixes.

$$\left.\begin{aligned} A + Bb + Cb^{3/2}I_{3/2}(kb) + Db^{3/2}K_{3/2}(kb) &= \tfrac{1}{2}Ub^3 \\ -2A + 0 + Cb^{3/2}L(kb) + Db^{3/2}M(kb) &= 2Ub^3 \\ A + Ba + Ca^{3/2}I_{3/2}(ka) + Da^{3/2}K_{3/2}(ka) &= \tfrac{1}{2}Ua^3 \\ -2A + 0 + Ca^{3/2}L(ka) + Da^{3/2}M(ka) &= 2Ua^3 \end{aligned}\right\} \tag{17}$$

where

$$\begin{aligned} L(kx) &= I_{3/2}(kx) + 2kx\, I'_{3/2}(kx) \\ M(kx) &= K_{3/2}(kx) + 2kx K'_{3/2}(kx) \end{aligned} \tag{18}$$

The constants A, B, C and D are evaluated as

$$\begin{aligned} A = -\frac{Ub^{5/2}a^{5/2}}{\Delta} \Big[& 2b^{3/2}\{PM(ka) - QL(ka)\} \\ & + 2a^{3/2}\{QL(kb) - PM(kb)\} \\ & + \tfrac{1}{2}(b^2 - a^2)\{L(kb)M(ka) - M(kb)L(ka)\} \Big] \end{aligned}$$

$$\begin{aligned} B = \frac{Ub^{3/2}a^{3/2}}{\Delta} \Big[& 3b^{3/2}\{P_1M_1(ka) - Q_1L_1(ka)\} \\ & + 3a^{3/2}\{Q_1L_1(kb) - P_1M_1(kb)\} \\ & + \tfrac{1}{2}(b^3 - a^3)\{L_1(kb)M_1(ka) - M_1(kb)L_1(ka)\} \Big] \end{aligned}$$

$$\begin{aligned} C = -\frac{Ub^{3/2}a^{3/2}}{\Delta} \Big[& b^{-1/2}\{6a^{3/2}Q_1 - (a^3 - b^3)M_1(ka)\} \\ & + a^{-1/2}\{(a^3 - b^3)M_1(kb) - 6b^{3/2}Q_1\} \\ & + 6(b - a)\{a^{3/2}M_1(kb) - b^{3/2}M_1(ka)\} \Big] \end{aligned}$$

$$D = \frac{Ub^{3/2}a^{3/2}}{\Delta}\Big[b^{-1/2}\{6a^{3/2}P_1 - (a^3 - b^3)\,L_1(ka)\}$$
$$+ a^{-1/2}\{(a^3 - b^3)\,L_1(kb) - 6b^{3/2}P_1\}$$
$$+ 3(b - a)\,\{a^{3/2}L_1(kb) - b^{3/2}L_1(ka)\}\Big]$$

where

$$\Delta = a^{3/2}b^{3/2}[2b^{-1/2}\{PM_1(ka) - QL_1(ka)\}$$
$$+ 2a^{-1/2}\{QL_1(kb) - PM_1(ka)\}$$
$$+ (b - a)\{L_1(kb)\,M_1(ka) - M_1(kb)\,L_1(ka)\}] \tag{20}$$

$$L_1(kx) = 3I_{3/2}(kx) + 2kxI'_{3/2}(kx)$$
$$M_1(kx) = 3K_{3/2}(kx) + 2kx\,K'_{3/2}(kx)$$
$$P = a^{1/2}I_{3/2}(ka) - b^{1/2}I_{3/2}(kb)$$
$$Q = a^{1/2}K_{3/2}(ka) - b^{1/2}K_{3/2}(kb)$$
$$P_1 = a^{3/2}I_{3/2}(ka) - b^{3/2}I_{3/2}(kb)$$
$$Q_1 = a^{3/2}K_{3/2}(ka) - b^{3/2}K_{3/2}(kb) \tag{21}$$

The stream functions and the velocity components for the dust and the fluid are given by

For dust

$$\psi^* = -\sin^2\theta\, e^{i\sigma t}(1 + i\sigma\tau)^{-1}\,[Ar^{-1} + B + Cr^{1/2}I_{3/2}(kr)$$
$$+ Dr^{1/2}K_{3/2}(kr)]$$
$$u^* = 2\cos\theta\, e^{i\sigma t}(1 + i\sigma\tau)^{-1}\,[Ar^{-3} + Br^{-2} + Cr^{-3/2}I_{3/2}(kr)$$
$$+ Dr^{-3/2}K_{3/2}(kr)]$$
$$v^* = -\sin\theta\, e^{i\sigma t}(1 + i\sigma\tau)^{-1}\,[-Ar^{-3} + \tfrac{1}{2}Cr^{-3/2}\{I_{3/2}(kr)$$
$$+2kr\,I'_{3/2}(kr)\} + \tfrac{1}{2}Dr^{-3/2}\{K_{3/2}(kr) + 2kr\,K'_{3/2}(kr)\}]$$

For fluid

$$\psi = -\sin^2\theta\, e^{i\sigma t}[Ar^{-1} + B + Cr^{1/2}I_{3/2}(kr) + Dr^{1/2}K_{3/2}(kr)]$$
$$u = 2\cos\theta\, e^{i\sigma t}[Ar^{-3} + Br^{-2} + Cr^{-3/2}I_{3/2}(kr) + Dr^{-3/2}K_{3/2}(kr)]$$
$$v = -\sin\theta\, e^{i\sigma t}[-Ar^{-3} + \tfrac{1}{2}Cr^{-3/2}\{I_{3/2}(kr) + 2kr\,I'_{3/2}(kr)\}$$
$$+ \tfrac{1}{2}Dr^{-3/2}\{K_{3/2}(kr) + 2kr\,K'_{3/2}(kr)\} \tag{23}$$

where the prime denotes differentiation with respect to the argument '*kr*'

3. Small Oscillations

When the magnitude of oscillation is small so that $|k|$ is small, we have up to $0(|k|^3)$

$$(u^*, v^*, 0) = (1 + i\sigma\tau)^{-1} (u, v, 0) \tag{24}$$

$$\psi = -\sin^2\theta\, e^{i\sigma t}(A_1 r^{-1} + A_2 + A_3 r + A_4 r^2 + A_5 r^4) \tag{25}$$

$$u = 2\cos\theta\, e^{i\sigma t}(A_1 r^{-3} + A_2 r^{-2} + A_3 r^{-1} + A_4 + A_5 r^2) \tag{26}$$

$$v = -\sin\theta\, e^{i\sigma t}(-A_1 r^{-3} + A_3 r^{-1} + 2A_4 + 4A_5 r^2)$$

where

$$\left.\begin{aligned}
A_1 &= -\frac{1}{9}\,U ab(a+b)\left[3 - \frac{7}{18}\,abk^2 + \frac{7}{18}\,ab(a+b)\,k^3\right] \\
A_2 &= \frac{U}{18}\left[6(a^2+ab+b^2) - ab(a+b)^2k^2 \right. \\
&\qquad \left. + \frac{7}{9}\,ab(a+b)(a^2+ab+b^2)\,k^3\right] \\
A_3 &= \frac{Uab}{18}\,(a+b)\,k^2 \\
A_4 &= \frac{U}{6}\left[1 - \frac{7}{27}\,(a+b)\,k^3\right];\; A_5 = \frac{U}{60}\,k^2
\end{aligned}\right\}$$

In real terms, when we neglect terms of $0(|k|^3)$, we have the following expressions for the velocity components, the pressure and the drag on the sphere $r = a$

$$\begin{aligned}
u/U = 12\cos\theta &\left[\left\{1 + \frac{2}{r^2}(a^2+ab+b^2) - \frac{2ab}{r^3}(a+b)\right\}\cos\sigma t\right. \\
&+ (k_1 + fk_2)\left\{\frac{1}{10}\,r^2 + \frac{ab(a+b)}{3r} - \frac{ab(a+b)^2}{3r^2}\right. \\
&\left.\left. + \frac{7a^2b^2(a+b)}{27r^3}\right\}\right]
\end{aligned}$$

$$\begin{aligned}
v/U = -3\sin\theta &\left[\left\{1 - \frac{ab(a+b)}{r^3}\right\}\cos\sigma t\right. \\
&\left. + (k_1 + fk_2)\left\{\frac{1}{5}\,r^2 + \frac{ab(a+b)}{6r} + \frac{7a^2b^2(a+b)}{54r^3}\right\}\right]
\end{aligned}$$

$$p = -\mu U\cos\theta \,.\, \frac{2ab(a+b)}{9r^2}\cdot(k_1 + fk_2)$$

$$\begin{aligned}
D_{a,t} = -\frac{4\pi\mu U}{27}&\left[12(a + b + b^2/a)\cos\sigma t\right. \\
&\left. - (k_1 + fk_2)\{2b(a+b)^2 + 3a\}\right]
\end{aligned}$$

and Eqn. (32) for the drag can be thrown into the form

$$D_{a,t} = [GH \cos(\sigma t - \gamma)]_c + [- GHfJ^{-1} \sin\gamma \cos(\sigma t + \beta)]_d$$

where [ ]$_c$ and]$_d$ denote respectively the clean and dusty viscous flow parts of the drag; and

$$k_1 = \frac{\sigma}{\nu} \sin \sigma t; \quad k_2 = \frac{\sigma}{\nu J} \cos(\sigma t + \beta)$$

$$\sigma\tau = J\cos\beta; \; 12(a + b + b^2/a) = H\cos\gamma$$

$$= J\sin\beta; \; \frac{\sigma}{\nu}[2b(a+b)^2 + 3a] = H\sin\gamma$$

$$J = \sqrt{1 + \sigma^2\tau^2}; \; H = \Big[144(a + b + b^2/a)^2 + \frac{\sigma^2}{\nu^2}\{2b(a+b)^2 + 3a\}^2\Big]^{1/2}$$

$$\beta = \cot^{-1}\sigma\tau; \quad \gamma = \tan^{-1}\frac{\sigma}{\nu}\left(\frac{2b(a+b)^2 + 3a}{12(a+b+b^2/a)}\right)$$

$$G = -4\pi\mu U/27;$$

4. Oscillation of a Single Sphere

For the oscillation of a single sphere, $r = a$, in an infinite expanse of a dusty fluid we deduce the velocity, the pressure and the drag as

$$u = 2\cos\theta e^{i\sigma t}\left[A\frac{1}{r^3} + B\frac{e^{-kr}}{r^2}\left(1 + \frac{1}{kr}\right)\right] \tag{35}$$

$$v = \sin\theta e^{i\sigma t}\left[A\frac{1}{r^3} + B\frac{ke^{-kr}}{r}\left(1 + \frac{1}{kr} + \frac{1}{k^2r^2}\right)\right]$$

$$p = p_0 + \mu Ak^2 e^{i\sigma t}\frac{\cos\theta}{r^2} \tag{37}$$

$$D_s = -\frac{2\pi}{3}\mu aUe^{i\sigma t}(k^2a^2 + 9ka + 9)$$

where

$$A = \tfrac{1}{2}Ua^3\left(1 + \frac{3}{ka} + \frac{3}{k^2a^2}\right), \; B = -\frac{3}{2}Ua\left(\frac{e^{ka}}{k}\right)$$

and p_0 is the pressure at infinity. The expression for the drag agrees with that given by Lamb[6] but k^2 is now as given by Eqn. (12)

The Eqn. (38) for the drag can be put as

$$D_s = M'U\sigma(k' \sin \sigma t - k'' \cos \sigma t)$$

where

$$k' = \tfrac{1}{2} R \sin 2\phi + \frac{9}{2a}\sqrt{\frac{\nu R}{\sigma}} \sin \phi$$

$$k'' = \tfrac{1}{2} R \cos 2\phi + \frac{9}{2a}\sqrt{\frac{\nu R}{\sigma}} \cos \phi + \frac{9\nu}{2a^2\sigma}$$

$$R = \left(1 + \frac{(2+f)f}{1+\sigma^2\tau^2}\right)^{1/2}, \quad \phi = \tfrac{1}{2}\tan^{-1}\left(\frac{1+f+\sigma^2\tau^2}{\sigma\tau f}\right)$$

and

$$M' = 4\pi a^3\rho/3,$$

is the mass of fluid displaced by the sphere.

4. Numerical Results and Discussion

Numerical calculations have been performed and the results presented graphically and in tables.

Figs. 1 and 2 show the components of velocity $\bar{u}$, $\bar{v}$ against the distance r in the case of oscillation of two concentric spheres of radii $a = 1$ and $b = 2$ respectively for the

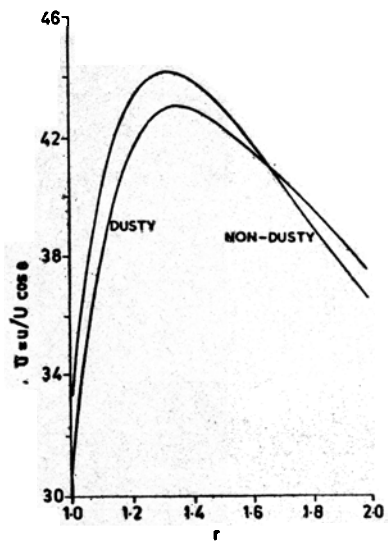

Figure 1. Variation of $\bar{u} = u/U \cos\theta$ across the annular space between two concentric spheres $r = a$, $r = b$.

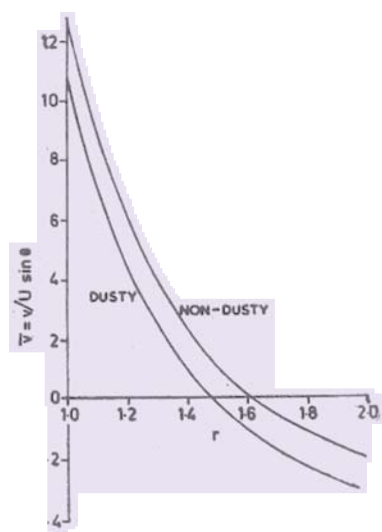

Figure 2. Variation of $\bar{v} = v/U \sin\theta$ across the annular spaces between two concentric spheres $r = a$, $r = b$.

value of the mass concentration of dust, $f = 0.4$ and time $t = 5$ secs. Fig. 1 shows that the clean fluid has greater velocity than the dusty fluid up to a value of $r = 1.66$ roughly and then it falls. It is evident from Fig. 2 that the velocity of the dusty fluid is smaller than that of the clean fluid.

Calculations of the components of velocity of the clean and dusty fluid performed for various values of t and for $f = 0.2$ have been omitted here to save space.

Fig. 3 depicts the drag ratio $D_{a,t}/D$ of the dusty fluid drag to the clean fluid drag on the sphere $r = a$ for the flow between concentric spheres. It is clear that the drag ratio decreases as time increases.

In Tables 1 and 2 are shown the values of $\bar{u}$ and $\bar{v}$ for certain values of t and r and for $f = 0.4$. It is seen that when $r = 1.2$ and 1.6 the numerical values of the

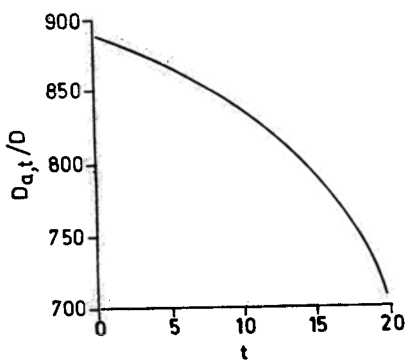

Figure 3. Variation of drag ratio $Da, t/D$ of dusty fluid drag to the clean fluid drag on sphere $r = a$.

Table 1. Values of $\bar{u} = u/U \cos\theta$ for flow between oscillating concentric spheres : $a = 1, b = 2; (f = 0.4)$.

r \ t	0 (Sec)	5 (Sec)	10 (Sec)	15 (Sec)	20 (Sec)
1.2	43.7268	41.5065	38.5588	34.9311	30.6854
1.4	44.3676	42.9760	40.8308	37.9659	35.2810
1.6	42.2453	41.6152	40.2546	38.1849	35.4399
1.8	39.4862	39.5259	38.8715	37.5325	35.5299

Table 2. Values of $\bar{v} = v/U \sin\theta$ for flow between oscillating concentric spheres $a = 1, b = 2 (f = 0.4)$.

r \ t	0 (Sec)	5 (Sec)	10 (Sec)	15 (Sec)	20 (Sec)
1.2	5.9611	4.2384	2.4425	0.6029	−1.2478
1.4	2.3545	0.9514	−0.4654	−1.8785	−3.2569
1.6	0.3018	−0.9541	−2.1922	−3.3822	−4.5326
1.8	−0.9717	−2.1780	−3.3452	−4.4540	−5.4843

dimensionless velocity $\bar{u}$ is smaller in the case of the dusty fluid than that of the clean fluid. It is interesting to note that, in general, the presence of dust in the fluid has the damping effect on the oscillations.

In Tables 3 and 4 the corresponding results pertaining to the oscillation of a single sphere of unit radius are presented for $f = 0.2$. The pressure distribution is tabulated in Table 5 which shows that the dimensionless pressure $\bar{p}$ is numerically large for the dusty fluid than for the clean fluid, at any time. The pressure $\bar{p}$ has a maximum at $t = 2$ secs., and a minimum at $t = 6$ secs.

Table 3. Values of $\bar{u} = u/U \cos\theta$ for the oscillation of a single sphere in a dusty fluid.

r \ t	0 (Sec)	2 (Sec)	4 (Sec)	6 (Sec)	8 (Sec)	10 (Sec)
			$f = 0.2$ (dusty fluid)			
1.2	0.8356	0.1587	−0.7456	−0.5817	0.4156	0.8175
1.4	0.5833	0.0349	−0.5635	−0.3546	0.3623	0.5602
1.6	0.3877	−0.0159	−0.3967	−0.2092	0.2781	0.3669
1.8	0.2612	−0.0257	−0.2758	−0.1308	0.2017	0.2452
2.0	0.1838	−0.0216	−0.1961	−0.0896	0.1452	0.1720
			$f = 0$ (clean fluid)			
1.2	0.8454	0.1620	−0.7517	−0.5917	0.4160	0.8277
1.4	0.5812	0.0344	−0.5611	−0.3530	0.3613	0.5580
1.6	0.4064	−0.0166	−0.4158	−0.2193	0.2914	0.3846
1.8	0.2751	0.0393	−0.2528	−0.1827	0.1491	0.2673
2.0	0.1925	−0.0168	−0.2020	−0.0978	0.1466	0.1809

Table 4. Values of $\bar{v} = v/U \sin\theta$ for the oscillation of a single sphere in a dusty fluid ($f = 0.2$).

r \ t	0 (Sec)	2 (Sec)	4 (Sec)	6 (Sec)	8 (Sec)	10 (Sec)
			$f = 0.2$ (dusty fluid)			
1.2	−0.0796	0.3171	0.2595	−0.1699	−0.3559	−0.0320
1.4	0.2290	0.2582	−0.0826	0.3050	−0.0905	0.2537
1.6	0.2466	0.1136	−0.2058	0.2170	0.0591	0.2505
1.8	0.1805	0.0248	−0.1665	−0.1192	0.0988	0.1753
2.0	0.1167	−0.0090	−0.1218	−0.0601	0.0877	0.1099
			$f = 0$ (clean fluid)			
1.2	−0.1283	0.2975	0.2971	−0.1289	−0.3702	−0.0811
1.4	0.1956	0.2708	−0.0420	0.2946	−0.1251	0.2236
1.6	0.2418	0.1382	−0.1634	0.2309	0.0324	0.2493
1.8	0.1916	0.0428	−0.1673	−0.1377	0.0892	0.1883
2.0	0.1302	−0.0019	−0.1312	−0.0726	0.0901	0.1237

Table 5. Values of $\bar{p} = \frac{p - p_0}{\frac{3}{2}\mu U \text{ Cos } \theta/a}$ for the oscillation of a single sphere in a dusty fluid ($f = 0.2$).

t \ r	0 (Sec)	2 (Sec)	4 (Sec)	6 (Sec)	8 (Sec)	10 (Sec)
			$f = 0.2$ (dusty fluid)			
1.2	9.2652	24.4615	4.6124	—21.8448	—17.0055	12.1972
1.4	6.8071	17.9717	3.3887	—16.0493	—12.4938	8.9612
1.6	5.2117	13.7596	2.5945	—12.2877	—9.5656	6.8609
1.8	4.1179	10.8718	2.0500	—9.7088	—7.5580	5.4210
2.0	3.3355	8.8062	1.6605	—7.8641	—6.1220	4.3910
			$f = 0$ (clean fluid)			
1.2	8.6576	21.1850	3.3612	—19.2782	—14.2982	11.1665
1.4	6.3607	15.5645	2.4694	—14.1635	—10.5048	8.2039
1.6	4.8699	11.9166	1.8907	—10.8440	—8.0427	6.2811
1.8	3.8478	9.4156	1.4939	—8.5681	—6.3547	4.9629
2.0	3.1167	7.6266	1.2100	—6.9401	—5.1473	4.0199

The values of drag $\bar{D}_s$ shown in Table 6 reveal the fact that a sphere experiences a greater drag in the dusty fluid than in the clean fluid. Now, the meaning of the two terms in the expression for the drag D_s at Eqn. (40) can be seen from the following reasoning. The force required to move the sphere of mass M in the absence of fluid stresses if $-MU\sigma$ Sin σt. Equation (40) shows that, in addition, a further force $-M'Uk^1\sigma$ Sin σt in phase with the acceleration is required. This arises because in the process of moving the sphere, fluid is necessarily moved as well. The quantity $k'M'$ is called the virtual mass of the sphere, and depends upon the frequency in a very complicated way. The second term in the Eqn. (40) always opposes the movement of the sphere, and is thus a damping force out of phase with the acceleration. This is the force that would produce the decay of the oscillation of the sphere if left free.

Table 6. Values of $\bar{D}_s = \frac{-D_s}{\frac{2}{3}\pi\mu U a^2}$ in the case of a single sphere oscillating in a dusty fluid ($f = 0.2$).

t \ r	0 (sec)	2 (sec)	4 (sec)	6 (sec)	8 (sec)	10 (sec)
0.2	39.6480	61.9974	—4.4754	—64.5364	—32.1377	46.3039
0	37.0230	55.0893	—5.7695	—58.3625	—27.3410	42.8513

The functions k' and k'' are given in graphical form in Figs. 4 and 5 respectively against σ for the values $f = 0$, 0.2 and 0.4. Both k' and k'' tend to infinity as the frequency σ tends to zero; they tend to 0 and 0.5 respectively as σ tends to infinity. It is interesting to note that in the former case the dusty fluid behaviour agrees with

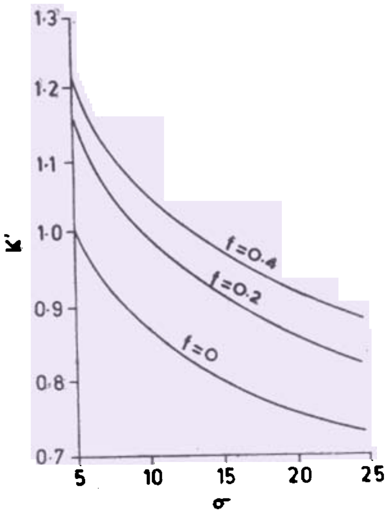

Figure 4. Variation of k' against frequency σ for different values of mass concentration of dust f (Eqn. 41).

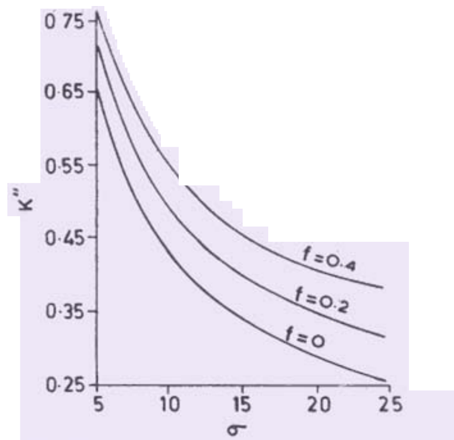

Figure 5. Variation of k'' against frequency σ for different values of mass concentration of dust f (Eqn. 42).

that of the clean fluid but in the latter case, that is, when σ tends to infinity in the case of the clean fluid, for which $f = 0$, the functions k' and k'' tend respectively to 0.5 and 0. It is seen from Figs. 4 and 5 that as the mass concentration of dust f increases the curves of both k' and k'' rise.

We conclude that the presence of dust in the fluid has the effect of increasing the virtual mass of the sphere and the magnitude of the damping force acting on the sphere.

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