HYDROMAGNETIC COUETTE FLOW WITH TIME DEPENDENT SUCTION OR INJECTION

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Hydromagnetic couette flow of an electrically conducting viscous and incompressible fluid in a transverse magnetic field is studied when the plates are non-magnetic and non-conducting. The suction velocity is assumed to be varying as (time)$^{-\frac{1}{2}}$. The analysis is valid for small values of hydromagnetic parameter $(mt)^{\frac{1}{2}}$ and for magnetic Prandtl number unity.

Recently Pandey[1] studied the Rayleigh's problem in magnetohydrodynamics with suction. The object of this paper is to extend the problem studied by Pandey to the case when the fluid is confined between two non-conducting and non-magnetic parallel porous flat plates. It is assumed that the lower plate is fixed while the upper one starts to move with constant velocity U_0 at time $t=0$.

The expression for velocity and induced magnetic field have been obtained in terms of known functions by expansion in series in ascending powers of hydromagnetic parameter $(mt)^{\frac{1}{2}}$.

Similar problems have also been solved by Katagiri[2], Mahuri[3], Agarwal[4], Ludford[5] and some others.

BASIC EQUATIONS AND BOUNDARY CONDITIONS

We assume that an electrically conducting incompressible fluid fills the space between two non-conducting, non-magnetic infinite parallel porous flat plates. One of the plates, for instance, the lower one is kept fixed while the upper one at a distance h from the lower is accelerated suddenly from rest and moves in its own plane in the direction of x increasing. The coordinate axes x and y have been taken along and perpendicular to the lower plate respectively.

Since the plates are infinite in extent the equations governing the hydromagnetic flow of an electrically conducting fluid in the presence of a uniform transverse magnetic field H_0 are

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\mu_e H_0}{\rho}\cdot\frac{\partial H_x}{\partial y}, \tag{2}$$

$$\frac{\partial H_x}{\partial t} + v\frac{\partial H_x}{\partial y} = H_0\frac{\partial u}{\partial y} + \frac{\nu}{P_m}\frac{\partial^2 H_x}{\partial y^2} \tag{3}$$

where H_x, t, ν, ρ, μ_e and P_m ($= \sigma\,\mu_e\,\nu$) are the induced magnetic field in the direction of x-axis, the time, the kinematic viscosity, the density, the magnetic permeability and magnetic Prandtl number respectively. σ being the electrical conductivity of the fluid. u, v are the velocity components parallel and perpendicular to the plate.

Since from (1) v is independent of y, we conclude that v is a function of t only. Let us take $v = -v_0(t) = -c\,(\nu/t)^{1/2}$, where c is a real positive constant.

Substituting $v = -c(\nu/t)^{\frac{1}{2}}$ and $\alpha = H_x(\mu_e/\rho)^{\frac{1}{2}}$ in (2) and (3), we get

$$\left.\begin{aligned}\frac{\partial u}{\partial t} - c(\nu/t)^{1/2}\frac{\partial u}{\partial y} &= \nu\frac{\partial^2 u}{\partial y^2} + H_0\left(\frac{\mu_e}{\rho}\right)^{1/2}\frac{\partial \alpha}{\partial y}\\ \frac{\partial \alpha}{\partial t} - c(\nu/t)^{1/2}\frac{\partial \alpha}{\partial y} &= \frac{\nu}{P_m}\frac{\partial^2 \alpha}{\partial y^2} + H_0\left(\frac{\mu_e}{\rho}\right)^{1/2}\frac{\partial u}{\partial y}\end{aligned}\right\} \tag{4}$$

where α stands for the Alfvén wave velocity and c for a real positive constant. The boundary conditions pertinent to the problem are

$$\begin{aligned} &u = 0, \quad \alpha = 0 \quad \text{for } t < 0,\\ &u = U_0, \quad \alpha = 0 \quad \text{at } y = h \quad \text{for } t \geqslant 0,\\ &u = 0, \quad \alpha = 0 \quad \text{at } y = 0 \quad \text{for } t \geqslant 0.\end{aligned} \tag{5}$$

Solution of the Problem when $P_m = 1$

Let us assume the solution of (4) in ascending powers of $(mt)^{\frac{1}{2}}$ as follows:

$$\left.\begin{aligned} u &= U_0\sum_{i=0}^{\infty}(mt)^{i/2}u_i(z),\\ \alpha &= U_0\sum_{i=0}^{\infty}(mt)^{\frac{i+1}{2}}\alpha_i(z)\end{aligned}\right\} \text{for } (mt)^{\frac{1}{2}} << 1, \tag{6}$$

$$\text{where } z = \frac{y}{2(\nu t)^{\frac{1}{2}}} \text{ and } m = \frac{\sigma H_0^2 \mu_e^2}{\rho}$$

Substituting the above expressions for u and α in (4), neglecting coefficients of $(mt)^2$ and higher powers of mt, and comparing harmonic terms, we get

$$\left.\begin{aligned} u_0'' + 2(z+c)u_0' &= 0,\\ u_1'' + 2(z+c)u_1' - 2u_1 &= 0,\\ u_2'' + 2(z+c)u_2' - 4u_2 &= -2\alpha_0',\\ u_3'' + 2(z+c)u_3' - 6u_3 &= -2\alpha_1'\end{aligned}\right\} \tag{7}$$

and

$$\left.\begin{aligned} \alpha_0'' + 2(z+c)\alpha_0' - 2\alpha_0 &= -2u_0',\\ \alpha_1'' + 2(z+c)\alpha_1' - 4\alpha_1 &= -2u_1',\\ \alpha_2'' + 2(z+c)\alpha_2' - 6\alpha_2 &= -2u_2',\end{aligned}\right\} \tag{8}$$

where dashes denote differentiation with respect to z.

The solution of system of (7) and (8) are given by

$$u_0(z) = K_1 \operatorname{erf}(z+c) + K_2, \tag{9}$$

$$u_1(z) = 0, \tag{10}$$

$$\begin{aligned} u_2(z) = {} & K_3\left[\sqrt{\pi}\operatorname{erf}(z+c) + 2(z+c)^2\sqrt{\pi}\operatorname{erf}(z+c) + \right.\\ & \left. + 2(z+c)\exp\{-(z+c)^2\}\right] + K_4\left[2 + 4(z+c)^2\right] +\\ & + \frac{K_5\sqrt{\pi}}{2}\operatorname{erf}(z+c) - \frac{K_1}{4}(z+c)\exp\{-(z+c)^2\} + K_6,\end{aligned} \tag{11}$$

$$u_3(z) = 0\,, \tag{12}$$

$$\alpha_0(z) = K_5\left[(z+c)\sqrt{\pi}\,\mathrm{erf}\,(z+c) + \exp\{-(z+c)^2\}\right] + 2K_6(z+c) + \frac{K_1}{2}\exp\{-(z+c)^2\}\,, \tag{13}$$

$$\alpha_1(z) = 0\,, \tag{14}$$

$$\alpha_2(z) = K_7\left[6(z+c)\sqrt{\pi}\,\mathrm{erf}\,(z+c) + 4\exp\{-(z+c)^2\} + 4(z+c)^3\sqrt{\pi}\,\mathrm{erf}\,(z+c) + 4(z+c)^2\right] + K_8\left[12(z+c) + 8(z+c)^3\right] + 2\left(K_3 + \frac{K_5}{4} - \frac{7K_1}{144}\right)\exp\{-(z+c)^2\} + 2K_3(z+c)\sqrt{\pi}\,\mathrm{erf}\,(z+c) + 4K_4(z+c) + \frac{K_1}{18}(z+c)^2\exp\{-(z+c)^2\}\,. \tag{15}$$

where the constants K_1, K_2, K_3, K_4, K_5, K_6, K_7 and K_8 are to be determined with the use of boundary conditions:

$$\left.\begin{array}{l} u_0(0) = u_1(0) = u_2(0) = u_3(0) = 0 = \alpha_0(0) = \alpha_1(0) = \alpha_2(0) \\ u_0(z_1) = 1\,; \\ u_1(z_1) = u_2(z_1) = u_3(z_1) = 0 = \alpha_0(z_1) = \alpha_1(z_1) = \alpha_2(z_1) \end{array}\right\} \tag{16}$$

where $z_1 = h/2(\nu t)^{\frac{1}{2}}$ and the function erf (x) is defined as

$$\mathrm{erf}\,(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-\eta^2)\,d\eta\,.$$

Thus we get the expression for u and α as

$$\frac{u}{U_0} = K_1\,\mathrm{erf}\,(z+c) + K_2 + (mt)\Big[K_3\sqrt{\pi}\,\mathrm{erf}\,(z+c) + 2K_3\sqrt{\pi}(z+c)^2 \times \mathrm{erf}\,(z+c) + 2K_3(z+c)\,\mathrm{erf}\,\{-(z+c)^2\} + K_4\{2+4(z+c)^2\} + \frac{K_5\sqrt{\pi}}{2}\mathrm{erf}\,(z+c) - \frac{K_1}{4}(z+c)\exp\{-(z+c)^2\} + K_6\Big]$$

and

$$\frac{\alpha}{U_0} = (mt)^{\frac{1}{2}}\left[K_5\sqrt{\pi}(z+c)\,\mathrm{erf}\,(z+c) + K_5\exp\left\{-(z+c)^2\right\} + 2K_6(z+c) + \frac{K_1}{2}\exp\left\{-(z+c)^2\right\}\right] + (mt)^{3/2}\Bigg[6K_7\sqrt{\pi}(z+c) \times \mathrm{erf}\,(z+c) + 4K_7\exp\left\{-(z+c)^2\right\} + 4K_7\sqrt{\pi}(z+c)^3\,\mathrm{erf}\,(z+c) + 4K_7(z+c)^2 + 12K_8(z+c) + 8K_8(z+c)^3 + 2\left(K_3 + \frac{K_5}{4} - \frac{7K_1}{144}\right) \times \exp\left\{-(z+c)^2\right\} + 2K_3\sqrt{\pi} \times (z+c)\,\mathrm{erf}\,(z+c) + 4K_4(z+c) + \frac{K_1}{18}(z+c)^2\exp\left\{-(z+c)^2\right\}\Bigg].$$

On transferring the origin from the lower plate to the upper plate and reversing the direction of y we find that the above expressions for velocity (on tending z_1 to ∞) reduce to those as obtained by Pandey[1] for his case ($P_m = 1$).

TABLE 1

NUMERICAL VALUES OF $\frac{a}{U_0}$

z	$(mt)^{\frac{1}{2}}=0\cdot1$			$(mt)^{\frac{1}{2}}=0\cdot5$		
	$c=-1$	$c=0$	$c=1$	$c=-1$	$c=0$	$c=1$
0	0	0	0	0	0	0
0·4	0·0145	0·2278	—0·1473	0·0741	0·1150	—1·4570
0·8	0·0253	0·2038	—0·1855	0·1259	0·1047	—3·1657
1·2	0·0252	0·1143	—0·2883	0·1252	0·0991	—3·8078
1·6	0·0145	0·0301	—0·2353	0·0728	0·0089	—2·8825
2·0	0	0	0	0	0	0

DISCUSSION

Curves have been drawn showing the variation of $\frac{u}{U_0}$ with z for $c = -1,\ 0,\ 1$ and $(mt)^{\frac{1}{2}} = 0,\ 0\cdot25$.

It is seen from Fig 1 that the effect of magnetic field is to damp the velocity for all values of c, while the velocity increases as we go from $c = -1$ to $c = 1$.

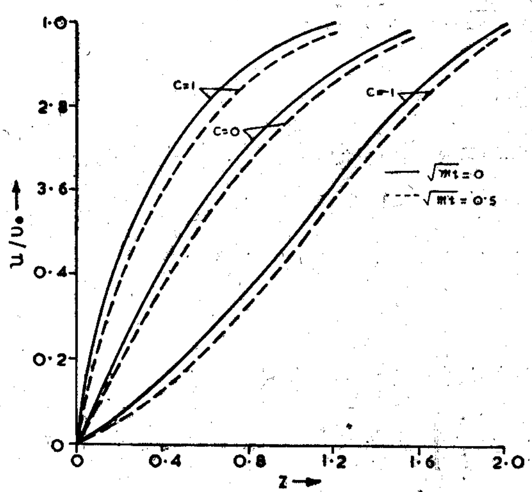

Fig 1—Variation of u/U_0 vs z.

Table 1 shows the variation of a/U_0 with z for $(mt)^{\frac{1}{2}} = 0\cdot1,\ 0\cdot5$ and for different values of c. It is found that as $(mt)^{\frac{1}{2}}$ increases the induced field decreases for $c = 0$ while it increases for $c = -1$ or 1.

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REFERENCES

1. PANDEY, G. C., *Def. Sci. J.*, 20 (1970), 71.
2. KATAGIRI, M., *J. Phys. Soc.*, Japan, 17 (1962), 393.
3. MAHURI, P. K., *J. Phys. Soc.* Japan, 18 (1963), 1671.
4. AGARWAL, J. P., *Appl. Sci. Res.*, 9B (1962), 255.
5. LUDFORD, G. S. S., *Arch. Rat. Mech. Anal.*, 3 (1959), 14.