A PHYSICAL INTERPRETATION OF THE WAVE DAMPING EQUATION OF DORRESTEIN

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The two methods of wave damping equation which provide a physical interpretation to Dorrestein equation for wave damping by surface active materials have been placed close to each other.

The effect of surface active agents on the damping of waves and ripples was known for a very long time and Pliny the Elder was able to calm the turbulent waters by pouring oil on the surface of the sea. Lamb[1] was able to treat the problem theoretically, but he assumed that the horizontal motion of the wave was annulled to zero by the presence of surface active agents *a piori*. Dorrestein[2] made the first direct approach to solving the problem theoretically by deriving a damping equation for the presence of surface active agents. Levich[3] also gives a direct approach using a mass balance equation for the film molecules. The purpose of this paper is to juxtapose the two methods of the wave damping equation which then provides a physical interpretation to the Dorrestein equation for wave damping by surface active materials.

NOMENCLATURE

T = shear stress

Pt = $-\dfrac{d\gamma}{dx}$ = grad γ

A = area available per molecule

N_s = concentration of film molecules

U_s = surface velocity

γ = surface tension

C_s^{-1} = surface compressional modulus of elasticity

j surf = surface flux of molecules (convectine)

j diff = diffusional flux of molecules = $- D_s$ grad N_s

J_n = flux of molecules across surface

METHOD

At an interface, which is contaminated, the boundary conditions are unaltered except for the continuity of the shear stresses. It is found that $T + Pt = T^1$, where T and T^1, are the tangential viscous stresses for the two phases forming the interface. In the damping of waves by films Dorrestein obtained the equation $\dfrac{1}{A}\dfrac{dA}{dt} = \dfrac{dU_s}{dt}$, that is the rate of change of area per unit area is equal to the spatial rate of change of the velocity parallel to the surface. By definition, $\dfrac{1}{A}\dfrac{dA}{dt} = \dfrac{1}{C_s^{-1}}\dfrac{d\gamma}{dt} = \dfrac{dU_s}{dx}$;

therefore

$$\frac{d\gamma}{dt} = C_s^{-1}\operatorname{div}(U_s).$$

This is the damping equation obtained by Dorrestein and incorporates the effect of the film through C_s^{-1}, the surface compressional modulus of the film.

The Levich approach uses the usual hydrodynamic equations for wave motion and the mass balance equation of the film molecules.

Let us consider the mass balance equation of the film molecules. In the most general form it is given as

$$\frac{\partial N_s}{\partial t} + \operatorname{div}(j\,\text{surf} + j\,\text{diff}) + J_n = 0 \ .$$

In this particular problem, the film is insoluble, so that $J_n = 0$. The distribution of film molecules is envisaged as sticking to the underlying liquid and moving with it.

With the above considerations in mind the mass balance equation becomes

$$\frac{\partial N_s}{\partial t} + \operatorname{div}(j\,\text{surf}) = 0. \text{ That is } \frac{\partial N_s}{\partial t} + \operatorname{div}(N_s\,U_s) = 0.$$

or $\dfrac{\partial N_s}{\partial t} + N_s \operatorname{div}(U_s) = 0$, analogous to the equation of continuity in fluid flow.

The surface tension γ is a function of the surface concentration of film molecules N_s, therefore

$$\gamma = \gamma\,(N_s),$$

so that

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dN_s} \cdot \frac{dN_s}{dt}$$

substituting for $\dfrac{dN_s}{dt}$, from above we get

$$\frac{d\gamma}{dt} = -\frac{d\gamma}{dN_s} \cdot N_s \operatorname{div}(U_s) \ .$$

By definition

$$C_s^{-1} = A\,\frac{d\gamma}{dA},$$

where A is the area available per molecule, therefore,

$$N_s = \frac{1}{A} \cdot$$

Hence,

$$C_s^{-1} = A\frac{d\gamma}{dA} = -N_s\frac{d\gamma}{dN_s}.$$

Therefore

$$\frac{d\gamma}{dt} = C_s^{-1}\operatorname{div}(U_s).$$

This is the same equation obtained by Dorrestein following the motion of the surface. This implies that the Dorrestein equation, in effect is a mass balance equation for the film molecules.

CONCLUSIONS

The Dorrestein damping equation is indeed a simple equation as opposed to the general equation of mass balance above. However, engineers are more familiar with mass balance equations and should find this paper very useful, especially in recent years, since they are concerned with the effect of surface films on liquid flows to a considerable extent.

REFERENCES

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3. LEVICH, V., "Physicochemical Hydrodynamics", (USSR Acad. Sci.), 1962.