MAGNETOHYDRODYNAMIC FLOW BETWEEN CONCENTRIC ROTATING POROUS CYLINDERS

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An attempt has been made to study the steady laminar flow of a viscous incompressible electrically conducting fluid between infinitely long concentric rotating porous cylinders under the influence of radial magnetic field. A solution has been obtained under the assumption of uniform conditions along the axis of the cylinders. The cylinders being porous, a hyperbolic radial velocity distribution has been superimposed over the circumferential velocity produced due to rotation. There is a Bernoulli-type pressure variation in the radial direction. When the inner cylinder is at rest, the shearing stress at it and the torque transmitted to it decrease as R $(=v_1 y_1/\nu = v_2 y_2/\nu)$ increases and the magnetic parameter λ $(=4\sigma\mu_e^2 A^2/\mu)$ will further decrease them.

Couette first obtained the exact solution of the Navier-Stokes equations for steady laminar flow of a viscous incompressible fluid between two coaxial rotating cylinders. Sinha & Choudhary[1] have discussed the same problem with uniform radial velocity imposed at the surfaces. They have also shown that $R = -2$ is a critical value at which the results take indeterminate forms. This difficulty will not arise when the magnetic field is present and that is why we have discussed the magnetohydrodynamic flow in the present paper. The present problem is treated for radial magnetic field A/y, where A is a constant and y is the radial distance from the axis and the solution obtained agrees with all special cases heretofore available.

The case when the inner cylinder is at rest and the outer rotates has some practical importance. The circumferential velocity distribution, the shearing stress at the inner cylinder and the torque transmitted to it decrease as R increases and the magnetic parameter λ will further decrease them.

NOTATIONS

(y, ϕ, x) = cylindrical polar coordinates

(v, w, u) = radial, azimuthal and axial velocity components

ρ = density of the fluid

ν = kinematic viscosity

y_1, y_2 = radii of the inner and outer cylinders

σ = electrical conductivity of the fluid

μ_e = magnetic permeability

B_0 = electromagnetic induction

λ = magnetic parameter = $\dfrac{4\sigma\,\mu_e^2\,A^2}{\mu}$

v_1, v_2 = cross-flow velocities at the walls of the inner and outer cylinders

ω_1, ω_2 = angular velocities of the inner and outer cylinders

p = pressure

$$R = \frac{v_1 y_1}{\nu} = \frac{v_2 y_2}{\nu} = \text{cross-flow Reynolds number}$$

$$\eta = \text{dimensionless } y \text{ coordinate} = \frac{y}{y_1}$$

$$k = \frac{y_2}{y_1} \; (>1)$$

BASIC EQUATIONS

The equation of motion for laminar flow of an incompressible, electrically conducting fluid is, in the usual notation,

$$\rho \frac{D\vec{V}}{Dt} = -\operatorname{grad} p + \mu \nabla^2 \vec{V} + (\vec{J} \times \vec{B}), \tag{1}$$

where $\vec{J}$ and $\vec{B}$ are given by Maxwell's equations and Ohm's law.

The equation of continuity is

$$\operatorname{div} \vec{V} = 0. \tag{2}$$

For steady flow between two rotating porous cylinders, the following assumptions are made :

(a) The total excess charge density, the imposed electric field intensity, and the induced magnetic field are zero (Rossow[2]),

(b) Electrical conductivity σ of the fluid is very large so that the displacement current is neglected,

(c) No external electric field is applied,

(d) A magnetic field of strength $H_0 = A/y$ is applied in the radial direction,

(e) $u = 0$, for motion due to rotation only,

(f) $\partial v/\partial x = 0$, for uniform suction and injection throughout the whole lengths,

(g) $\partial w/\partial x = 0$, for circumferential velocity produced due to rotation only.

With these assumptions, (1) and (2) are reduced simply to

$$\rho \left(v \frac{\partial v}{\partial y} - \frac{w^2}{y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial y^2} + \frac{1}{y} \frac{\partial v}{\partial y} - \frac{v}{y^2} \right), \tag{3}$$

$$\rho v \left(\frac{\partial w}{\partial y} + \frac{w}{y} \right) = \mu \left(\frac{\partial^2 w}{\partial y^2} + \frac{1}{y} \frac{\partial w}{\partial y} - \frac{w}{y^2} \right) - \sigma \mu_e^2 H_0^2 w \tag{4}$$

$$0 = \frac{\partial p}{\partial x}, \tag{5}$$

$$\frac{\partial v}{\partial y} + \frac{v}{y} = 0 \tag{6}$$

Equation (5) states the condition of uniform pressure distribution along the axis of the cylinders.

Substituting from (6) into (3), we get

$$\rho \left(\frac{v^2 + w^2}{y} \right) = \frac{\partial p}{\partial y}. \tag{7}$$

Equation (7) gives Bernoulli-type pressure variation in the radial direction, which will not be discussed further in the present investigation.

We now assume that the suction rate at one wall is equal to the injection rate at the other.

Therefore
$$v_2 y_2 = v_1 y_1. \tag{8}$$

From (6) and (8), we get
$$vy = v_2 y_2 = v_1 y_1. \tag{9}$$

Substituting from (9) into (4), we get
$$\frac{d^2 w}{dy^2} + \frac{(1-R)}{y}\frac{dw}{dy} - \left(1 + R + \frac{\lambda}{4}\right)\frac{w}{y^2} = 0 . \tag{10}$$

The boundary conditions are
$$\left.\begin{aligned} w &= y_1 \omega_1 \quad \text{when} \quad y = y_1 \\ \text{and} \quad w &= y_2 \omega_2 \quad \text{when} \quad y = y_2 \end{aligned}\right\}. \tag{11}$$

Let us introduce the following dimensionless quantities :
$$\eta = \frac{y}{y_1}, \quad U = \frac{w}{y_2 \omega_2}, \quad N = \frac{y_1 \omega_1}{y_2 \omega_2}.$$

Equation (10) then becomes
$$\eta^2 \frac{d^2 U}{d\eta^2} + (1-R)\,\eta \frac{dU}{d\eta} - \left(1 + R + \frac{\lambda}{4}\right) U = 0, \tag{12}$$

with the boundary conditions
$$\left.\begin{aligned} U &= N \quad \text{when} \quad \eta = 1 \\ U &= 1 \quad \text{when} \quad \eta = k \end{aligned}\right\}. \tag{13}$$

The solution of (12) with the boundary conditions (13) is
$$U = \frac{(1 - Nk^n)\,\eta^m + (Nk^m - 1)\,\eta^n}{(k^m - k^n)}, \tag{14}$$

where
$$m = \frac{R + \{ (R+2)^2 + \lambda \}^{\frac{1}{2}}}{2},$$

and
$$n = \frac{R - \{ (R+2)^2 + \lambda \}^{\frac{1}{2}}}{2}.$$

When the inner cylinder is at rest and the outer rotates
$$N = 0$$

and
$$\frac{w}{y_1 \omega_2} = \frac{k\,(\eta^m - \eta^n)}{(k^m - k^n)}. \tag{15}$$

The distribution of circumferential velocity for various values of R and λ and $k = 1{\cdot}5$ is shown in Fig 1. It is noted that the circumferential velocity at any point inside the channel decreases as R increases and λ will further decrease it.

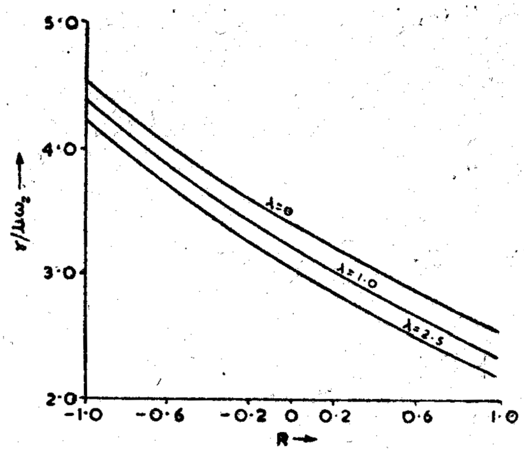

Fig 1—Circumferential velocity distribution $w/y_1\omega_2$ vs $\eta = y/y_1$ for various values of R and λ when the inner cylinder is at rest.

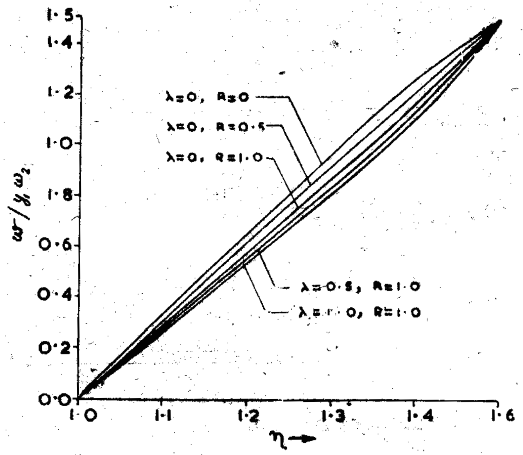

Fig 2—Shearing stress at the inner cylinder $\tau/\mu\,\omega_2$ vs R when the inner cylinder is at rest for various values of λ.

The shearing stress at the inner cylinder is

$$\frac{\tau}{\mu\omega_2} = \frac{k\,\{\,(R+2)^2+\lambda\,\}^{\frac{1}{2}}}{(k^m - k^n)} \qquad (16)$$

The shearing stress has been calculated for various values of R, λ and $k = 1{\cdot}5$ and the results of calculation have been plotted in Fig 2. From this figure it is obvious that the shearing stress at the inner cylinder decreases as R increases and the magnetic parameter λ will further decrease it.

The torque transmitted by the fluid to unit length of the inner cylinder is

$$\frac{M}{2\pi\,\mu\,\omega_2\,y_2^2} = \frac{\{\,(R+2)^2+\lambda\,\}^{\frac{1}{2}}}{k\,(k^m - k^n)} \qquad (17)$$

From (16) and (17), we get

$$\left(\frac{M}{2\pi\,\mu\,\omega_2\,y_2^2}\right) = \frac{1}{k^2}\left(\frac{\tau}{\mu\,\omega_2}\right),$$

and hence the torque transmitted by the fluid to unit length of the inner cylinder also decreases as R and λ increase.

Proceeding to the limit when $R = 0$, we get the results of Rammamorthy[3]. The results of Sinha & Choudhary[1] are obtained when $\lambda = 0$.

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