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Art in Internal Ballistics

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Abstract. A review of the computations in Internal Ballistic Systems for developing pressure and velocity space curves, called primary problem and differential variations due to change in initial phase space of loading conditions, called secondary problem, is presented. In the concluding part, the general aspects of the secondary problem are analysed and reported.

PRIMARY PROBLEM

1. Introduction

Systems of Internal Ballistics can be broadly classified into three types, namely "Emperical, Semi-Emperical and Nearly-Exact". The emperical systems[1–4] derive little support from the thermo-dynamic theory, but assume algebraic relations between ballistic parameters. The semi-emperical systems[5–10] are built up with ballistic parameters, derived from extensive fired data, coupled with emperical rules and some assumption of thermo-dynamic theory. The nearly-exact systems[11–23] attempt to include all main ballistic phenomena, using thermo-dynamic theory, upto a certain degree of magnitude. These methods also draw the support of fired data, to arrive at some assumptions, which simplifies mathematical complexities, of the problem. Refinement to physical plausibility, depicting more realistic situations, had led to various mathematical models, which, however, complex, could be solved by differential analyzers and high-speed computers of to-day. Treatments of this nature are associated with many researchers[24–52].

2. Statement of the Primary Problem

The primary problem of a system of Internal Ballistics is to evaluate spatial, as well as time curves of velocity and pressure, with physical plausibility and possible precision, under a given set of loading conditions.

For a given phase space of loading conditions, it admits a single valued solution—a single-pressure curve with maximum pressure (P_{max}) and a single-velocity curve for a projectile with muzzle velocity (MV).

Conversely for a given P_{max} and MV of a given mass of projectile, the determination of design data of barrel and loading conditions admits a series of solutions. The converse problem of internal ballistics leads to the ballistic design of a gun.

The variation of a set of given loading conditions leads to a series of solutions of a particular system. Thus the accuracy and precision of the solutions of any system depend upon the correct description of the initial conditions as well as assumptions about realistic ballistic phenomena. Different systems have been evolved as a result of differences in degree of complexity and in sophistication of treatment in details of mathematical procedures, interpretation and in representation of some thermo-dynamical, physical and mechanical phenomena, such as density function, resistance function, form function, heat transfer, laws of burning and dissociation products, initial conditions and consideration of secondary energy losses. A brief review of some of the systems reveals the following salient features.

3. Le Duc Approach

The emperical-system of Le Duc[57] published by Challeat[4] assummes the velocity (V)—space (x) curve in the form

$$V = \frac{ax}{b + x}$$

The constant 'a' and 'b' represent thrice the velocity at maximum pressure ((P_{max}) and twice the shot-travel to P_{max} respectively. 'a', asymptotic velocity in an infinitely long-gun, is a function of propellant (type, mass) projectile (mass) and loading density. 'b' is a function of relative-propellant quickness, initial-air-space, chamber-volume and projectile-mass. Out of the five equations of the system, only one equation contains properties of propellant other than density. Most of the secondary energy-losses are ignored. But an emperical correction factor 1.12 derived from extensive firings, is introduced in estimating pressure, to account for pressure-gradient in the bore, shot-start pressure and resistances due to band-engraving and bore. In this system, advance knowledge of P_{max} and muzzle velocity is necessary to evaluate 'a' and 'b'. This is a great disadvantage. For pre-determining 'a' and 'b' in conformity with conditions of loading and propellant parameters, Le Duc introduced two quantities "α and β".

$$a = \alpha \left[\frac{(w)}{q}\right]^{1/2} \Delta^{1/12}$$

$$b = \beta \left[\frac{(W_0)}{q}\right]^{3/8} \left[1 - \frac{3}{4}\Delta\right]$$

where 'w' and 'q' are masses of propellant and project, 'w_0' initial chamber Volume and 'Δ' the loading density. The quantity 'α' characterises the propellant potential, depends mainly on propellant-type and fluctuates within narrow limits, whereas 'β'

characterises rate of burning, depends mainly on propellant-web and fluctuates within wider limits. The dependence of 'β' on propellant-web was established by M. E. Serebraikov[81,82] who also proposed simplified relations for 'a' and 'b' for Soviet propellants.

$$a = 0.16 \left(2gf \frac{w}{q} \right)^{1/2}$$

$$b = 2x_0 \, \triangle$$

where 'f' is the propellant-force and 'x_0' is the initial equivalent length of total volumes of chamber and bore. Thus the so-called constants '*a*' and '*b*' are functions of loading conditions and differ from propellant to propellant. Moreover, the derived values of '*b*' from P_{max} and MV are not consistent. The ratio of specific heats 'γ' is 7/6. This method was in use by the U. S. Navy as late as 1942.

4. Oerlikon Approach

The semi-emperical system of Oerlikon[9] is based on methods, developed by Vallier-Heydenreich[53], Kratz[54] and F. Herlach[83]. The spatial curves of velocity, pressure and time are obtained by using Heydenreich-tables. These tables were formulated on the basis of treatment of a large number of velocimetric-recoil curves, obtained by firing different calibre-guns, under varying loading conditions. The system uses modified Resal's energy equation, established by Bergmann (C. Cranz[55]) with ratio of specific heats 'γ' equals to 1.2 and fictitious projectile-mass 'M' as

$$M = 1.07 \left(1 + \frac{C}{4W} \right) W$$

where W and C are actual projectile and propelling-charge masses. The estimation of either P_{max} or design of propellant-web is by Sarru's formulae[56], in which the constant K determined by experimental firings, takes into account the conditions of ignition and obturation, as well as losses due to friction. The specific charge-mass (charge mass per unit projectile-muzzle energy) is estimated, using graphical relation established by Herlach between P_{max} and specific charge-mass on muzzle velocity. Thermal efficiency with specific charge-mass takes into account some of the secondary-energy losses, neglecting rotational energy of projectile. Kratz method is used to evaluate after effects of powder-gases after shot-ejection.

As Heydenreich had not considered the influence of propellant-web, Kisnemsky[80] eliminated this disadvantage and formulated tables based on the results of firings of gun-propellant combinations of Soviet Union. Moreover, the differential variations of propellant (mass, temperature, ignition) and projectile (mass, crimping force), inherent in round-to-round, were found to cause variation in gas-pressure curve within wider limits compared to Heydenreich method. Oerlikon used this system, whereas Rheinmetall used Heydenreich's method until 1945.

5. Bofors Approach

The semi-emperical system of Bofors[10] assumes that the order of pressures, encountered in the gun is the same as that of closed vessel (CV) and the all-burnt-point (ABP) coincides with P_{max}. The laws of combustion gases and propellant burning, form the basis of this system. The P_{max} is derived from the equation of expansion of combustion-gases[58], whereas the mean-gas-pressure (Pm) is derived from propellant-burning rate[59]. The energy equation is worked out from pressure-working diagrams[60] which are computed using polytropic-laws with certain assumptions. The maximum-loading density (Δ) is regarded as a function of propellant energy and is related to calorimetric-value emperically. Based on calorimetric value, propellants are classified into eight types : double-base propellants (seven types) and single base (one). The propellant constants, derived from CV for each type are compared and corrected with that of gun-barrels. The design of propellant is by similitudes, using Sarru's formula. The specific gas-emission parameter (σ) is a function of form-factor (α) and propellant-web (e). The degree of progressivity of propellant-grain (π), expressed in percentage, is :

$$\pi = \left(\frac{Y_2}{Y_1} - 1 \right)$$

where Y_1 and Y_2 are original free surface before the start of burning and final surface at the end of burning respectively. Based on the value of π, the type of burning is indicated. The secondary energy losses due to band-engraving and initial resistance were accounted by web-size adjustments. These adjustments were evaluated by static-firings[61]. Muzzle-velocity was estimated using a proportionality factor μ, which varies inversely as loading density. The factor μ becomes more sensitive to variations in some internal ballistic magnitudes such as calibre of weapon, engraving resistance of driving-band, crimping force of projectile and type and arrangement of ignition-charge. Thus the system works for those propellants for which the propellant constants are known and has proved to provide a satisfactory solution to the common-interior ballistic problems in artillery.

6. Hunt-Hinds Approach

The nearly-exact system adopting shot-start model, comprises equations of modified Resals energy, motion, form-function and linear law of burning. The pressure-gradient is replaced by mean-pressure. The secondary energy losses, more than or equal to one per cent of Kinetic Energy of projectile, coupled with initial resistance and engraving of driving band, are accounted by increasing shot-mass by 5%. Heat-transfer to gun is omitted and correction is made to theremo-dynamic efficiency. The form-function is quadratic, based on a constant form coefficient. Initial velocity of shot and increase of volume during engraving is neglected. Aggarwal[24] and his associates[27,28] solved the Hunt-Hinds problem with spatial-dynamic density functions. More realistic situations of resistance, form, heat-transfer and burning law were considered by these workers. Narvilkar[44,45] solved the problem by introducing the hydro-dynamic aspects into the problem and density gradients in the flow.

7. Sugot Approach

The system of Sugot[18,62] adopts Resal's energy equation Charbonniers form function, which implies no theoretical assumption of law of burning and uses shot-start model. The system differs from Hunt-Hinds in defining central-ballistic parameter M and velocity parameter η. A dependent variable y was introduced with an arbitrary constant to express shot-travel in terms of the fraction of charge mass burnt 'z'. This facilitated practical application. The secondary energy losses were accounted by coefficient representing secondary-work ϕ

$$\phi = 1.05\left(1 + \frac{1}{4}\frac{W}{q}\right)$$

This was a particular case of general expression, established by Slukhotsky[84], discussed later in Drozdovs approach.

8. Crow Approach

The system of Crow[17] differs from Hunt-Hinds in the following respects:

(a) Kinetic energy term and co-volume term of Resal's energy equation are neglected and correction is applied to propellant force F, which is further corrected to account for heat losses.

(b) Shot-start pressure is assumed as zero and an emperical correction is applied to propellant web.

(c) The value of the ratio of specific heats is taken as unity.

Thus the system failed to give pressure-space curve in reasonable agreement with experiment, as well as when extrapolation was required beyond experimental data available for use.

9. Coppack Approach

The system of Coppack[18] is an improvement of 'Crows' approach, with an intention of obtaining a more-realistic pressure-space curve. The system is further modified by Lacey and Ruston[91,92]. The modified system considers complete Resals energy equation. The shot-start pressure is assumed to be zero and band-engraving is accounted by web adiustments emperically. The mean-value of ratio of specific heats 'γ' is taken as 1.25.

10. Goldie Approach

The system differs slightly from modified system of Coppack[18]. It assumes a shot-start pressure for analysis.

Patnaik[88] made a comparison of 5 methods—RD-38[94] GM II[95], Goldie[19], Hirchfelders[23] and Bennet[96]— and the differences observed had the following salient features.

(a) Kinetic energy (K. E.) of projectile is separately accounted in all methods, except in RD-38. In RD-38, K. E. of shot is accounted for by using a constant gas-temperature, smaller than the explosion-temperature.

(b) The co-volume term is included in all, except in RD-38.

(c) The rate of burning of propellant is assumed to be proportional to the pressure (p) in all except in Bennets method where it has been taken as $p^{2/3}$.

(d) The allowance for band-engraving is made as follows:

(i) In RD-38, propellant-web is reduced.

(ii) In Goldie and GM II, shot-start pressure is assumed.

(iii) In Hirchfelder's and Bennet, shot-start pressure is assumed as $\left(\frac{2500}{d}\right)$ lb/(in)$^{1/3}$ and 2500 lb/in^2 where d is the calibre in inches.

(e) The emperical allowance for heat-loss is not included in Goldie and Bennet whereas the same is included in GM II and Hirchfelders.

By a numerical analysis of a set of firing results in a number of American Naval guns with multitude propellant, Goldie[19] made a comparison of the methods of RD-38, Goldie, Le Duc[57], Hirchfelders[23], and Bennet and concluded that only a slight trend towards greater accuracy of estimation was revealed with better theoretical methods and on the whole the results of the test suggested that the use of single emperical procedure for the comparison of different ballistic systems is unsatisfactory. A better comparison should result, if the methods were used in conjunction with the particular emperical procedures, employed with them in practice.

11. Taylors Approach

The system of Taylors[85] considers three basic equations, involving four variables. The energy equation is derived by assuming the specific volume of solid—propellant is equal to specific co-volume of gas. The secondary energy losses due to unburnt propellant and gas, frictional resistance to the motion of projectile including shot-start pressure and engraving of driving band are accounted by effective mass (M).

$$M = \frac{(1 + \theta)\, W + C/3}{g}$$

where C is the weight of propellant and θ a factor, accounting energy losses due to frictional resistance and rotation. The kinetic energy expended on recoiling parts is neglected. The heat-lost to the barrel is accounted by increasing the ratio of specific heats γ to 1.30. The equation of motion is formulated, using approximate relation among the pressures of breech, base of projectile, and space-mean, based on a special solution of Lagrange problem. The burning rate is regarded as a function of combustion pressure and reacting surface. Since the latter is assumed to be constant, the form-function is not considered. The burning rate coefficient is estimated from experimental firings, using statistical methods. The pressure index is taken as 0.8. The four variables considered are work-done by gas, including energy lost by heating barrel (K), space-mean pressure (P), shot-travel (X) and energy released by the amount of charge-burnt (T). Thus the system comprises two first order differential equations in K and P with T as independent variable, besides a relation between K, P and X.

12. Drozdovs Approach

The equations of Drozdovs[15] system slightly differ from that of Hunt-Hinds. The propellant charge is assumed to burn under an average pressure as per geometrical law of combustion and rate of burning is proportional to the pressure. The composition of combustion products is assumed to be constant. The equivalence equation leads to Resal's energy-equation. The ratio of specific heats γ is taken as the mean γ between gas temperatures at combustion and at muzzle. The secondary work-coefficient ϕ accounts for kinetic energy for projectile-translation (E_1), coupled with secondary energy losses due to rotational (E_2), frictional resistance as a result of translation and rotation (E_3), displacement of gases of charge itself and unburnt propellant (E_4), and recoil (E_5). The energies E_2 to E_5 are expressed in terms of E_1. The numerical value of work-coefficient for classical weapons varies between 1.05 and 1.20, depending upon loading conditions and may exceed these values. Slukhotsky[84] offers a general formula for ϕ.

$$\phi = K + \frac{1}{3}\frac{W}{q}$$

where K is sum of E_1 to E_5 excluding E_4 and $\frac{W}{q}$ is the relative charge mass with respect to projectile mass. The value of K is a function of the type of weapon. The system adopts shot-start model and shot-start pressure is evaluated by static tests. A shot-start pressure of 300 kg/cm^2 is assumed while compiling the tables. The energy expended for heating the walls of barrel, cartride case and projectile is accounted either by increasing the value of γ or by decreasing the propellant force. This problem was studied by Muraur[93] by considering pressure-time and space-time curves. Serevyakov[89] has established a method to compute the heat lost in the absence of these cuves. Based on the general equations of gas-dynamics for one-dimensional unstable gas motion, established by Shkvornikov[90], the relations among pressures at breech, base of projectile and mean-gas pressure have been utilised in deriving equation of motion. The important deduction from equation of motion is that the ratio of P mean to work-coefficient in energy equation is equal to the ratio of pressure at the base of projectile to the resistance offered during its translatory motion (ϕ_1) provided the work done due to recoil is neglected. The numerical value of ϕ_1 is 1.02 for Howitzer. The form-function is a relation between the volumetric burnt portion of propellant (ψ) and the relative thickness of propellant (z) consumed at the same instant. Thus the form-function is a cubic, involving powder constants as coefficients. But the system uses a quadratic with modified coefficients. Extensive work on the function ψ and its uses in internal ballistics has been done by Serevyakov. The initial value of this function ψ_0 is used to estimate shot-start pressure by static tests. In Hunt-Hinds method, the form function is a relation between the portion of charge mass consumed and that of propellant-web remaining at the same instant, whereas in this system the portion of charge mass consumed is related to the portion of web consumed at the same instant. The Drozdovs loading parameter is analogous to central-ballistic parameter of Hunt-Hinds. A variation in Drozdovs solution of internal ballistic equations is suggested by Oppokov[86].

13. Okunevs Approach

The system of Okunevs[16] differs from Drozdovs in reducing the number of variables in the four basic equations. The introduction of four constants *P*, *L*, *V*, *T* indicating pressure, shot-travel, velocity and time under certain assumptions, to get relative variables, has reduced the number of basic variables from seven to four. The propellant parameters namely force, 'f', co-volume, 'α' density, 'δ' and shot-start pressure, p_0 can take any value. The disadvantage is that the main parameters namely expansion, charging, combustion and shot-start pressure do not cover all the cases expected in practice. The introduction of relative variables is associated with other Soviet researchers Drozdovs[97], Oppokov[98], Okunevs[99], Gorokhov, and Sviridov[100].

SECONDARY PROBLEM

14. Introduction

Systems of differential variations in internal ballistics can be studied either qualitatively or quantitatively. The qualitative treatment is associated with Tranter[63]. The quantitative treatment hither-to can be broadly classified into three types namely Emperical, Semi-Emperical and Nearly Exact. The emperical systems of Ikopz[64], Slukhotsky[65], Pidduck[66] and Vickers Armstrong[67] derive little support from internal ballistic theory, but consider important loading parameters with constant numerical values, derived from extensive firings. The semi-emperical systems of Hunt-Hinds[68], Hitchcock[69,70] and Bofors[79] are built up with some important loading parameters, with reference to a certain degree of internal ballistic theory, under certain assumptions. The nearly exact systems of Sugot[71], Tawakley[72,73] Kapur[74,75], Winter[76], Corner[77] and Puchenkin[78] attempt to include most of the important loading parameters, as well as ballistic theory, with, fewer assumptions. Thus the secondary problem so far studied is restricted to monomial variations of loading parameters, with reference to either initial conditions, P_{max}, *MV*, *ABP* or some of them.

15. Statement of Secondary Problem

The secondary problem of internal ballistics, we shall assume, is to follow physical principle of smoothness and continuity which states that small causes produce small effects. Here again these small effects are sub-classified to evaluate the extent of rigidity and effective non-rigidity of any system of primary problem.

Here the rigidity of internal ballistic system is defined as those differential variations in loading parameters that do not significantly change both P_{max} and *MV*. The non-rigidity is the departure in central tendency and dispersion from the system. Thus the effective non-rigidity is the region in which P_{max} and *MV* distribute around the values of the system, due to small differential variations.

Mathematically, the secondary problem can be stated as follows :

Let A_{io} indicate different loading initial conditions for parameters suffixed $i = 1, 2, \ldots\ldots n$. Let $\phi(A_{io})$ be a function of loading conditions and suppose that l parameters have not changed. Then

$(A_{k,0})$ – represents loading conditions that do not vary for values of :

$$k = 1, 2\ldots\ldots l \text{ and } 0 \leqslant l \leqslant n$$

and

$(A_{j,0})$ – represents loading conditions that vary for values of $j = 1, 2, \ldots\ldots (n - l)$.

Let j be varied by small increments and the increments be denoted by $\delta A(k + j)$, 0. Further let ϵ be a positive number satisfying

$$\epsilon_0 < | \phi (A_{i,o}) - \phi [A_{k,o}, A_{(k+j),0} + \delta A_{(k+j),0}] | < \epsilon$$

where ϵ_0 is the function of significance. The function ϵ is dependent on η such that :

$$| \delta A_{(k+j),0} | < \eta; \epsilon(\eta) < \epsilon_0 \text{ and } \left(\frac{d\phi}{dx}\right)^2 << 1$$

The function of significance can be evaluated as follows : If solution set $S = (p, v, T, f, x)$ for any given epoch 't' of internal ballistics is dependent on 'n' initial loading conditions and if an element of Schanges, 'm' of these 'n' conditions, 'm' is a proper sub-set of 'n'. Let further 'l' ($l \leqslant m$) of these conditions do not contribute to this element of S to the significance level, then for any change of 'j' ($n \geqslant j \geqslant l$), conditions, the probability of 'l' is a hypergeometric distribution.

$$P(l) = \frac{C(m, l), C(n - m, j - l)}{C(n, j)}.$$

where $C(r, s) = rC_s$

Firstly an analysis of the work by researchers is made and then the general aspects of this problem are presented.

16. Tranter Approach

The qualitative study of Tranter[63] is with reference to a spatial pressure curve of a standard propellant, manufactured by solvent process, indicating the positional change of P_{max} and ABP due to variations in loading paramters. The study is limited only to the loading paramters of propellant (mass, shape, size and constants), projectile (mass, start pressure) and gun (chamber capacity and bore-area). A good number of important parameters are excluded from the study.

17. Ikopz Approach

As a result of extensive firings conducted at Ikopz[64] the first differential correction formulae of internal ballistics, with numerical constants came into existence.

N. A. Zabudsky and G. P. Kisnemsky[87], were associated with these firings. P_{max} and MV are expressed as functions of propellant (mass, size, volatile matter, temperature), projectile (mass) and gun (chamber capacity). The numerical constants in these formulae were determined with reference to 1902 Model 76 mm gun and its ammunition. As such for loading conditions, different from the conditions of 1902 Model 76 mm gun and its ammunition, these formulae provide approximate values of the variation. A general form of functions of the relative changes in P_{max} and $MV (v_m)$ with respect to relative changes in loading parameters can be expressed in the following manner :

$$\frac{\delta P_{max}}{P_{max}} = m_x \frac{\delta x}{x} \; ; \; \frac{\delta v_m}{v_m} = l_x \frac{\delta x}{x}$$

where 'x' is the loading parameter; 'm_x' and 'l_x' are known as correction factors for pressure and velocity respectively.

18. Slukhotsky Approach

The study of Slukhotsky[65] pertains to propellant-wise and is restricted to loading parameters of propellant (mass, force, loading density, muzzle pressure), projectile (mass) and gun (chamber capacity). Even though these are great step-forward to Ikopz formulae, Slukhotsky has neither considered shot-start pressure, nor internal ballistic equations, nor change in temperature through propellant force. Moreover the experimental factors, connecting the concerned P_{max} and MV, have different values regarding temperature variation *i.e.* the value differs from propellant to propellant. The general formulae adopted in case of change in temperature of charge is

$$\frac{\delta P_{max}}{P_{max}} = m_t \, \delta t^{\circ} \; ; \; \frac{\delta v_m}{v_m} = l_t \, \delta t^{\circ}$$

19. Corner Approach

The study of Corner[77] is restricted to the spatial curves of one particular calibre-projectile primer-propellant combination of an isothermal sytem. The loading parameters considered are propellant (nature, mass, shape, size), projectile (mass) and gun (chamber capacity, shot travel). In all these cases, the basic paramaters have been calculated abintio for ballistic effects of charge and design variables due to small variations. Corner considered the ratio of D/β and assumed the distribution around its mean.

20. Hunt-Hinds Approach

The study of Hunt-Hinds[68] is limited to derive an analytical expression for change in MV in terms of basic propellant parameters (force 'F', burning coefficient β) due to temperature variation only. Two sets of indices to estimate the effect due to small monomial variations, due to Messers Vickers Armstrong Ltd and Pidduck are given with a remark that indices are mean-values and reliable values should be obtained by calculations abinitio.

21. Irwin Roman and Hitchcock Approach

Differential coefficients of Irwin Roman for Le Duc system has been improved by Hitchcock[69,70], based on Bennets system[96], eventhough the fundamental hypothesis differs from Le Duc. The study is limited to parameters of propellant (mass, size, specific energy), projectile (mass) and gun (chamber capacity and shot-travel). Maximum pressure is assumed as a function of propellant quickness, loading density and specific energy. Muzzle velocity is assumed as a function of quickness, loading-density, space ratio and velocity ratio. These independent variables are expressed in terms of differential coefficients of loading parameters of velocity and pressure separately. With the help of two auxillary differential coefficients, algebric quotients of each loading parameter, have been obtained. Thus these formulae are applicable for certain guns, having standard propellant, under standard conditions, and can be used for differential variations from these conditions. Coefficients for other conditions should be computed, as these will give approximate-values.

22. Bofors Approach

Differential coefficients q of Bofors[79] are defined by the expression

$$\frac{dE}{E} = q \frac{dx}{x}$$

By partial derivation of basic formulae, expression for q can be obtained. The variables included in the basic formulae have in most cases exponents that may change somewhat, though rules of variation have not been more precisely determined. Consequently, expressions for q do not become exact enough. As such Bofors constructed emperical formulae with variable coefficients and with a few principal variables. The loading parameters considered are propellant (mass, size, temperature, moisture content), projectile (mass, start pressure), and gun (chamber capacity, bore capacity, shot-travel). Appointed values of differential coefficients of standard propellants for varying loading densities are tabulated. These tables are only applicable to Bofors propellants.

23. Sugot Approach

Secondary tables of Sugot[62] give differential coefficients for variation in initial conditions. The differential coefficients are derived by differentiating the expressions for P_{max} and MV and numerical differentiation of relevant tabulated functions. The loading parameters considered are propellant (vivacity, force, mass), projectile (mass, start pressure) and gun (chamber capacity, shot travel). These tables can be used for different characteristics of propellant, assumed in primary tables, with varying shot-start pressure.

24. Tawakley's Approach

Tawakley[72] study is limited to loading parameters of propellant (mass, size, shape, force, rate of burning coefficient), projectile (mass and start pressure) and gun (chamber capacity, shot-travel, bore-area) with reference to alternate ballistic equations. The central ballistic parameter M is expressed as a pressure ratio of two quantities defined. Tawakley has derived analytical expressions for percentage change in P_{max}, MV and muzzle-energy (ME), due to unit percentage change in any one of the loading parameters. Tawakley[73] further studied and corrected these expressions and compared them with that of Vickers Armstrong[67]. As Tawakley has not used ballistic similitudes, all the partial derivatives are to be calculated in each case.

25. Winter Approach

Winter[76] studied the problem with reference to French ballistic system. He grouped the differential coefficients of 12 parameters of loading conditions in four series – two series for pressure and two series for velocity. One of the two series contains differential coefficients when combustion is complete and the other series, when combustion is incomplete. The four series of differential coefficients are expressed as functions of principal coefficients—four in case of velocity and three in case of pressure. The principal coefficients considered are quickness, shot-start pressure, chamber—volume and volume behind base of projectile at time t. The principal coefficients of finite combustion are consistent with that of incomplete combustion upto a point where combustion takes place. Winter has derived analytical expressions for percentage change in MV, velocity at P_{max}. P_{max}. and ABP due to unit percentage change in any one of the other loading parameters, adopting the method of similitudes.

26. Kapur Approach

Kapur[74,75] study pertains to Hunt-Hinds system, on similar lines as that of Winter[76] but with transformations simpler than that of Winter. Though 18 parameters of loading conditions are listed, only 12 parameters are considered. Nine parameters are expressed in terms of three parameters. Kapur has derived analytical expressions for the percentage change in MV, P_{max} and ABP due to unit percentage change in any one of the other loading parameters. The second and third order partial derivatives are expressed in terms of first rank partial derivatives which can be expressed in terms of 9 basic loading parameters. The effect of ratio of specific heats 'γ', position of ABP and its effect on P_{max} and MV, for the simplified model was also studied. Tables for different parameters were compiled.

27. Puchenkin Approach

Puchenkin[78] has studied the problem with reference to Okunev's system[16]. Eight loading parameters are considered. Complex analytical expressions are obtained, adopting the normal method in ballistics, the principle of which is that P_{max} and MV change only when loading parameters on which P_{max} and MV depend vary. The analytical expressions are obtained by quadratures. The common parameters in these formulae are grouped with additional functions for which tables are compiled. As the auxillary tables have been compiled for narrow limits, they cannot cover all the cases likely to encounter in practice. Correlations, between correction factors, calculated on the basis of main ordnance divisions internal ballistic tables, differ from that of correlations, obtained by Puchenkin[78] only because of Okunev's hypothetical factor (ϕ). If values of correction factors for certain parameters are obtained by experiment, then Puchenkins correlations will permit calculation, under same loading conditions, of parameters that could not be determined experimentally. Puchenkin has considered the effect of temperature on P_{max} and MV separately taking the propellant parameters—propellant force F, specific co-volume (α), velocity of combustion of unit pressure (u_1), heat of explosive transformation, specific weight of propellant (δ) and expansion parameter ($\theta = \gamma - 1$).

28. Analysis of Secondary Problems

In all cases of these studies, the researchers had calculated the variations of P_{max} and MV forms under restricted conditions. These results do not encompass the dynamic motion-variations, nor through insight whether a change in initial parameter changes only the initial conditions, or the motion equations, or introduces a small force, or both. What is the quantitative effect of a particular variation in pressure/velocity space curve throughout is not known. An effort therefore is needed to firstly classify parameters according the effect they cause and secondly to develop variations in the complete space-curve and thirdly to estimate the desired changes in P_{max} and MV.

Work is now in progress to formulate and solve the problem of variations due to initial conditions or parameters.

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Table 1.

SOURCE OF VARIATION

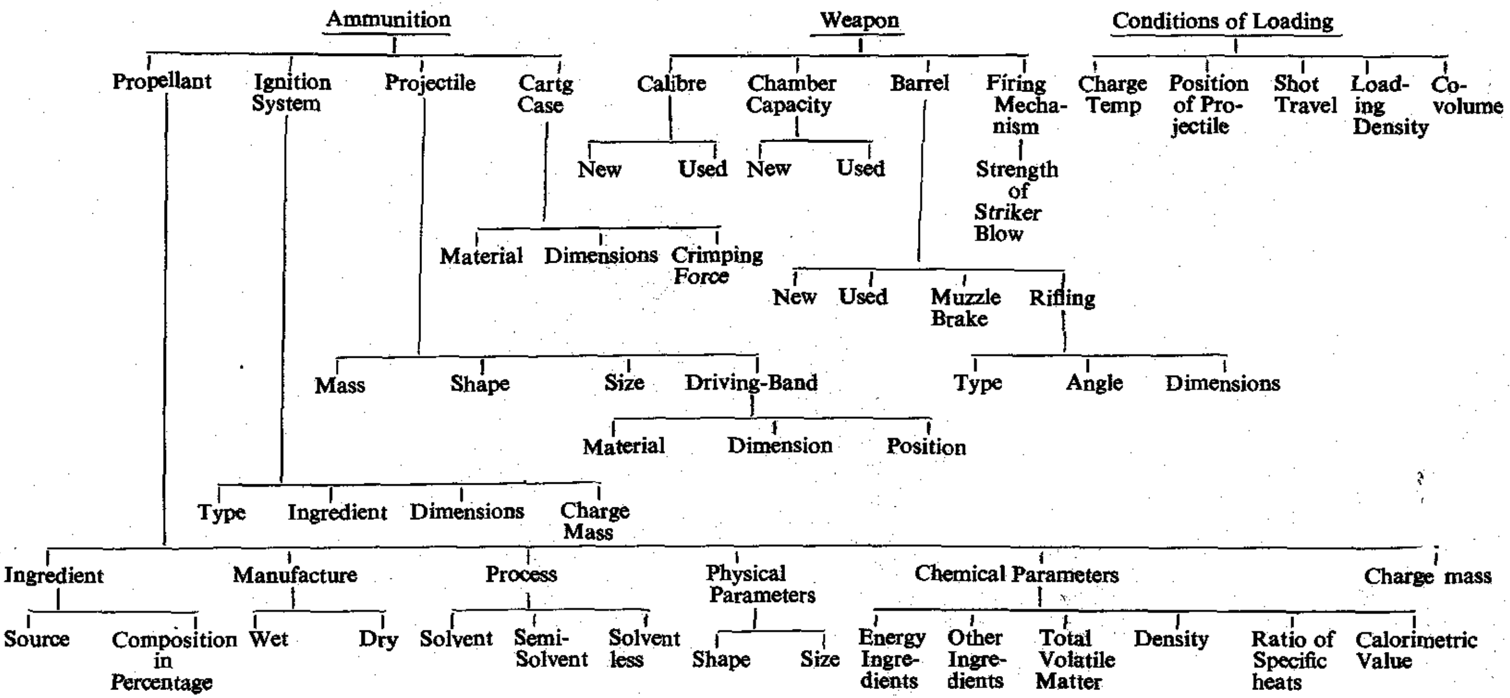

Table 2.

Factor	ξ_0	η_0	ζ_0	z_0	M	B	θ	γ	Δ	Remarks
1. *Propellant*										
Ingredients	—	—	—	√	√	√	—	√	√	
Source of ingredients	—	—	—	√	√	√	—	—	√	
Process	—	—	—	√	√	√	—	—	√	
Type	—	—	—	√	√	√	—	√	√	
Shape	—	—	—	√	—	—	√	—	—	
Size	—	—	—	√	√	—	—	—	—	
Moisture/VM	—	—	—	√	√	√	—	—	√	
Density	—	—	—	√	—	√	—	—	√	
Chargemass	—	—	—	√	√	√	—	—	√	
Calorimetric Value	—	—	—	√	√	√	—	√	√	
2. *Ignition system*	—	—	—	—	—	—	—	—	—	
3. *Projectile*										
Mass	—	—	√	√	√	—	—	—	—	
Shape	—	—	—	—	—	—	—	—	—	
Size	—	—	—	—	—	—	—	—	—	
Driving Band	—	—	√	√	√	—	—	—	—	
4. *Cartridge case*										
Mass,	—	—	—	√	—	√	—	—	—	
Crimping Force	—	—	√	√	—	—	—	—	—	
5. *Weapon*										
Calibre	—	—	√	√	√	√	—	—	—	
Chamber capacity	—	—	—	√	—	√	—	—	√	
Shot travel	—	—	—	—	—	—	—	—	—	
Bore area	—	—	√	√	√	√	—	—	—	
6. *Firing Mechanism*										
Strength of striker blow	—	—	—	—	—	—	—	—	—	
Mass of Weapon	—	—	—	—	—	—	—	—	—	
7. *Charge Temperature*	—	—	—	—	√	—	—	√	√	
8. *Projectile position*	√	—	—	√	—	—	—	—	—	
9. *Density of Loading*	—	—	—	√	√	√	—	—	√	
10. *Co-volume*	—	—	—	√	—	√	—	—	√	

'—' Indicates 'No Change'
'√' Indicates 'Change'

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