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Unsteady Flow of a Viscous Fluid Through an Annulus

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Abstract. One boundary of an annulus is fixed, and other boundary is subjected to a series of pulses. The fluid in the annulus, therefore acquires a velocity, relative to the annulus, due to the transfer of momentum from the boundary by viscous stresses. The flow relative to the annulus is determined when a constant pressure gradient and a series of pulses act together. The velocity profiles for unsteady motion, are plotted for various times and for a fixed radii ratio.

1. Introduction

Most of the flow problems through an annulus are studied under time dependent pressure gradient. In particular, we have some engineering problems in which the boundaries are moving like the motion of a piston, cylindrical bearings whereas the problem of the engines of certain aircraft fail, when guns are fired[1].

In this study, one boundary of an annulus is fixed and the other boundary is subjected to a series of pulses. The fluid in the annulus, therefore acquires a velocity, relative to the annulus, due to the transfer of momentum from the boundary by viscous stresses. The axially symmetrical flow relative to the annulus is determined, when a constant pressure gradient and a series of pulses act together. Two cases are considered. In the first case, an impulsive motion is given, and in the second case, the boundary is under the action of a series of continuous pulses. Velocity profiles are plotted in both the cases for various times and for a fixed radii ratio.

2. Basic Equations

Let (r, θ, z) be the cylindrical polar coordinates, a and b be respectively, the internal and external radii of the annulus, and $\sigma = b/a$ be the radii ratio. Let the axis of the annulus be along z-axis, v be the axial velocity and v_m be the mean velocity of the steady flow. We introduce the following dimensionless parameters

$$\xi = \frac{r}{a}, \; z^* = \frac{z}{a}, \; V^* = \frac{v}{v_m}, \; p^* = \frac{pa}{\mu v_m}, \; T = \frac{t\nu}{a^2} \tag{1}$$

Let $$P = \frac{-\partial p^*}{\partial z^*} \tag{2}$$

be the dimensionless constant pressure gradient.
Then the governing equations continuity and momentum in the dimensionless form are

$$\frac{\partial V^*}{\partial z^*} = 0 \tag{3}$$

$$\frac{\partial V^*}{\partial T} \quad P + \left(\frac{\partial^2 V^*}{\partial \xi^2} + \frac{}{\xi}\,\frac{\partial V^*}{\partial \xi}\right) \tag{4}$$

The flow velocity due to the constant pressure gradient and the action of pulses can be taken as

$$V^*(\xi, T) = V_1^*(\xi) \qquad V_2^*(\xi, T) \tag{5}$$

The differential equation for steady motion is

$$\frac{\partial^2 V_1^*}{\partial \xi^2} + \frac{1}{\xi}\,\frac{\partial V_1^*}{\partial \xi} = \qquad P \tag{6}$$

with the boundary conditions

$$V_1^* = 0 \text{ for } \xi =$$

$$V_1^* = 0 \text{ for } \xi = \sigma \tag{7}$$

The differential equation for unsteady motion is

$$\frac{\partial^2 V_2^*}{\partial \xi^2} + \frac{1}{\xi}\,\frac{\partial V_2^*}{\partial \xi} \qquad \frac{\partial V_2^*}{\partial T}$$

with the boundary conditions for,

Case I

$$V_2^*(1, T) \qquad 0 \text{ for all } T$$

$$V_2^*(\sigma, T) = 0 \text{ for all } T < 0 \tag{9}$$

$$\sum_{n=0}^{N} A_n \delta(T - nD) \text{ for all } T \geqslant nD$$

Case II

$$V_2^*(1, T) = 0 \text{ for all } T$$

$$V_2^*(\sigma, T) = 0 \text{ for all } T < 0$$

$$\sum_{n=0}^{N} A_n \times H(T - nD) \text{ for all } T > 0$$

where A_n is the strength of the nth pulse, $\delta(T)$ is the Dirac delta function and $H(T)$ is the Heaviside unit step function.

3. Solution

Solution of the steady motion from (6) or (7) is

$$V_1^*(\xi) = \frac{P}{4}\left(1 - \frac{2}{\xi} + \frac{(\sigma^2 - 1)}{\log \sigma} \log \xi\right) \tag{11}$$

Solution of the unsteady motion is obtained with the help of Laplace transform technique.

Case I

From (8) and (9), we have

$$V_2^*(\xi, T) = \sum_{m=1}^{\infty} \sum_{n=0}^{N} \left[(A_n \pi (\alpha_m)^2 e^{-(\alpha_m)^2 (T - nD)}) \left(\frac{J_0(\alpha_m) J_0(\alpha_m \sigma)}{J_0^2(\alpha_m) - J_0^2(\alpha_m \sigma)} \right) \left\{ (J_0(\alpha_m \xi) G_0(\alpha_m) - J_0(\alpha_m) G_0(\alpha_m \xi) \right\} H(T - nD) \right] \tag{12}$$

Case II

From (8) and (10), we have

$$V_2^*(\xi, T) = \sum_{n=0}^{N} A_n \left[\left(\frac{\log \xi}{\log \sigma} \right) \quad \sum_{m=1}^{\infty} \left\{ (\pi e^{-(\alpha_m)^2 (T - nD)}) \times \left(\frac{J_0(\alpha_m) J_0(\alpha_m \sigma)}{J_0^2(\alpha_m) - J_0^2(\alpha_m \sigma)} \right) (J_0(\alpha_m \xi) G_0(\alpha_m) - J_0(\alpha_m) G_0(\alpha_m \xi) \right\} \right] \times H(T - nD)$$

where α_m is the positive root of the transendental equation

$$J_0(\alpha_m \sigma) G_0(\alpha_m) - G_0(\alpha_m \sigma) J_0(\alpha_m) = 0$$

4. Flux Through The Channel

Quantity of the fluid flowing per unit time relative to the channel is

$$Q^*(T) = 2\pi \int_1^{\sigma} \xi V_2^*(\xi, T)\, d\xi$$

Case I

From (12) and (15), we have

$$Q^*(T) = 4\pi \sum_{n=0}^{N} \sum_{m=1}^{\infty} \left[(A_n e^{-(\alpha_m)^2 (T-nD)}) \left(\frac{J_0(\alpha_m)}{J_0(\alpha_m) + J_0(\sigma_m \sigma)} \right) \times H(T - nD) \right]$$

Case II

From (13) and (15), we have

$$Q^* (T) = \sum_{n=0}^{N} \pi A_n \left[\left(\frac{2\sigma^2 \log \sigma - \sigma^2 + 1}{2 \log \sigma} \right) - 4 \sum_{n=1}^{\infty} \left\{ (e^{-(\alpha_m)^2 (T-nD)}) \times \frac{1}{(\alpha_m)^2} \left(\frac{J_0(\alpha_m)}{J_0(\alpha_m) + J_0(\alpha_m \sigma)} \right) \right\} \right] H(T - nD)$$

The steady flow causes a discharge given by

$$q^*(T) = \pi V_m^* (\sigma^2 - 1)$$

where

$$V_m^* = \frac{P}{8} \left[(1 + \sigma^2) - \frac{(\sigma^2 - 1)}{\log \sigma} \right]$$

5. Numerical Discussion

Velocity profiles for unsteady motion for $\sigma = 10$, are plotted against various values of time, viz (2, 4, 6, 8), when the outer boundary is under the action of a single pulse (Fig. 1 & Fig. 2).

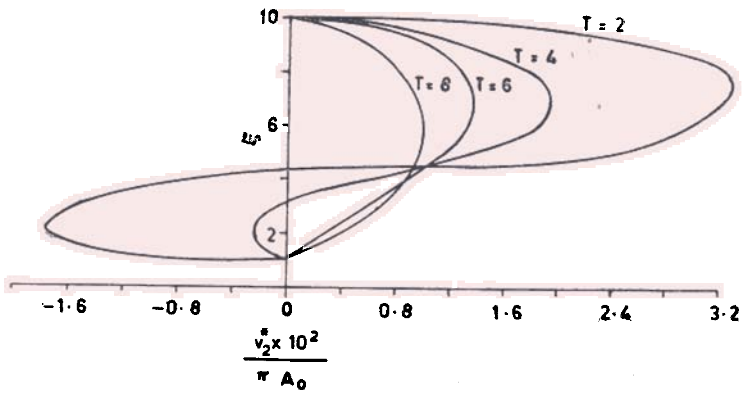

Figure 1. Velocity distribution for unsteady motion in case I for $t = (2, 4, 6, 8)$; $\sigma = 10$.

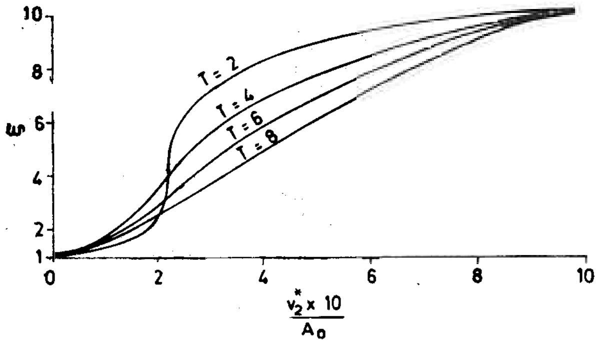

Figure 2. Velocity distribution for unsteady motion in case II for $t = (2, 4, 6, 8)$; $\sigma = 10$.

It can be seen that in both the cases, the disturbance due to the outer boundary reaches the inner boundary completely approximately after $T = 5$. It is therefore obvious that if the pulse is travelling in a direction opposing the flow, it can check the forward motion completely if the strength and the frequency of the pulse is large.

Hence it is inferred that, adjusting σ and T, and on application of a series of pulses. the phenomenon of fluid starvation can occur.

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Reference

1. Lance, G. N., Quarterly of Applied Mathematics, 14 (1956).