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On Availability of a Series-System with Imperfect Detectors

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Abstract. An n-unit series system with exponential distributions for life-times and repair-times has been considered. Each unit is equipped with a detector to detect failures. Detectors are subject to two failure modes : viz. (i) instantaneous failure i.e. it fails at the time of need when a unit fails; (ii) gradual failure i.e. it fails and gives false alarm for system failure. Steady-state availability of the system is obtained by studying the underlying system equations. Behaviour of steady-state unavailability has also been studied analytically.

1. Introduction

To have a control over failure and hence over operational time of electrical and telecommunication systems, the use of monitoring devices is quite prevalent. Existence of such devices is emphasised further because modern systems are quite sophisticated and all failures are not covered under automatic detection. In the context of digital computer systems the faults, which are automatically covered[1] are called 'coverage'. There may be faults which are not covered under 'coverage' and they require some mechanism for their detection.

In the literature of reliability there are not many articles which concentrate on the aforesaid aspects. However, the necessity of analysis of reliability systems incorporating monitoring devices/detectors is beyond description. Kumar[2] discussed the detection of failures in an n-unit system at an optimal cost. The detectors are subject to one failure mode viz., they detect the failed component wrongly. But, generally detectors are subject to two failure modes :

(i) *Failure Mode 1* (*Instantaneous Failure*). A detector fails at the time of use i.e. when a unit is failed. This failure is operationally similar to 'transfer switch failure' in Kumar[3].

(ii) *Failure Mode 2* (*Gradual Failure*). A detector fails gradually like units 'Connect Switch' in Kumar[3].

Recently Takami and others[4] discussed the problem of allocation of detectors in an n-unit series system assuming failure mode 1 only for the detectors.

In the present note we incorporate both failure modes of detectors in an n-unit series system and obtain steady-state unavailability.

2. Development of the Model

(i) There is an n-unit series system. Each unit has separate detector.

(ii) Each detector is subject to 2 failure modes defined above.

(iii) When a detector fails in failure mode 1, the system enters the state of identification. After identification, the failed unit goes to repair. When it fails in mode 2, it is immediately detected and it goes to repair.

(iv) When the system is down, failure rate of each unit/detector is zero.

(v) The identification-time, units/detectors failure time, and repair–time are exponential.

(vi) All random variables defined to model the system are s-independent.

(vii) System states and transitions between them are given in Fig. 1.

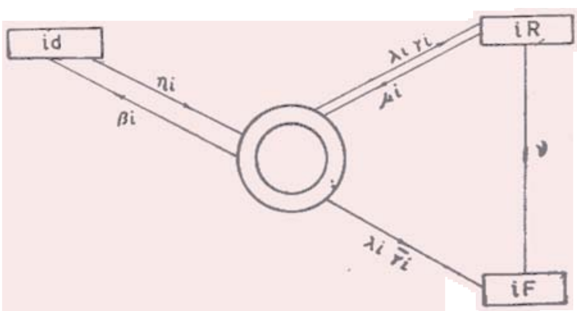

Figure 1. Transition diagram for the model.

3. Notation

n	number of units in the system
λ_i μ_i	failure, repair rate of the ith unit
β_i, η_i	failure, repair rate of the ith detector
r_i	reliability of the ith detector
ν	constant identification rate
$P_i(t)$	probability that the system is in state i at time t
$P_{iR}(t)$, $P_{id}(t)$	probability that the ith unit, detector is in repair at time t
$P_{iF}(t)$	probability that the system is under identification after failure of ith unit and ith detector in mode 1

Q_0	steady state unavailability of the system
—	denotes complementary function e.g. $\bar{Q}_0 = 1 - Q_0$
$'$	denotes differential e.g. $P'_i(t) = d/dt\, P_i(t)$
iF	system under identification due to failure of ith unit
iR	ith unit under repair
id	ith detector under repair
Σ	$\sum_{i=1}^{n}$

4. Steady-state Unavailability of the System

Following usual arguments the following equations governing system behaviour are straight forward

$$P'_0(t) = -\Sigma(\lambda_i + \beta_i)\, P_0(t) + \Sigma\, \mu_i P_{iR}(t) + \Sigma\, \eta_i P_{id}(t) \quad (1)$$

$$P'_{iR}(t) = -\mu_i P_{iR}(t) + \lambda_i r_i P_0(t) + \nu P_{iF}(t) \quad (2)$$

$$P'_{iF}(t) = -\nu P_{iF}(t) + \lambda_i \bar{r}_i P_0(t) \quad (3)$$

$$P'_{id}(t) = -\eta_i P_{id}(t) + \beta_i P_0(t) \quad (4)$$

for $i = 1, 2, \ldots, n$

For steady-state solution of Eqns (1) – (4), we must have

$$P'_0(t) = P'_{iR}(t) = P'_{iF}(t) = P'_{id}(t) = 0 \quad (5)$$

So, taking $P_0(t)$, $P_{iR}(t)$, $P_{iF}(t)$ and $P_{id}(t)$ as independent of t and solving them, we get

$$P_0 = 1/[1 + \Sigma\{\beta_i/\eta_i + \lambda_i(1/\mu_i + \bar{r}_i/\nu)\}] \quad (6)$$

So, steady state unavailability is given by

$$Q_0 = 1 - P_0 = A/B$$

$$A \equiv \Sigma\ [\beta_i/\eta_i + \lambda_i(1/\mu_i + \bar{r}_i/\nu)]$$

$$B \equiv 1 + A$$

If $\lambda_i \ll \mu_i,\ \lambda_i \ll \nu,\ \beta_i \ll \eta_i,$

then $Q_0 \simeq A$ (8)

Further, if $r_i = 1$ for all $i = 1, 2, \ldots, n$, then

$$Q_0 = \frac{\Sigma(\beta_i/\eta_i + \lambda_i/\mu_i)}{1 + \Sigma(\beta_i/\eta_i + \lambda_i/\mu_i)} \quad (9)$$

If $\beta_i = 0$, $r_i = r$ for all $i = 1, 2, \ldots, n$, then Eqn. (7) reduces to

$$Q_0 = \frac{\Sigma \lambda_i (1/\mu_i + \bar{r}/\nu)}{1 + \Sigma \lambda_i (1/\mu_i + \bar{r}/\nu)} \tag{10}$$

which agrees with Eqn. (6) in (4), if $\alpha_i = 1$, for $i = 1, 2, \ldots, n$.

The following theorem gives behaviour of Q_0. The proof is simple

Theorem

(*i*) $Q_0 \uparrow$ as $r_i \downarrow$ for any $i = 1, 2, \ldots, n$

(*ii*) $Q_0 \uparrow$ as $\beta_i/\eta_i \uparrow$ for any $i = 1, 2, \ldots, n$

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References

1. Arnold, T. F., *IEEE Trans. Computers*, **C-20** (1973), 251–254.
2. Kumar, A., *ZAMM*, **55** (1975), 679–680.
3. Kumar, A., *IEEE Trans. Reliability*, **R-26** (1977), 366–369.
4. Takami, I., *IEEE Trans. Reliability*, **R-27** (1978), 360–362.