SOME FINITE INTEGRALS INVOLVING GENERALIZED HYPERGEOMETRIC POLYNOMIALS

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Some finite integrals involving generalized hypergeometric polynomials have been evaluated by defining the polynomial in the form

$$F_n(x) = x^{(m-1)n} {}_{p+m}F_q \left[\begin{matrix} \triangle(m, -n), a_1, a_2, \ldots, a_p \\ b_1, b_2, \ldots, b_q \end{matrix} ; kx^c \right]$$

where m and n are positive integers. Some particular cases have also been obtained with proper choice of parameters.

Fred[1] had introduced a generalized result in the hypergeometric polynomial as

$$f_n \left[k ; \begin{matrix} \alpha_1, \alpha_2, \ldots, \alpha_p \\ \beta_1, \beta_2, \ldots, \beta_q \end{matrix} ; x\,v ; x \right]$$

$$= {}_{k+p}F_q \left[-\frac{n}{k}, \frac{-n+1}{k}, \ldots, \frac{-n+k-1}{k}, \begin{matrix} a_1, a_2, \ldots, a_p \\ \beta_1, \beta_2, \ldots, \beta_q \end{matrix} ; x \right]$$

where $k = 1, 2, 3 ; \ldots\ldots\ldots\ldots\ldots\ldots$

Recently Zeitlin[2] and other workers have also defined the hypergeometric polynomials and studied their various properties.

The author[3] has introduced the polynomial in a more generalized form

$$F_n(x) = x^{(\delta - 1)n} {}_{p+\delta}F_q \left[\begin{matrix} \triangle(\delta, -n), a_1, a_2, \ldots, a_p \\ b_1, b_2, \ldots, b_q \end{matrix} ; \lambda x^c \right] \qquad (1)$$

where δ and n are positive integers.

To economise space, notation $\triangle(m, n)$ stands for m-parameters;

$$\frac{n}{m}, \frac{n+1}{m}, \ldots, \frac{n+m-1}{m} ; \quad a_p(b_q) \text{ for } p(q) \text{ parameters } a_1, \ldots, a_p$$

$(b_1, \ldots, b_q)$; $(a_p)_r$ denotes $\prod_{j=1}^{p} (a_j)_r$ and similarly for $(b_q)_r$.

The symbol $\triangle(m, a \pm b)$ stands for $\triangle(m, a+b)$ and $\triangle(m, a-b)$ with similar interpretation for $\Gamma(a \pm b)$ as $\Gamma(a+b)$ and $\Gamma(a-b)$.

This polynomial has been investigated in the course of an attempt to unify and to extend the study of almost the wellknown sets of classical and other hypergeometric polynomials. The polynomial (1) which by proper choice of parameters and for different values of argument $\lambda\, x^c$ includes not only the polynomials of Sister Celine, Rice, Jacobi, Laguerre, Bessel but also yields the polynomials such as Bedient, Hermite, Lommel and Legendre.

The polynomial (1) defined here is terminating and unrestricted. The terminating nature of the polynomial is governed by the numerator parameters $\triangle\,(\delta, -n)$. The parameters a_p and b_q are all independent of x but these can be functions of n so that the polynomial always remains well defined and as such the polynomial has attracted considerable attention in the field of pure and applied mathematics.

The aim of the paper is to evaluate certain results of finite integrals involving generalized hypergeometric polynomials and associated Legendre function or Tchebicheff polynomial by using the known formulae. Results obtained here yield many particular cases on specializing the parameters, some of which are known and others are believed to be new and interesting.

We have employed the following results in the present investigation.

(a) *Integrals*[4]

$$\int_0^1 x^{s-1}\,(1-x^2)^{\frac{1}{2}m}\; P_\gamma^{\,m}\,(x)\,dx$$

$$= \frac{(-1)^m\,\pi^{\frac{1}{2}}\,\Gamma\,(s)\,\Gamma\,(1+m+\gamma)\,\Gamma\,(\tfrac{1}{2}+\tfrac{1}{2}s+\tfrac{1}{2}m-\tfrac{1}{2}\gamma)}{2^{m+s}\,\Gamma\,(1-m+\gamma)\;\Gamma\,(1+\tfrac{1}{2}s+\tfrac{1}{2}m+\tfrac{1}{2}\gamma)}\,,\; Re\,(s) > 0\,, \qquad (2)$$

$$\int_0^2 x^{s-1}\,(4-x^2)^{-\frac{1}{2}}\;T_n\,(\tfrac{1}{2}x)\,dx$$

$$= \frac{\pi\,\Gamma\,(s)}{2\,\Gamma\,(\tfrac{1}{2}+\tfrac{1}{2}s\pm\tfrac{1}{2}n)}\;,\; Re\,(s) > 0\,, \qquad (3)$$

where $P_\gamma{}^m(x)$ and $T_n\,(x)$ are the associated Legendre function and Tchebicheff polynomial.

(b) *Relations*

$$(a)_n = \frac{\Gamma\,(a+n)}{\Gamma\,(a)}\,,\; \frac{\Gamma\,(1-a-n)}{\Gamma\,(1-a)} = \frac{(-1)^n}{(a)_n}\,,\; (a)_{nk} = k^{nk}\prod_{i=0}^{k-1}\left(\frac{a+i}{k}\right)_n . \qquad (4)$$

INTEGRALS

The integrals to be established are :

$$\int_0^1 x^{s-1} (1-x^2)^{\frac{1}{2}m} P_\gamma^m(x) \left\{ x^{(\delta-1)n} {}_{p+\delta}F_q \left[\begin{matrix} \triangle(\delta,-n), a_p \\ b_q \end{matrix} ; \lambda x^{2c} \right] \right\} dx$$

$$= \frac{(-1)^m \pi^{\frac{1}{2}} \Gamma(1+m+\gamma) \Gamma[s+(\delta-1)n] \Gamma[\frac{1}{2}\{1+s+(\delta-1)n+m-\gamma\}]}{2^{m+s+(\delta-1)n} \Gamma(1-m+\gamma) \Gamma[1+\frac{1}{2}\{s+(\delta-1)n+m+\gamma\}]}$$

$${}_{p+\delta+3c}F_{q+c} \left[\begin{matrix} \triangle(\delta,-n), ap, \triangle(2c, s+(\delta-1)n), \triangle(c, \frac{1}{2}\{1+s+(\delta-1)n+m-\gamma\}) \\ bq, \triangle(c, 1+\frac{1}{2}\{s+(\delta-1)n+m+\gamma\}) \end{matrix} ; \lambda c^{2c} \right] \quad (5)$$

where $Re[s+(\delta-1)n]>0$, δ, n and c are positive integers.

$$\int_0^1 x^{s-1} (1-x^2)^{\frac{1}{2}m} P_\gamma^m(x) \left\{ x^{(\delta-1)n} {}_{p+\delta}F_q \left[\begin{matrix} \triangle(\delta,-n), a_p \\ b_q \end{matrix} ; \lambda x^{-2c} \right] \right\} dx$$

$$= \frac{(-1)^m \pi^{\frac{1}{2}} \Gamma(1+m+\gamma) \Gamma[s+(\delta-1)n] \Gamma[\frac{1}{2}\{1+s+(\delta-1)n+m-\gamma\}]}{2^{m+s+(\delta-1)n} \Gamma(1-m+\gamma) \Gamma[1+\frac{1}{2}\{s+(\delta-1)n+m+\gamma\}]}$$

$${}_{p+\delta+c}F_{q+3c} \left[\begin{matrix} \triangle(\delta,-n), a_p, \triangle(c, -\frac{1}{2}\ s+(\delta-1)n+m+\gamma\ ) \\ b_q, \triangle(2c, 1-s-(\delta-1)n), \triangle(c, \frac{1}{2}\{1-s-(\delta-1)n-m+\gamma\}) \end{matrix} ; \frac{\lambda}{c^{2c}} \right] \quad (6)$$

valid for δ, n and c are positive integers, $Re[s+(\delta-1)n]>0$.

$$\int_0^2 x^{s-1} (4-x^2)^{-\frac{1}{2}} T_\gamma(\tfrac{1}{2}x) \left\{ x^{(\delta-1)n} {}_{p+\delta}F_q \left[\begin{matrix} \triangle(\delta,-n), a_p \\ b_q \end{matrix} ; \lambda x^{2c} \right] \right\} dx$$

$$= \frac{\pi \Gamma[s+(\delta-1)n]}{2\Gamma[\frac{1}{2}\{1+s+(\delta-1)n\pm\gamma\}]} \times$$

$$\times {}_{p+\delta+2c}F_{q+2c} \left[\begin{matrix} \triangle(\delta,-n), a_p, \triangle(2c, s+(\delta-1)n) \\ b_q, \triangle(c, \frac{1}{2}\{1+s+(\delta-1)n\pm\gamma\}) \end{matrix} ; \gamma 2^{2c} \right] \quad (7)$$

where $Re[s+(\delta-1)n]>o$, δ, n and c are positive integers.

$$\int_0^2 x^{s-1} (4-x^2)^{-\frac{1}{2}} T_\gamma(\tfrac{1}{2}x) \left\{ x^{(\delta-1)n} {}_{p+\delta}F_q \left[\begin{matrix} \triangle(\delta,-n), a_p; \\ b_q \end{matrix} \lambda x^{-2c} \right] \right\} dx$$

$$= \frac{\pi \Gamma[s+(\delta-1)n]}{2\Gamma[\frac{1}{2}\{1+s+(\delta-1)n\pm\gamma\}]} \times$$

$$\times {}_{p+\delta+2c}F_{q+2c} \left[\begin{matrix} \triangle(\delta,-n), a_p, \triangle(c, \frac{1}{2}\{1-s-(\delta-1)n\pm\gamma\}) \\ b_q, \triangle(2c, 1-s-(\delta-1)n) \end{matrix} ; \frac{\lambda}{2^{2c}} \right] \quad (8)$$

where δ, n and c are positive integers and $Re[s+(\delta-1)n]>0$.

Proof

To prove the integral (5), we substitute the series for the polynomial in the integrand and change the order of integration and summation which is easily seen to be justified due to the absolute convergence of the integral and summation involved in the process, we obtain

$$\sum_{r=0}^{\infty} \frac{\prod_{i=0}^{\delta-1}\left(\frac{-n+i}{\delta}\right)_r (a_p)_r \lambda^r}{r!\,(b_q)_r} \int_0^1 x^{s+(\delta-1)n+2cr-1} (1-x^2)^{\frac{1}{2}m} P_\gamma^m(x)\,dx.$$

Now evaluating the integral with the help of (2) and using (4), we get

$$\frac{(-1)^m \pi^{\frac{1}{2}} \Gamma[s+(\delta-1)n]\,\Gamma(1+m+\gamma)\,\Gamma[\frac{1}{2}\{1+s+(\delta-1)n+m-\gamma\}]}{2^{m+s+(\delta-1)n}\,\Gamma(1-m+\gamma)\,\Gamma[1+\frac{1}{2}\{s+(\delta-1)n+m+\gamma\}]}$$

$$\sum_{r=0}^{\infty} \frac{\prod_{i=0}^{\delta-1}\left(\frac{-n+i}{\delta}\right)_r (a_p)_r \prod_{i=0}^{2c-1}\left(\frac{s+(\delta-1)n+i}{c}\right)_r \prod_{i=0}^{C-1}\left(\frac{\frac{1}{2}(1+s+(\delta-1)n+m-\gamma)+i}{c}\right)_r \lambda^r 2^{2cr}}{r!\,(b_q)_r \prod_{i=0}^{c-1}\left(\frac{1+\frac{1}{2}(s+(\delta-1)i+m+\gamma)+i}{c}\right)_r}$$

which yields the value of the integral (5).

The integral (6) is similarly established on applying the same procedure as above.

Following on the same lines as above, the integrals (7) and (8) can easily be evaluated in view of (3) and (4).

PARTICULAR CASES

When $\delta = c = 1$ in (5) and (7): —

(a) Setting $a_1 = n+\alpha+\beta+1$, $b_1 = 1+\alpha$, $b_2 = \frac{1}{2}$, $\lambda = 1$ and multiplying both sides by $\frac{(1+\alpha)_n}{n!}$, we obtain,

$$\int_0^1 x^{s-1} (1-x^2)^{\frac{1}{2}m} P_\gamma^m(x)\, f_n^{(\alpha,\beta)}\left(\begin{matrix} a_2,\ldots,a_p \\ b_3,\ldots,b_q \end{matrix}; x^2\right) dx$$

$$= \frac{(-1)^m \pi^{\frac{1}{2}} \Gamma(s)\,\Gamma(1+m+\gamma)\,\Gamma[\frac{1}{2}(1+m+s-\gamma)]}{2^{m+s}\,\Gamma(1-m+\gamma)\,\Gamma[1+\frac{1}{2}(s+m+\gamma)]} \cdot$$

$$\cdot f_n^{(\alpha,\beta)}\left(\begin{matrix} a_2,\ldots,a_p, \triangle(2,s), \frac{1}{2}+\frac{1}{2}s+\frac{1}{2}m-\frac{1}{2}\gamma \\ b_3,\ldots,b_q, \quad 1+\frac{1}{2}s+\frac{1}{2}m+\frac{1}{2}\gamma \end{matrix}; 1\right), \; Re(s) > 0 \qquad (9)$$

$$\int_0 x^{s-1} (4-x^2)^{-\frac{1}{2}}\, T_\gamma(\tfrac{1}{2}x)\, f_n^{(\alpha,\beta)}\left(\begin{matrix} a_2,\ldots,a_p \\ b_3,\ldots,b_q \end{matrix}; x^2\right) dx$$

$$= \frac{\pi\,\Gamma(s)}{2\,\Gamma(\frac{1}{2}+\frac{1}{2}s\pm\frac{1}{2}\gamma)} f_n^{(\alpha,\beta)}\left(\begin{matrix} a_2,\ldots,a_p, \triangle(2,s) \\ b_3,\ldots,b_q, \frac{1}{2}+\frac{1}{2}s\pm\frac{1}{2}\gamma \end{matrix}; 2^2\right), \; Re(s) > 0 \qquad (10)$$

where

$$f_n^{(\alpha,\beta)}\begin{pmatrix} a_2,\ldots, a_p & ; x \\ b_3,\ldots, b_q \end{pmatrix} = \frac{(1+\alpha)_n}{n\,!}\ {}_{p+1}F_q\left[\begin{matrix} -n, n+\alpha+\beta+1, a_2,\ldots, a_p \\ 1+\alpha, \frac{1}{2}, \qquad b_3,\ldots, b_q \end{matrix} ; x\right]$$

is a generalized Sister Celine polynomial[3] which for $\alpha = \beta = 0$ yie ds Sister Celine polynomial[5].

In (9) and (10), taking $p = q = 3$, $a_2 = \rho$, $a_3 = \frac{1}{2}$ and $b_3 = \sigma$, we have

$$\int_0^1 x^{s-1} (1-x^2)^{\frac{1}{2}m}\, P_\gamma^m (x)\, H_n^{(\alpha,\beta)} (\rho, \sigma, x^2)\, dx$$

$$= \frac{(-1)^m \pi^{\frac{1}{2}}\, \Gamma (s)\, \Gamma (1+m+\gamma)\, \Gamma [\frac{1}{2}(1+s+m-\gamma)](1+\alpha)_n}{2^{m+s}\, \Gamma (1-m+\gamma)\, \Gamma [1+\frac{1}{2}(s+m+\gamma)]\, n\,!} \,.$$

$$\cdot\, {}_6F_3 \left[\begin{matrix} -n, n+\alpha+\beta+1, \rho, \triangle (2, s), \frac{1}{2}+\frac{1}{2}s+\frac{1}{2}m-\frac{1}{2}\gamma \\ 1+\alpha, \sigma, 1+\frac{1}{2}s+\frac{1}{2}m+\frac{1}{2}\gamma \end{matrix} ; 1\right], \; Re(s)>0. \qquad (11)$$

$$\int_0^2 x^{s-1} (4-x^2)^{-\frac{1}{2}}\, T_\gamma (\tfrac{1}{2}x)\, H_n^{(\alpha,\beta)} (\rho, \sigma, x^2)\, dx$$

$$= \frac{\pi\, \Gamma(s)(1+\alpha)_n}{2\, \Gamma [\frac{1}{2}(1+s\pm\gamma)]\, n\,!}\ {}_5F_4 \left[\begin{matrix} -n, n+\alpha+\beta+1, \rho, \triangle (2, s) \\ 1+\alpha, \sigma, \frac{1}{2}+\frac{1}{2}s\pm\frac{1}{2}\gamma \end{matrix} ; 2^2\right] \qquad (12)$$

where Re $(s) > 0$ and

$$H_n^{(\alpha,\beta)} (\rho, \sigma, x) = \frac{(1+\alpha)_n}{n\,!}\ {}_3F_2 \left[\begin{matrix} -n, n+\alpha+\beta+1, \rho \\ 1+\alpha, \sigma \end{matrix} ; x\right]$$

is the generalized Rice's polynomial[6] which also satisfies the differential equation[7] :

$$\Big\{ x^2 (1-x)\, D^3 + [(\sigma+\alpha+2)\, x - (4+\rho+\alpha+\beta)\, x^2]\, D^2 + [\sigma (1+\alpha) +$$

$$+ \{n (n+1) - (1+\rho)(\alpha+\beta+2) + n (\alpha+\beta)\}\, x]\, D + n\, \rho (n+\alpha+\beta+1) \Big\}.$$

$$.\ H_n^{(\alpha,\beta)} (\rho, \sigma, x) = 0,\ D \equiv \frac{d}{dx}.$$

With $\alpha = \beta = 0$, it reduces to Rice's polynomial[8].

Further setting $\rho = \sigma$ in (11) and (12), we get

$$\int_0^1 x^{s-1} (1-x^2)^{\frac{1}{2}m}\, P_\gamma^m (x)\, P_n^{(\alpha,\beta)} (1-2x^2)\, dx$$

$$= \frac{(-1)^m\, \pi^{\frac{1}{2}}\, \Gamma (s)\, \Gamma (1+m+\gamma)\, \Gamma [\frac{1}{2}(1+s+m-\gamma)]\, (1+\alpha)_n}{2^{m+s}\, \Gamma(1-m+\gamma)\, \Gamma[1+\frac{1}{2}(s+m+\gamma)]\, n\,!} \,.$$

$$\cdot\ {}_5F_2 \left[\begin{matrix} -n, n+\alpha+\beta+1, \triangle (2, s), \frac{1}{2}+\frac{1}{2}s+\frac{1}{2}m-\frac{1}{2}\gamma \\ 1+\alpha, 1+\frac{1}{2}s+\frac{1}{2}m+\frac{1}{2}\gamma \end{matrix} ; 1\right], \; Re(s)>0 \qquad (13)$$

$$\int_0^2 x^{s-1} (4 - x^2)^{-\frac{1}{2}} T_\gamma(\tfrac{1}{2}x) P_n^{(\alpha,\beta)}(1 - 2x^2)\, dx$$

$$= \frac{\pi \Gamma(s)(1+\alpha)_n}{2\Gamma[\frac{1}{2}(1+s\pm\gamma)]\, n!}\, {}_4F_3 \left[\begin{matrix} -n, n+\alpha+\beta+1, \triangle(2,s) \\ 1+\alpha, \frac{1}{2}+\frac{1}{2}s \pm \frac{1}{2}\gamma \end{matrix} ; 2^2 \right] \tag{14}$$

where $Re\ (s) > 0$ and $P_n^{\alpha,\beta}(x)$ is the Jacobi polynomial which leads to either Gegenbauer, Legendre or Tchebicheff polynomials on specializing the parameters.

(b) Substituting $p = 0$, $q = 1$, $b_1 = 1 + \alpha$, $\lambda = 1$ and multiplying both sides by $\frac{(1+\alpha)_n}{n!}$, we obtain

$$\int_0^1 x^{s-1} (1 - x^2)^{\frac{1}{2}m} P_\gamma^m(x) L_n^{(\alpha)}(x^2)\, dx$$

$$= \frac{(-1)^m \pi^{\frac{1}{2}} \Gamma(s) \Gamma(1+m+\gamma) \Gamma[\frac{1}{2}(1+m+s-\gamma)]\, (1+\alpha)_n}{\Gamma(1-m+\gamma)\, \Gamma[1+\frac{1}{2}(m+s+\gamma)]\, n!} \cdot$$

$$\cdot\ {}_4F_2 \left[\begin{matrix} -n, \triangle(2,s), \frac{1}{2}+\frac{1}{2}s+\frac{1}{2}m-\frac{1}{2}\gamma \\ (1+\alpha, 1+\frac{1}{2}s+\frac{1}{2}m+\frac{1}{2}\gamma \end{matrix} ; 1 \right],\ Re\ (s) > 0 \tag{15}$$

$$\int_0^2 x^{s-1} (4 - x^2)^{-\frac{1}{2}} T_\gamma(\tfrac{1}{2}x) L_n^{(\alpha)}(x^2)\, dx$$

$$= \frac{\pi \Gamma(s)(1+\alpha)_n}{2\Gamma(\frac{1}{2}+\frac{1}{2}s\pm\frac{1}{2}\gamma)\, n!}\ {}_3F_3 \left[\begin{matrix} -n, \triangle(2,s) \\ 1+\alpha, \frac{1}{2}+\frac{1}{2}s \pm \frac{1}{2}\gamma \end{matrix} ; 2^2 \right] \tag{16}$$

where $Re\ (s) > 0$ and $L_n^{(\alpha)}(x)$ is the generalized Laguerre polynomial.

(c) With $p = 1$, $q = 0$, $a_1 = n + a - 1$, $\lambda = -\frac{1}{b}$, we have

$$\int_0^1 x^{s-1} (1 - x^2)^{\frac{1}{2}m} P_\gamma^m(x) Y_n(x^2, a, b)\, dx$$

$$= \frac{(-1)^m \pi^{\frac{1}{2}} \Gamma(s)\, \Gamma(1+m+\gamma)\, \Gamma[\frac{1}{2}(1+s+m-\gamma)]}{2^{m+s}\, \Gamma(1-m+\gamma)\, \Gamma(1+\frac{1}{2}s+\frac{1}{2}m+\frac{1}{2}\gamma)} \cdot$$

$$\cdot\ {}_5F_1 \left[\begin{matrix} -n, n+a-1, \triangle(2,s), \frac{1}{2}+\frac{1}{2}s+\frac{1}{2}m-\frac{1}{2}\gamma \\ 1+\frac{1}{2}s+\frac{1}{2}m+\frac{1}{2}\gamma \end{matrix} ; -\frac{1}{b} \right], Re\ (s) > 0 \tag{17}$$

$$\int_0^2 x^{s-1} (4 - x^2)^{-\frac{1}{2}} T_\gamma(\tfrac{1}{2}x) Y_n(x^2, a, b)\, dx$$

$$= \frac{\pi\ \Gamma(s)}{2\ \Gamma(\frac{1}{2}+\frac{1}{2}s\pm\frac{1}{2}\gamma)}\ {}_4F_2 \left[\begin{matrix} -n, n+a-1, \triangle(2,s) \\ \frac{1}{2}+\frac{1}{2}s \pm \frac{1}{2}\gamma \end{matrix} ; -\frac{1}{b} 2^2 \right] \tag{18}$$

where $\operatorname{Re}(s) > 0$ and

$$Y_n(x, a, b) = {}_2F_0\left[\begin{matrix}-n, n+a-1 \\ \text{---}\end{matrix} ; -\frac{1}{b}x\right]$$

is the generalized Bessel polynomial introduced by Krall & Frink[9] which reduces to a simple Bessel polynomial when $a = b = 2$ and is represented by $Y_n(x)$.

PARTICULAR CASES

When $\delta=2$, $c=1$ in (6) and (8):—

Setting $p=1$, $q=2$, $a_1 = Y-\beta$, $b_1 = Y$, $b_2 = 1-\beta-n$, $\lambda=1$ and multiplying both sides by $\dfrac{2^n (\beta)_n}{n!}$, we obtain

$$\int_0^1 x^{s-1}(1-x^2)^{\frac{1}{2}m} P_\gamma^m(x) R_n(\beta, Y; x)\, dx$$

$$\frac{(-1)^m \pi^{\frac{1}{2}} \Gamma(1+m+\gamma)\Gamma(s+n)\Gamma\left[\frac{1}{2}(1+s+n+m-\gamma)\right](\beta)_n}{2^{m+s}\Gamma(1-m+\gamma)\Gamma(1+\frac{1}{2}s+\frac{1}{2}n+\frac{1}{2}m+\frac{1}{2}\gamma)\, n!}\,.$$

$$\cdot\ {}_4F_5\left[\begin{matrix}\triangle(2,-n), Y-\beta, -\frac{1}{2}s-\frac{1}{2}n-\frac{1}{2}m-\frac{1}{2}\gamma \\ Y, 1-\beta-n, \triangle(2, 1-s-n), \frac{1}{2}-\frac{1}{2}s-\frac{1}{2}n-\frac{1}{2}m+\frac{1}{2}\gamma\end{matrix} ; 1\right], \operatorname{Re}(s+n) > 0 \qquad (19)$$

$$\int_0^2 x^{s-1}(4-x^2)^{-\frac{1}{2}} T_\gamma(\tfrac{1}{2}x)\, R_n(\beta, Y; x)dx$$

$$= \frac{\pi\,\Gamma(s+n)2^n(\beta)_n}{2\,\Gamma(\frac{1}{2}+\frac{1}{2}s+\frac{1}{2}n\pm\frac{1}{2}\gamma)\, n!}\ {}_5F_4\left[\begin{matrix}\triangle(2,-n), Y-\beta, \frac{1}{2}-\frac{1}{2}s-\frac{1}{2}n\pm\frac{1}{2}\gamma \\ Y, 1-\beta-n, \triangle(2, 1-s-n)\end{matrix} ; \frac{1}{2^2}\right] \qquad (20)$$

where $\operatorname{Re}(s+n) > 0$ and

$$R_n(\beta, Y; x) = \frac{(2x)^n(\beta)_n}{n!}\ {}_3F_2\left[\begin{matrix}\triangle(2,-n), Y-\beta \\ Y, 1-\beta-n\end{matrix} ; x^{-2}\right]$$

is a Bedient's polynomial[10] which reduces to the Gegenbauer polynomial $C_n^{(\beta)}(x)$ when $\lim\limits_{Y\to\infty} R_n(\beta, Y; x)$.

If we put $m=0$ in (19), we obtain a known result[11].

Similarly by particular choice of parameters in (6) and (8), we can easily obtain the results involving Hermite, Lommel and Legendre polynomials.

Meijer's G-function, MacRobert's E-function and Fox's H-function can be reduced to the generalized hypergeometric polynomial by appropriately specializing the parameters. But the converse, however, is not necessarily true. As the various classical and other polynomials have immediate use in the different fields of applications, generalization of the results for these polynomials should have a compact and simplified form.

This is achieved when the results are expressed in terms of a generalized hypergeometric polynomial. Moreover, it is quite obvious that investigations with the generalized hypergeometric polynomial will have a wider region of freedom compared to the investigations with the generalized functions.

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