Def Sci J, Vol 32, No 3, July 1982, pp 241-249

Computerized System for Evaluating Small Arm Projectile Trajectory Parameters Involving Space/Time Functions*

M. JAUHARI & S. M. CHATTERJEE

Central Forensic Science Laboratory, Calcutta-700 014

Received 30 October 1980; revised 23 July 1981

Abstract. Evaluation of certain parameters of the trajectory of a small arm projectile on the basis of Siacci approximation requires the values of Space (S) and Time (T) functions as tabulated in the Ingalls and Hodsock ballistic tables. The development is reported of a computerized system, whereby the necessity of referring to thesc tables has been completely obviated. Programme flow-chart has been presented and the logic behind the flow of programme has been made explicit. The programme has been executed successfully on the DCM Microsystem 1121.

1. Introduction

While reconstructing a shooting incident, a firearms expert may be required to undertake trajectory computations involving the determination of remaining velocity, range, etc. of small arm projectiles fired under specified conditions. In practice, such calculations are performed with the help of ballistic tables available in literature. A ballistic table essentially consists of a summary of the pertinent information gained from a collection of solutions of the normal equations of motion of a projectile. It is computed for a set of conditions which assume that (a) the earth is flat and non-rotating with one fixed value of acceleration due to gravity, (b) the drag and the gravity are the only forces acting on the projectile, (c) there is no wind, and (d) the standard distribution of density and temperature prevails.

In so far as the small arm projectiles are concerned, their flat trajectories based on Siacci approximation can be evaluated with the help of several ballistic tables available in literature.

Two important ballistic tables often used in small arm trajectory computation are the Ingalls[1] and Hodsock[2] tables. The Ingalls tables were computed by Col. Ingalls and are found to be most suitable for the ordinary hunting bullet, both rounded nose type and modified spitzer shape. The Hodsock tables give better results with sharp pointed spitzer bullets, especially of service variety. The two important functions tabulated in

*Presented at "The 9th International Meeting of the International Association of Forensic Sciences, held at Bergen, Norway."

these tables are the Space (S) and Time (T) functions which are needed for computing the various ballistic parameters of a trajectory. The values of these functions in the tables are tabulated against velocity at suitable intervals. An interpolation is necessary for evaluating the functions at an intermediate velocity. The object of the present paper is to obviate the necessity of referring to the ballistic tables by developing a computer programme whereby the values of the functions S and T for a given velocity and vice-versa are automatically computed as an intermediate step while evaluating the various parameters of the trajectory. The system that has resulted in this attempt has all the desirable features of speed, elegance and automation and it may, therefore, prove to be of positive help to a firearms expert in speeding up his work.

2. Basis of Computer Programme

The functions S and T are defined as below

$$\int_{u}^{U} \frac{du}{G(u)}, \tag{1}$$

$$\int_{u}^{U} \frac{du}{uG(u)}, \tag{2}$$

where $G(u) = \frac{R}{u}$; R being the retardation experienced by a projectile of ballistic coefficient unity moving with a pseudo-velocity u. For a flat trajectory, there is practically little difference in the pseudo-velocity and the actual velocity of the projectile and, therefore, it is perfectly in order to use the actual velocity of the projectile in place of u. This approximation is valid in forensic work where mostly short flat trajectories are required to be computed.

Ingalls Table: Col. Ingalls prepared his ballistic tables by relying on the Mayevski drag function, according to which the retardation (R) in ft/sec^2 in the different velocity zones is given by

$$R = Au^2,\ \log A = 5.6698914\text{–}10;\ 0 < V < 790 \text{ ft/sec}, \tag{3}$$

$$R = Au^3,\ \log A = 2.7734430\text{–}10;\ 790 \leqslant V < 970 \text{ ft/sec}, \tag{4}$$

$$R = Au^5,\ \log A = 6.8018712\text{–}20;\ 970 \leqslant V < 1230 \text{ ft/sec}, \tag{5}$$

$$R = Au^3,\ \log A = 2.9809023\text{–}10;\ 1230 \leqslant V < 1370 \text{ ft/sec}, \tag{6}$$

$$R = Au^2,\ \log A = 6.1192596\text{–}10;\ 1370 \leqslant V < 1800 \text{ ft/sec}, \tag{7}$$

$$R = Au^{1.7},\ \log A = 7.0961978\text{–}10;\ 1800 \leqslant V < 2600 \text{ ft/sec}, \tag{8}$$

$$R = Au^{1.55},\ \log A = 7.6090480\text{–}10;\ 2600 \leqslant V \text{ ft/sec}. \tag{9}$$

In computing the ballistic tables, the number U, viz., the upper limit of the integrals in Eqns. (1) and (2) is selected and fixed, larger than the greatest velocity at which the projectile is likely to be fired and for values of $u < U$, the above integrals are computed. Col. Ingalls took $U = 3600$ ft/sec. Therefore, if one takes $U = 3600$ ft/sec and compute the integrals (1) and (2), one gets the following expressions for S and T functions in the diflerent velocity zones :—

$$S = 158436.8384 - 21384.96776 \, l_n u \; ; \; 0 < u < 790 \text{ ft/sec},$$

$$S = \frac{16848335.42}{u} - 5571.37779 \; ; \; 790 \leqslant u < 970 \text{ ft/sec},$$

$$S = 6034.45995 + \frac{5.260264054}{u^3} \times 10^{12} \; ; \; 970 \leqslant u < 1230 \text{ ft/sec},$$

$$S = 365.6699735 + \frac{10449552.68}{u} \; ; \; 1230 \leqslant u < 1370 \text{ ft/sec},$$

$$S = 62875.28240 - 7598.712364 \, l_n u \; ; \; 1370 \leqslant u < 1800 \text{ ft/sec},$$

$$S = 31227.09901 - 2671.043404 \, u^{0.3} \; ; \; 1800 \leqslant u < 2600 \text{ ft/sec},$$

$$S = 21780.88949 - 546.6879309 \, u^{0.45} \; ; \; u \geqslant 2600 \text{ ft/sec},$$

$$T = \frac{21384.96776}{u} - 15.45952607 \; ; \; 0 < u < 790 \text{ ft/sec},$$

$$T = \frac{8424167.710}{u^2} - 1.888052223 \; ; \; 790 \leqslant u < 970 \text{ ft/sec},$$

$$T = \frac{3.945198040}{u^4} \times 10^{12} + 2.608879543 \; ; \; 970 \leqslant u < 1230 \text{ ft/sec},$$

$$T = \frac{5224776.342}{u^2} + 8.790424198 \times 10^{-1} \; ; \; 1230 \leqslant u < 1370 \text{ ft/sec},$$

$$T = \frac{7598.719270}{u} - 1.883741456 \; ; \; 1370 \leqslant u < 1800 \text{ ft/sec},$$

$$T = \frac{1144.732887}{u^{0.7}} - 3.688012618 \; ; \; 1800 \leqslant u < 2600 \text{ ft/sec},$$

$$T \quad \frac{447.2901253}{u^{0.55}} - 4.950202157 \; ; \; u \geqslant 2600 \text{ ft/sec}.$$

Hodsock Table: On the basis of the experiments carried out at Hodsock, Jones evolved a new relation between air-resistance and velocity for rifle bullets. According to Jones, the retardation (R) in ft/sec for a bullet of ballistic coefficient unity is given by the following formulae :

$$R = \frac{gu^2}{643680} \; ; \; u < 800 \text{ ft/sec},$$

$$R = \frac{gu}{804.6}; \; 800 \leqslant u < 1040 \text{ ft/sec},$$

$$R = \frac{g(u - 980)}{46.4186}; \; 1040 \leqslant u < 1120 \text{ ft/sec},$$

$$R = \frac{g(u - 950)^{0.7}}{12.07474}; \; u \geqslant 1120 \text{ ft/sec},$$

In the above formulae, g is taken as 32.19 ft/sec². Unlike the Ingalls tables, the Hodsock tables start with $u = 100$ ft/sec at which the values of the functions S and T are taken as zero. The values of these functions then increase with the increase of velocity. This would simply mean that the lower limit of integrals in Eqns. (1) and (2) is taken as 100 and then the integrals are computed for various velocity zones using the retardation formulae vide Eqns. (24) to (27). The functions S and T are thus determined by the following expressions :

$$S = 19996.27 \, l_n \frac{u}{100}; \; u < 800 \text{ ft/sec},$$

$$S = 21584.8 + 24.99534 \, u; \; 800 \leqslant u < 1040 \text{ ft/sec},$$

$$S = 46080.25 + 1.44202 \left[u + 980 \, l_n \frac{(u - 980)}{60} \right]; \quad 1040 \leqslant u < 1120 \text{ ft/sec},$$

$$S = 43118.82 + 0.3751084 \left[\frac{(u - 950)^{1.3}}{1.3} + \frac{950 \, (u - 950)^{0.3}}{0.3} \right]; \quad u \geqslant 1120 \text{ ft/sec},$$

$$T = 19996.2721 \left[0.01 - \frac{1}{u} \right]; \; u < 800 \text{ ft/sec},$$

$$T = 174.9674 + 24.9953 \, l_n \frac{u}{800}; \; 800 \leqslant u < 1040 \text{ ft/sec},$$

$$T = 181.5233 + 1.442 \, l_n \frac{(u - 980)}{60}; \; 1040 \leqslant u < 1120 \text{ ft/sec},$$

$$T = 176.9105 + 1.2503 \, (u - 950)^{.3}; \; u \geqslant 1120 \text{ ft/sec}, \tag{35}$$

The basic formulae involving the use of S and T functions of the Hodsock Table for evaluating the ballistic parameters of a trajectory are given below :

$$\frac{X}{C} = S(V_I) - S(V_R) \tag{36}$$

and

$$\frac{t}{C} = T(V_I) - T(V_R);$$

where

X = Range in feet,

C = Ballistic Coefficient,

t = Time of flight in seconds,

V_I = Initial velocity in ft/sec,

V_R = Remaining velocity in ft/sec,

$S(V)$ = Space function for velocity V,

$T(V)$ — Time function for velocity V.

While using the Ingalls Table, V_I and V_R need to be interchanged in Eqns. (36) and (37).

Thus the Eqns. (36) and (37) in conjunction with Eqns. (10) to (23) and Eqns. (28) to (35) can provide the basis of a computer programme which will eliminate the use of Ingalls or Hodsock ballistic Tables.

3. Programme Flow Chart

The Eqns. (36) and (37) involve consulting the ballistic tables for determining the values of S and T functions for a given velocity and vice-versa. The variables involved being (X, C, V_I, V_R) in Eqn. (36) and (t, C, V_I, V_R) in Eqn. (37). Knowing the values of any three, the fourth one can be calculated. A comprehensive programme flow chart for the various combinations of known/unknown variables involving the use of S function of the Hodsock ballistic Table is given in Fig. 1. While using the Ingalls Table, minor changes will have to be made in the flow chart because the Eqns. (36) and (37) will stand modified due to the interchange of V_I and V_R as mentioned above. Thus, instead of calculating $\{S(V_I) - S(V_R)\}$ in the $\{\alpha_1, \beta_{(1,2)}\}$ branch, one will calculate $\{S(V_R) - S(V_I)\}$, and the inequalities in the various 'decision boxes' will be reversed. The Eqns. (36) and (37) will be used after interchanging V_I and V_R. The individual flow charts as obtained by joining the appropriate connectors (α_1, β_1, γ_1, α_2, β_2, γ_2) represent the flow of programme in different determinations. For example, if one joins the connectors α_1 and also the connectors β_1, it results in a flow chart which can be used to calculate X provided V_I, V_R and C are known. If instead of the connectors β_1, the connectors β_2 are joined, one gets the flow chart for calculating C from a knowledge of V_I, V_R and X. Thus depending upon the known and unknown variables, the appropriate path for the flow of programme can be chosen. When T functions are involved, the flow chart will remain the same as in Fig. 1, with the difference that S is to be substituted by T and Eqn. (36) by Eqn. (37) in the flow chart. A reference to Fig. 1 will show that the programme flow charted therein requires the frequent calculation of S or T functions. Accordingly, subroutines for these two functions corresponding to the Ingalls and Hodsock ballistic Tables have been flow charted vide Figs. 2a to 2d.

The logic of the programme flow charted vide Figs. 1, 2c & 2d (Hodsock) can be easily understood by taking specific examples. Let us suppose that a projectile having a ballistic coefficient C is fired with an initial velocity V_I. One is now required to determine the range which it will attain when its initial velocity is reduced to V_R. As mentioned earlier, the programme path obtained by joining the connectros α_1 and the connectors β_1 will be chosen for this problem (Fig. 1). The initial velocity V_I will be entered first. The computer will print this figure and store it in the memory. It will

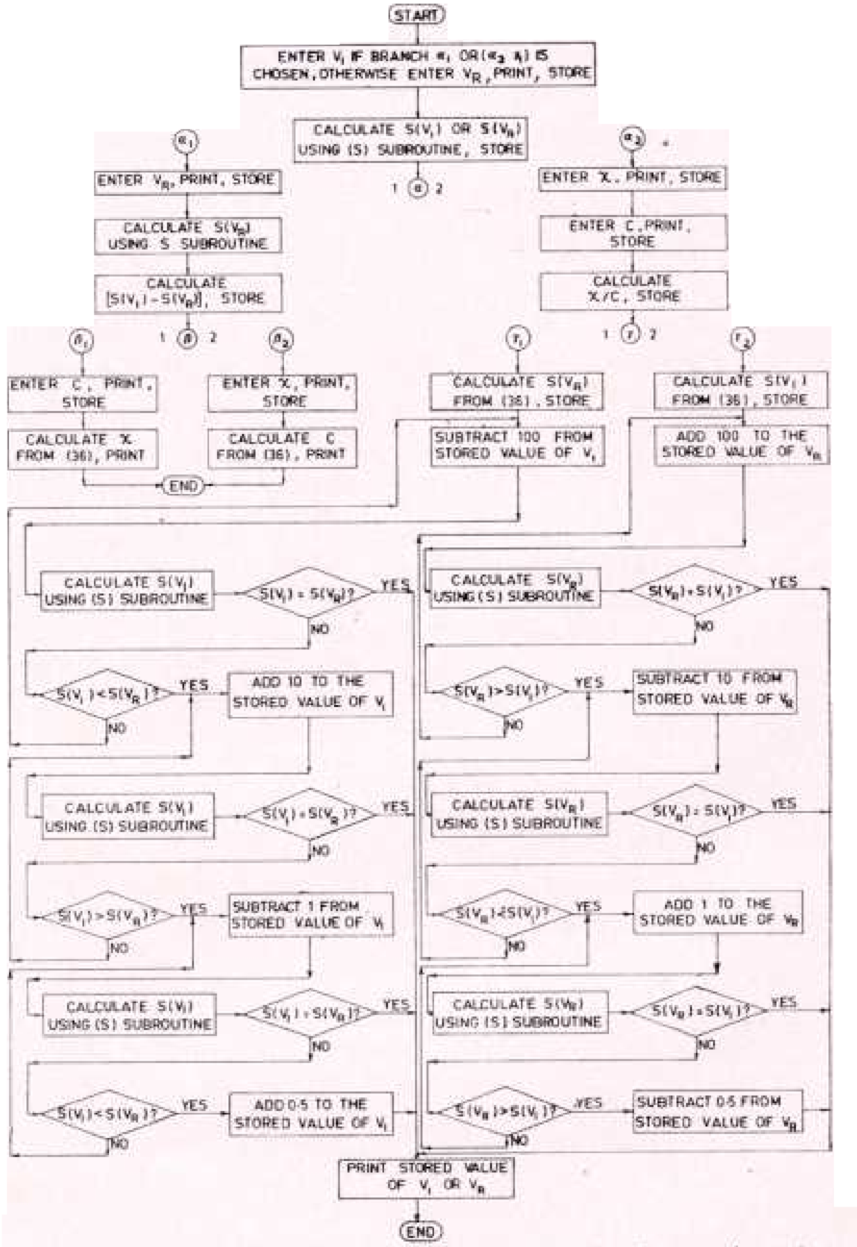
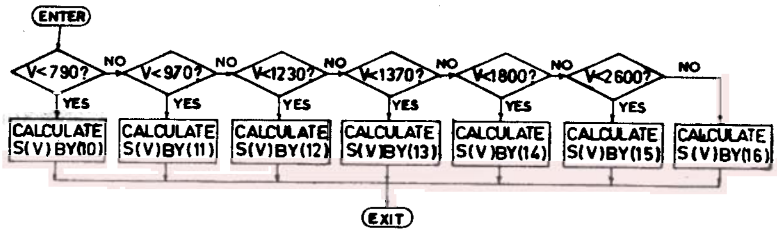
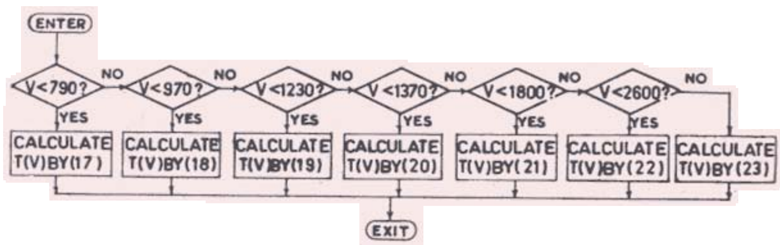


Figure 1. Programme flow chart involving the use of S functions incomputing trajectory parameters. When T functions are involved, S is to be replaced by T, and (36) by (37).

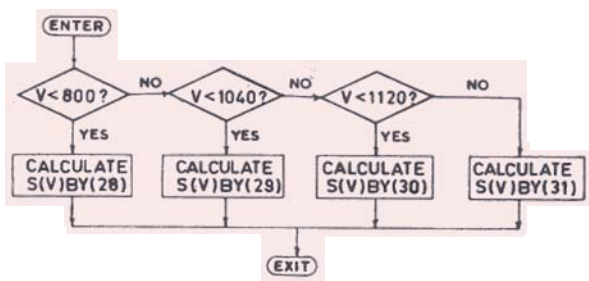
then call the S subroutine of the Hodsock ballistic Table. Having called the subroutine, the range in which V_1 falls will be determined followed by the calculation of $S(V_I)$ using the corresponding expression (Fig. 2c). The value of $S(V_I)$ will be stored in the memory. After this, the value of V_R is entered. The computer will follow exactly similar steps for calculating of $S(V_R)$ which will also be stored in the memory. Lastly, the value of C is entered. It is also printed and stored. With the values of $S(V_I)$,



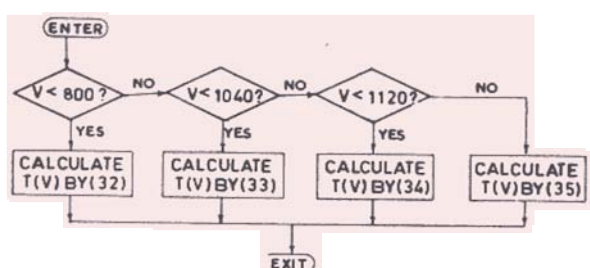
(a)



(b)



(c)



(d)

Figure 2(a) Subroutine for evaluating the *S* function of the Ingalls Table in conjuction with the programme flow.
(b) Subroutine for evaluating the *T* function of the Ingalls Table in conjuction with the programme flow.
(c) Subroutine for evaluating the *S* function of the Hodsock Table in conjunction with the programme flow.
(d) Subroutine for evaluating the *T* function of the Hodsock Table in conjuction with the progress flow.

$S(V_R)$ and C available in the storage, the computer calculates the value of X using Eqn. (36) and prints the result. At this stage, the programme terminates. Another example could be the calculation of the remaining velocity (V_R) from a knowledge of

V_I, X and C. For this purpose, the programme path obtained by joining the connectors α_2 and the connectors γ_1 is to be used. The operation of the computer upto the calculation of $S(V_I)$ and its storage in the memory remains the same as before. The values of X and C are now entered, printed and stored. This is followed by the calculation of X/C and then of $S(V_R)$ using Eqn. (36). This calculated value of $S(V_R)$ is stored in the memory. To determine the value of V_R corresponding to the stored value of $S(V_R)$, the stored value of V_I is reduced in steps of 100, the value of $S(V_I)$ is calculated at each step and compared with the stored value of $S(V_R)$. This cycle is repeated until

$$S(V_I) \leqslant S(V_R).$$

In case $S(V_I) = S(V_R)$, the stored value of V_I is printed as the required value of V_R and the programme terminates; otherwise the stored value of V_I is increased in steps of 10 until

$$S(V_I) \geqslant S(V_R).$$

Again for $S(V_I) = S(V_R)$, the stored value of V_I is printed as the desired value of V_R and the progrmme terminates, otherwise the stored value of V_I is reduced in steps of 1 until

$$S(V_I) \leqslant S(V_R).$$

Corresponding to $S(V_I) = S(V_R)$, the stored value of V_I as usual is printed as the value of V_R and the programme terminates. For $S(V_I) < S(V_R)$, the stored value of V_I is increased by 0.5 and is printed as the required value of V_R bringing the programme finally to an end. This looping procedure to arrive at the value of V from $S(V)$ or $T(V)$ correct within 1 ft/sec is considered to be simpler then the solving of the various expressions for V in terms of S or T (which could in certain cases be quite cumbersome) and then using these expressions for calculating V. It is obvious that one can enhance the accuracy of this determination by continuing this looping procedure to any extent.

4. Discussion

The solution of the normal equations of motion in the form (1) and (2) is based on the Siacci assumption of a flat trajectory. This requirement can be deemed to be met if the tangent to the trajectory does not turn through an angle more than about 15 degrees. In case the trajectory is markedly curved, the results of computation based on the ballistic tables of Ingalls or Hodsock may be in error. In forensic problems, one is mostly concerned with short trajectories which are flat and as such the use of the ballistic tables based on Siacci assumption is quite in order. Further, the accuracy required in forensic work need not be of the same order as that required in defence problems.

The computer programme developed above can be executed on any computer. However, at the present stage of development, the cost of buying and maintaining a computer is often astronomical in proportion especially in a small set up of a forensic science laboratory. The availability of micro-systems or programmable calculators has, however, solved this problem. The authors have executed the above programme on the DCM-Microsystem 1121 facility available in the Central Forensic Science Laboratory, Calcutta. The programme flow charted vide Fig. 1 and its modification (Ingalls) are stored on a set of magnetic cards. The various subroutines flow charted vide Figs. 2a to 2d are similarly stored separately. Depending upon the nature of the problem, the main programme (Fig. 1) or its modified version (Ingalls) along with the appropriate subroutine (Figs. 2a to 2d) can be loaded in the microsystem for execution. The selection of different routes is achieved by the 'FLAG' key. In case a different subroutine is to be used, the loaded subroutine need be superseded without altering the main programme. In this way, a powerful computerised system has been achieved for performing trajectory computations without the assistance of the ballistic tables. This can be deemed to be a step forward in achieving automation in firearms identification.

References

1. Hatcher, J. S., 'Hatcher's Notebook' (The Stackpole Co., Harrisburg, Pennsylvania), 1957, p. 583.
2. Burrard, G., 'Notes on Sporting Rifles' (Edward Arnold & Co., London), 1953, p. 163.