UNSTEADY TEMPERATURE DISTRIBUTION FOR LAMINAR FLOW IN A POROUS STRAIGHT CHANNEL

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(*Received* 27 *January* 1971)

Temperature distribution in a viscous incompressible fluid flowing between two parallel porous flat plates has been investigated when the lower plate is injecting the fluid while the upper one is sucking it with the same rate. Viscous dissipation has been neglected and the rate of heat generation per unit volume per unit time has been taken to be a function of time. Laplace transform technique has been used to obtain an expression for the temperature distribution. Taking the rate of heat generation as an oscillating function of time, numerical work has been done to demonstrate the effects of the variation of the cross-flow velocity on the temperature distribution.

Berman[1] has obtained an exact solution of the problem of steady flow of a viscous incompressible fluid through a porous annulus, when the rate of injection at one boundary is equal to the rate of suction at the other. Satya Prakash[2] has discussed the unsteady flow of a viscous incompressible fluid between two porous flat plates, when the rate of injection at one plate is equal to the rate of suction at the other. Krishna Lal[3] obtained the temperature distribution in a channel bounded by two coaxial circular pipes with time dependent boundary temperatures. He, however, made a physically unrealistic assumption that the temperature increases unboundly, which was corrected by Bhatnagar & Tikekar[4].

In this paper we obtain the temperature distribution in a viscous incompressible fluid between two porous flat plates, maintained at constant temperatures, with equal rates of injection and suction at the lower and upper plate respectively.

FORMULATION OF THE PROBLEM

For the two dimensional incompressible fluid flow, the energy equation neglecting the viscous dissipation is

$$\rho C_p \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \frac{\partial Q}{\partial t} + K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \tag{1}$$

where the lower plate is taken to be the axis of x and y is measured at right angles to it; u and v are the velocity components in the directions of x and y; ρ, the density; C_p, the specific heat at constant pressure; K, the coefficient of thermal conductivity and $\partial Q/\partial t$, the rate of heat generation per unit volume per unit time in the fluid.

Because of the condition that the suction rate at one plate be equal to the injection rate at the other and the assumption of constant wall temperatures, the fluid temperature T does not depend on x; hence the (1) is reduced to

$$\rho C_p \left[\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right] = \frac{\partial Q}{\partial t} + K \frac{\partial^2 T}{\partial y^2} . \tag{2}$$

Satya Prakash[2] has shown that the velocity component in the direction of x does not depend on x and the cross-flow velocity is given as

$$v = v_0 , \tag{3}$$

where v_0 is the constant velocity of injection and suction.

Equation (2) therefore becomes

$$\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial y} = \frac{1}{\rho C_p} \frac{\partial Q}{\partial t} + K' \frac{\partial^2 T}{\partial y^2} , \tag{4}$$

where
$$K' = \frac{K}{\rho\, C_p} .$$

The initial and boundary conditions are

$$\left.\begin{array}{ll} t = 0 \;:\; T = T_1 & \text{for} \quad 0 \leqslant y \leqslant h \\ t > 0 \;:\; T = T_1 & \text{for} \quad y = 0 \\ \qquad\quad\; T = T_2 & \text{for} \quad y = h \end{array}\right\} \tag{5}$$

where h is the distance between the plates.

METHOD OF SOLUTION

Introducing the following dimensionless quantities,

$$\left.\begin{array}{l} \eta = \dfrac{y}{h} \;,\; t' = \dfrac{t\nu}{h^2} \;,\; \theta = \dfrac{T - T_1}{T_2 - T_1}, \\ f(t') = \dfrac{\nu Q}{K\,(T_2 - T_1)} \;, \end{array}\right\} \tag{6}$$

and

in (4), we have

$$\frac{\partial \theta}{\partial t'} + R\,\frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}\left(\frac{\partial f}{\partial t'} + \frac{\partial^2 \theta}{\partial \eta^2}\right), \tag{7}$$

where
$$R = \frac{h v_0}{\nu} \quad \text{and} \quad Pr = \frac{\mu C_p}{K}$$

are the Reynolds number and Prandtl number respectively.

The initial and boundary conditions in terms of the non-dimensional parameters are

$$\left.\begin{array}{ll} t' = 0 \;:\; \theta = 0 & \text{for} \quad 0 \leqslant \eta \leqslant 1 \\ t' > 0 \;:\; \theta = 0 & \text{for} \quad \eta = 0 \\ \qquad\quad\;\; \theta = 1 & \text{for} \quad \eta = 1 \end{array}\right\} \tag{8}$$

Let us define
$$\bar{\theta}(\eta, s) = \int_0^\infty \theta(\eta, t')\, e^{-st'}\, dt' ,$$

and
$$\bar{f}(s) = \int_0^\infty f(t')\, e^{-st'}\, dt' ,$$

where $\bar{\theta}(\eta, s)$ and $\bar{f}(s)$ are Laplace transforms of $\theta(\eta, t')$ and $f(t')$ respectively.

Taking Laplace transform of (7), we have

$$\frac{d^2\bar{\theta}}{d\eta^2} - Pe\,\frac{d\bar{\theta}}{d\eta} - s\,Pr\left[\bar{\theta} - \left(\frac{s\bar{f} - f_0}{s\,Pr}\right)\right] = 0 , \tag{9}$$

where $Pe = RPr$ is the Peclet number and f_0 is the value of $f(t')$ at $t' = 0$, and the boundary conditions (8) are reduced to

$$\left.\begin{array}{ll} \bar{\theta} = 0 & \text{for} \quad \eta = 0 \\ \bar{\theta} = \dfrac{1}{s} & \text{for} \quad \eta = 1. \end{array}\right\} \tag{10}$$

The solution of (9) in view of (10) is

$$\theta(\eta, s) = \frac{e^{(Pe/2)\eta}}{\sinh \frac{\sqrt{Pe^2 + 4\,s\,Pr}}{2}} \begin{bmatrix} \left(\frac{1}{s} - M\right) e^{-Pe/2} \sinh \frac{\sqrt{Pe^2 + 4\,s\,Pr}}{2}\,\eta - \\ - M \sinh \frac{\sqrt{Pe^2 + 4\,s\,Pr}}{2}\,(1-\eta) \end{bmatrix} + M\,, \quad (11)$$

where

$$M = \frac{s\bar{f} - f_0}{s\,Pr}\,.$$

Now

$$L^{-1}(M) = \frac{1}{Pr}\left[f(t') - f_0\right].$$

Taking inverse Laplace transform of (11), we get

$$\theta(\eta, t') = \frac{1}{Pr}\left[f(t') - f_0\right] + e^{-(Pe/2)(1-\eta)} \int_0^{t'} \left[1 + \frac{1}{Pr}\left\{f_0 - f(u)\right\}\right] F(t'-u)\,du +$$

$$+ e^{(Pe/2)\eta} \int_0^{t'} \frac{1}{Pr}\left\{f_0 - f(u)\right\} G(t'-u)\,du, \quad (12)$$

where

$$F(t') = L^{-1}\left[F(s) \equiv \frac{\sinh \frac{\sqrt{Pe^2 + 4\,s\,Pr}}{2}\,\eta}{\sinh \frac{\sqrt{Pe^2 + 4\,s\,Pr}}{2}}\right],$$

and

$$G(t') = L^{-1}\left[G(s) \equiv \frac{\sinh \frac{\sqrt{Pe^2 + 4\,s\,Pr}}{2}\,(1-\eta)}{\sinh \frac{\sqrt{Pe^2 + 4\,s\,Pr}}{2}}\right].$$

$F(s)$ and $G(s)$ admit simple poles at

$$s = -\frac{Pe^2 + 4\,\pi^2\,n^2}{4\,Pr} \quad (n = 0, 1, 2, \ldots\ldots)$$

Evaluating residues at these simple poles, (12) becomes

$$\theta(\eta, t') = \frac{1}{Pr}\left[f(t') - f_0\right] - e^{-(Pe/2)(1-\eta)} \sum_{n=0}^{\infty} (-1)^n \frac{2\,\pi\,n \sin(n\,\pi\,\eta)}{Pr} \times$$

$$\times \int_0^{t'} \left[1 + \frac{1}{Pr}\left\{f_0 - f(u)\right\}\right] \exp\left\{-\frac{Pe^2 + 4\,\pi^2\,n^2}{4\,Pr}(t'-u)\right\} du +$$

$$+ e^{(Pe/2)\eta} \sum_{n=0}^{\infty} \frac{2\,\pi\,n \sin(n\,\pi\,\eta)}{Pr} \int_0^{t'} \left[\frac{1}{Pr}\left\{f_0 - f(u)\right\}\right] \times$$

$$\times \exp\left\{-\frac{Pe^2 + 4\,\pi^2\,n^2}{4\,Pr}(t'-u)\right\} du, \quad (13)$$

which gives the temperature distribution of the fluid in the channel.

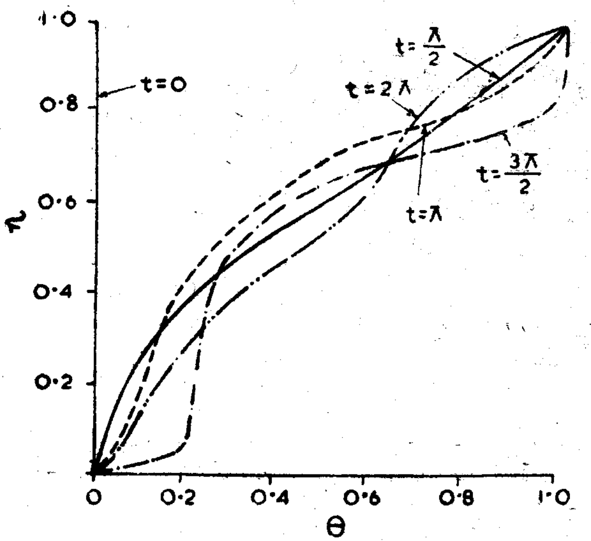

Fig. 1—Temperature distribution for $Pr = 1$, $R = 1$ and $t' = 0, \pi/2, \pi, 3\pi/2$ and 2π.

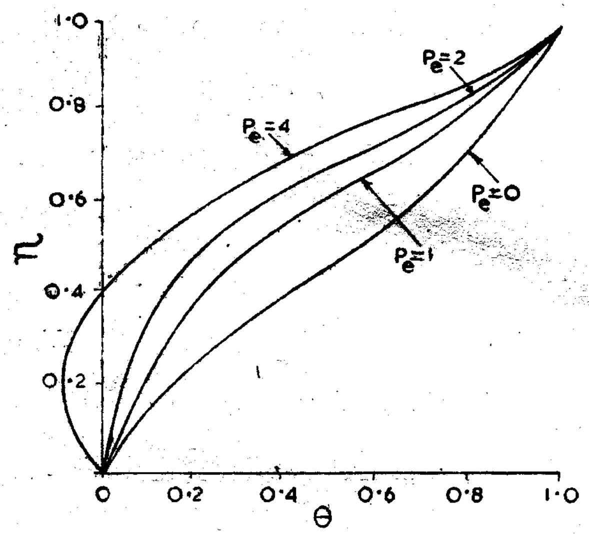

Fig. 2—Temperature distribution for $Pr=1$ and $t'=\pi/2$ for $R = 0, 1, 2$ and 4.

NUMERICAL DISCUSSION

Taking $f(t') \equiv \sin t'$, (13) is reduced to

$$\theta(\eta, t') = \frac{1}{Pr} \sin t' - e^{-(Pe/2)(1-\eta)} \sum_{n=0}^{\infty} (-1)^n \frac{2\pi n \sin(n \pi \eta)}{Pr} \times$$

$$\times \left[\frac{1}{A}\left(1 - e^{-At'}\right) - \frac{1}{A^2+1}\left(A \sin t' - \cos t' + e^{-At'}\right) \right] -$$

$$- e^{(Pe/2)\eta} \sum_{n=0}^{\infty} \frac{2\pi n \sin(n\pi\eta)}{Pr} \left[\frac{1}{A^2+1}\left(A \sin t' - \cos t' + e^{-At'}\right) \right], \quad (14)$$

where

$$A = \frac{Pe^2 + 4\pi^2 n^2}{4 Pr}.$$

For numerical work, we take $Pr = 1$ and $R = 1$ and obtain different values of θ for values of η varying from 0 to 1 for $t' = 0, \pi/2, \pi, 3\pi/2$ and 2π; which have been shown in Fig. 1. We note that the temperature in the neighbourhood of the plates fluctuates according to heat addition, as the time increases. Fig. 2 shows the effects of the variation of the cross-flow velocity ($R = 0, 1, 2$ and 4) on the temperature distribution when $t' = \pi/2$. It is found that due to porosity of the plates the temperature decreases as the cross-flow velocity increases.

ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Dr. P. D. Verma for his kind guidance during this investigation. Thanks are also due to Prof. P. L. Bhatnagar for his stimulating encouragement.

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