

Performance of Optical Systems Employing Linear Polarisation Masks under Partially Polarised Light Illumination

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Received 14 August 1980

Abstract. The modification in the imaging characteristics, achieved by employing linear polarisation masks at different zones of the optical system is considered. This modification technique is comparatively easy to realise into practice. The imaging characteristics of the optical system employing linear polarisation masks illuminated with partially polarised light in terms of point image intensity distributions by making use of coherency matrix approach is studied. The effect of degree of polarisation and polarisation form of the incident light and relative sizes of the three zones, and orientation of the linear polarisation masks has been considered.

1. Introduction

In general, it can be stated that for different applications of an optical system, different imaging characteristics are required. The use of apodisers at the pupil of an optical system for improvement of the imaging characteristics is well known¹. Apodisation can be achieved by employing amplitude, phase and complex amplitude filters on the pupil or on different zones of the pupil of the optical system. In previous papers we have discussed the performance of the optical system employing above type of coatings on the pupil of the optical system²⁻¹². It is very difficult to realise accurately circularly symmetric amplitude filters. However, the fabrication technique is available which approximately realises a circularly symmetric amplitude filter in the form of transparent rings on an opaque background¹³. Besides use of different types of coatings on the pupil of the optical system, there are other ways also of achieving apodisation. Among various other techniques of achieving apodisation, use of polarisation masks on different zones of the optical system is one of the recent techniques which can be comparatively realised into practice easily.

In the present paper, we study the effect of using polarisation masks on different zones of the optical system. This process of achieving apodisation is based on the principle that if the pupil transmission of an optical system is made dependent on the state of polarisation of the in-coming beam by placing polarisation masks at different orientations in different zones, then the contribution to the diffraction pattern would be different for different zones. Therefore, by making suitable choice of the state

of polarisation and with proper orientation of the polarisation masks one can simulate the conditions of different amplitude and phase coatings at different zones of the optical system. Moreover, with an analyser placed in the image plane, the diffraction pattern of the optical system can be continuously modified. The incident beam considered is partially polarised quasi-monochromatic light. The study of partial polarisation is analogous to the study of partial coherence since both are related to statistical nature of incident radiation.

Several matrix methods are available to take the effect of polarisation of the incident beam into account. Jones¹⁴ considered the problem of fully polarised beams and for the first time introduced matrix methods by developing the theory in terms of field components and specifying the polarising optical system by a 2×2 complex matrix. The treatment of partial polarisation employs the concept of correlation functions and coherency matrices¹⁵. Parrent & Roman¹⁶ applied the coherency matrix representation of the analytic signal associated with the electric field to certain polarising optical systems by deriving the transformation law in the quasi-monochromatic case for the coherency matrix in terms of the optical system operators. In general, polarising optical systems are frequency dependent and they introduce different effects on different frequency components of the field. For quasi-monochromatic fields, Wolf¹⁷ has shown that the effect of the system is the same as if X and Y components of the field were themselves effected by the same amount as the mean frequency components. Thus in this approximation all frequency dependent quantities may be evaluated at the mean frequency and operate directly on X and Y components of the field instead of the respectively Fourier frequency components separately. An excellent review of all the matrix methods has been given by O'Neill¹⁸. A comprehensive review of different types of polarisers, retarders etc. is that of Bennet & Bennet¹⁹ and Shurcliff²⁰. Most useful polarisers available employ dichroism effects. These polarisers come usually in sheet form and they are thin, light in weight and rugged and can be made in any desired shape.

The performance of the optical system masked with three polarisation masks in terms of image intensity distributions using coherency matrix formulations is studied. The effect of degree of polarisation and polarisation form of the incident illumination and orientation of the analyser has also been considered. Besides the image intensity distributions, the polarisation form and degree of polarisation which characterise a beam of partially polarised beam have also been worked out.

2. Theory

In this section we derive the results of the image intensity distribution of the optical system masked by three linear polarisation masks P_1, P_2, P_3 placed at orientations $\theta_1, \theta_2, \theta_3$ in inner, central and outer zones respectively. Let the radius of the full circular aperture of the optical system R is unity and $R_1 = \epsilon_1, R_2 = \epsilon_2$, are the outer radii of the central and inner zones respectively. The general layout of the optical system divided into three zones and coordinate axes used is shown in Fig. 1. Here (x, y) define the cartesian coordinates and (r, θ) as polar coordinates in the focal plane

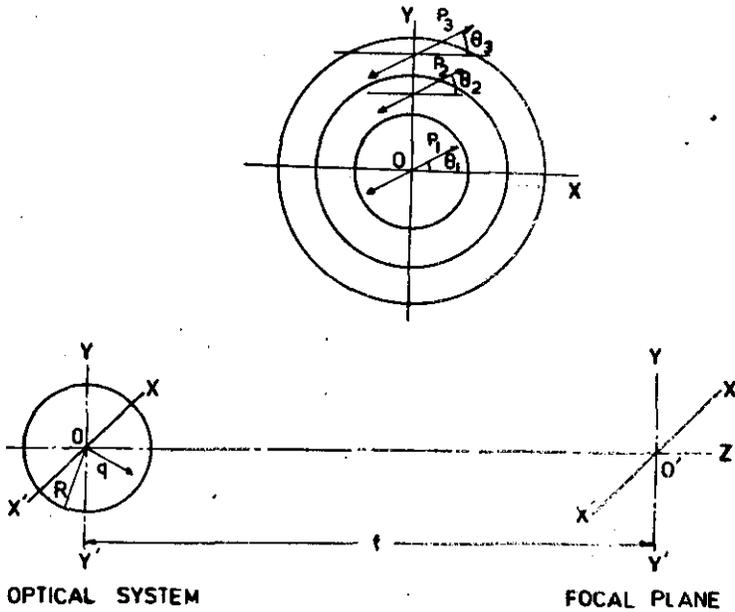


Figure 1. Schematic diagram for coordinates system.

of the optical system. We also define a reduced coordinate $v = \frac{kr}{f}$, where $r = (x^2 + y^2)^{1/2}$, $k = \frac{2\pi}{\lambda}$ and f is the focal length of the optical system.

Let us consider a uniform transverse electric (TE) plane wave, which is partially polarised, is incident on the above optical system. To deal with the propagation of partially polarised light through polarising optical system, coherency matrix formulation has been employed with the state of the polarisation is represented by coherency matrix J and the optical system by its Jones matrix P . The coherency matrix J° of the emergent wave is obtained by matrix multiplication²¹.

$$J^\circ = PJ^iP^+ \tag{1}$$

Where J^i represents the coherency matrix of the incident beam. The coherency matrix J is defined as ;

$$J = \langle E \times E^+ \rangle = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} \tag{2}$$

Where E^+ is the Hermitian conjugate of E and is the row matrix. The sharp brackets denote statistical averaging. The coherency matrix J of partially polarized wave can be expressed as sum of the coherency matrices of totally polarised and unpolarised beam,

$$J = J^{ip} + J^{up} \tag{3}$$

Further, the coherency matrix of totally polarised beam J^{tp} having intensity unity, azimuth θ' and ellipticity ϵ' is expressed as²¹

$$J^{tp} = \frac{1}{2} \begin{bmatrix} (1 + \cos 2\theta' \cos 2\epsilon') & (\sin 2\theta' \cos 2\epsilon' - i \sin 2\epsilon') \\ (\sin 2\theta' \cos 2\epsilon' + i \sin 2\epsilon') & (1 - \cos 2\theta' \cos 2\epsilon') \end{bmatrix} \quad (4)$$

For unpolarised light of intensity unity, the coherency matrix J^{up} is expressed as²¹

$$J^{up} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

The coherency matrix of partially polarised light may be obtained by adding P' times Eqn. (4) and $(1 - P')$ times Eqn. (5)

$$J = \frac{1}{2} \begin{bmatrix} (1 + P' \cos 2\theta' \cos 2\epsilon') & P'(\sin 2\theta' \cos 2\epsilon' - i \sin 2\epsilon') \\ P'(\sin 2\theta' \cos 2\epsilon' + i \sin 2\epsilon') & (1 - P' \cos 2\theta' \cos 2\epsilon') \end{bmatrix} \quad (6)$$

Let J_{xx} , J_{xy} , J_{yx} and J_{yy} are the elements of matrix J in which E_x , E_y are the two components of the Jones vector E representing the incident vector field. On passing through the optical system, the incident beam is differently modified in the three zones and let E_1^p , E_2^p , E_3^p represent the Jones vector in the inner, central and outer zones respectively.

A polariser whose transmission axis makes an angle θ with X axis is represented by the Jones matrix P of the following form²¹

$$P(\theta) = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \quad (7)$$

This equation represents corresponding Jones matrices for P_1 , P_2 and P_3 when θ is replaced by θ_1 , θ_2 and θ_3 respectively. The modified Jones vector E_1^p , E_2^p and E_3^p are related to E and P_s in the following way²¹

$$\begin{aligned} E_1^p &= \begin{bmatrix} \cos^2 \theta_1 & \sin \theta_1 \cos \theta_1 \\ \sin \theta_1 \cos \theta_1 & \sin^2 \theta_1 \end{bmatrix} \times \begin{bmatrix} E_x \\ E_y \end{bmatrix} \\ &= (E_x \cos \theta_1 + E_y \sin \theta_1) \begin{bmatrix} \cos \theta_1 \\ \sin \theta_1 \end{bmatrix}, \\ E_2^p &= P_2 E = (E_x \cos \theta_2 + E_y \sin \theta_2) \times \begin{bmatrix} \cos \theta_2 \\ \sin \theta_2 \end{bmatrix} \text{ and} \\ E_3^p &= P_3 E = (E_x \cos \theta_3 + E_y \sin \theta_3) \times \begin{bmatrix} \cos \theta_3 \\ \sin \theta_3 \end{bmatrix} \end{aligned} \quad (8)$$

Now we determine the contribution in the Fraunhofer diffraction plane from the three zones. Let E_1^d , E_2^d and E_3^d be the contributions to the Jones vector in this plane corresponding to E_1^p , E_2^p and E_3^p in the pupil plane and these are related in the following way²¹

$$\begin{aligned}
 E_1^d &= \epsilon_2 \frac{J_1(v\epsilon_2)}{v} E_1^p, \\
 E_2^d &= \left[\epsilon_1 \frac{J_1(v\epsilon_1)}{v} - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right] E_2^p \text{ and} \\
 E_3^d &= \left[\frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right] E_3^p
 \end{aligned} \tag{9}$$

The total coherency matrix J^d may be written in terms of E_1^d , E_2^d and E_3^d as

$$\begin{aligned}
 J^d &= \langle (E_1^d + E_2^d + E_3^d) \times (E_1^d + E_2^d + E_3^d)^+ \rangle \\
 &= J_{11}^d + J_{22}^d + J_{33}^d + J_{12}^d + J_{21}^d + J_{23}^d + J_{32}^d + J_{13}^d + J_{31}^d,
 \end{aligned}$$

Where

$$J_{mn}^d = \langle E_m^d \times E_n^{d+} \rangle \tag{10}$$

The expression for J_{11}^d can be written as

$$\begin{aligned}
 J_{11}^d &= \langle E_1^d \times E_1^{d+} \rangle \\
 &= \langle E_1^p \times E_1^{p+} \rangle E_2^2 \frac{J_1^2(v\epsilon_2)}{v^2} \\
 &= A_1 \begin{bmatrix} \cos^2 \theta_1 & \sin \theta_1 \cos \theta_1 \\ \sin \theta_1 \cos \theta_1 & \sin^2 \theta_1 \end{bmatrix} \epsilon_2^2 \frac{J_1^2(v\epsilon_2)}{v^2}
 \end{aligned} \tag{11}$$

Where

$$A_1 = J_{xx} \cos^2 \theta_1 + J_{yy} \sin^2 \theta_1 + (J_{xy} + J_{yx}) \sin \theta_1 \cos \theta_1.$$

In a similar way, expressions for J_{22}^d , J_{33}^d , J_{12}^d etc can be written as

$$\begin{aligned}
 J_{22}^d &= A_2 \begin{bmatrix} \cos^2 \theta_2 & \sin \theta_2 \cos \theta_2 \\ \sin \theta_2 \cos \theta_2 & \sin^2 \theta_2 \end{bmatrix} \times \left[\epsilon_1 \frac{J_1(v\epsilon_1)}{v} - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right]^2, \\
 J_{33}^d &= A_3 \begin{bmatrix} \cos^2 \theta_3 & \sin \theta_3 \cos \theta_3 \\ \sin \theta_3 \cos \theta_3 & \sin^2 \theta_3 \end{bmatrix} \times \left[\frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right]^2, \\
 J_{12}^d &= A_4 \begin{bmatrix} \cos \theta_1 \cos \theta_2 & \sin \theta_2 \cos \theta_1 \\ \cos \theta_2 \sin \theta_1 & \sin \theta_1 \sin \theta_2 \end{bmatrix} \times \left[\left\{ \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \right. \\
 &\quad \left. \times \left\{ \epsilon_1 \frac{J_1(v\epsilon_1)}{v} - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \right], \\
 J_{21}^d &= B_4 \begin{bmatrix} \cos \theta_1 \cos \theta_2 & \sin \theta_1 \cos \theta_2 \\ \cos \theta_1 \sin \theta_2 & \sin \theta_1 \sin \theta_2 \end{bmatrix} \times \left[\left\{ \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \right. \\
 &\quad \left. \times \left\{ \epsilon_1 \frac{J_1(v\epsilon_1)}{v} - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \right],
 \end{aligned}$$

$$\begin{aligned}
 J_{23}^d &= A_5 \begin{bmatrix} \cos \theta_2 \cos \theta_3 & \cos \theta_2 \sin \theta_3 \\ \sin \theta_2 \cos \theta_3 & \sin \theta_2 \sin \theta_3 \end{bmatrix} \times \left[\left\{ \epsilon_1 \frac{J_1(v)}{v} - \epsilon_2 \right. \right. \\
 &\quad \left. \left. \times \frac{J_1(v\epsilon_2)}{v} \right\} \times \left\{ \frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right\} \right], \\
 J_{32}^d &= B_5 \begin{bmatrix} \cos \theta_2 \cos \theta_3 & \sin \theta_2 \cos \theta_3 \\ \cos \theta_2 \sin \theta_3 & \sin \theta_2 \sin \theta_3 \end{bmatrix} \times \left[\left\{ \frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right\} \right. \\
 &\quad \left. \times \left\{ \epsilon_1 \frac{J_1(v\epsilon_1)}{v} - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \right], \\
 J_{31}^d &= A_6 \begin{bmatrix} \cos \theta_3 \cos \theta_1 & \cos \theta_3 \sin \theta_1 \\ \sin \theta_3 \cos \theta_1 & \sin \theta_3 \sin \theta_1 \end{bmatrix} \times \left[\left\{ \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \right. \\
 &\quad \left. \times \left\{ \frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right\} \right], \\
 J_{13}^d &= B_6 \begin{bmatrix} \cos \theta_3 \cos \theta_1 & \cos \theta_1 \sin \theta_3 \\ \sin \theta_1 \cos \theta_3 & \sin \theta_1 \sin \theta_3 \end{bmatrix} \times \left[\left\{ \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \right. \\
 &\quad \left. \times \left\{ \frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right\} \right] \quad (12)
 \end{aligned}$$

Where

$$\begin{aligned}
 A_2 &= [J_{xx} \cos^2 \theta_2 + J_{yy} \sin^2 \theta_2 + (J_{xy} + J_{yx}) \times \sin \theta_2 \cos \theta_2], \\
 A_3 &= [J_{xx} \cos^2 \theta_3 + J_{yy} \sin^2 \theta_3 + (J_{xy} + J_{yx}) \times \sin \theta_3 \cos \theta_3], \\
 A_4 &= [J_{xx} \cos \theta_1 \cos \theta_2 + J_{yy} \sin \theta_1 \sin \theta_2 + J_{yx} \sin \theta_2 \cos \theta_1 \\
 &\quad + J_{xy} \cos \theta_2 \sin \theta_1], \\
 B_4 &= [J_{xx} \cos \theta_1 \cos \theta_2 + J_{yy} \sin \theta_1 \sin \theta_2 + J_{xy} \sin \theta_1 \cos \theta_2 \\
 &\quad + J_{yx} \cos \theta_1 \sin \theta_2], \\
 A_5 &= [J_{xx} \cos \theta_2 \cos \theta_3 + J_{yy} \sin \theta_2 \sin \theta_3 + J_{xy} \sin \theta_3 \cos \theta_2 \\
 &\quad + J_{yx} \sin \theta_2 \cos \theta_3], \\
 A_6 &= [J_{xx} \cos \theta_1 \cos \theta_3 + J_{yy} \sin \theta_1 \sin \theta_3 + J_{xy} \sin \theta_1 \cos \theta_3 \\
 &\quad + J_{yx} \sin \theta_3 \cos \theta_1], \\
 B_5 &= [J_{xx} \cos \theta_2 \cos \theta_3 + J_{yy} \sin \theta_2 \sin \theta_3 + J_{xy} \sin \theta_2 \cos \theta_3 \\
 &\quad + J_{yx} \sin \theta_3 \cos \theta_2],
 \end{aligned}$$

and

$$\begin{aligned}
 B_6 &= [J_{xx} \cos \theta_1 \cos \theta_3 + J_{yy} \sin \theta_3 \sin \theta_1 + J_{xy} \sin \theta_3 \cos \theta_1 \\
 &\quad + J_{yx} \cos \theta_3 \sin \theta_1].
 \end{aligned}$$

If however an analyser is placed at an azimuth θ , the total coherency matrix in the image plane according to Eqn. (1) will be given by

$$J^a = P \cdot J^d P^+ \quad (13)$$

The coherency matrix J^a is the sum of nine component matrices J_{ij}^a . The expression for J_{11}^a comes out to be

$$J_{11}^a = A_1 \epsilon_2^2 \frac{J_1^2(v\epsilon_2)}{v^2} \cos^2(\theta - \theta_1) \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \quad (14)$$

In a similar way, expressions for other component matrices can be expressed. The total intensity in the image plane is the sum of the traces of J_{11}^a , J_{22}^a , J_{33}^a etc. and is given by,

$$\begin{aligned} B(v, \theta) = & \left[A_1 \epsilon_2^2 \frac{J_1^2(v\epsilon_2)}{v^2} \cos^2(\theta - \theta_1) \right. \\ & + A_2 \left\{ \epsilon_1 \frac{J_1(v\epsilon_1)}{v} - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\}^2 \cos^2(\theta - \theta_2) \\ & + A_3 \left\{ \frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right\}^2 \cos^2(\theta - \theta_3) \\ & + (A_4 + B_4) \left\{ \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \times \left\{ \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right. \\ & \left. - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \times \cos(\theta - \theta_1) \cos(\theta - \theta_2) \\ & + (A_5 + B_5) \left\{ \epsilon_1 \frac{J_1(v\epsilon_1)}{v} - \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \\ & \times \left\{ \frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right\} \times \cos(\theta - \theta_2) \cos(\theta - \theta_3) \\ & + (A_6 + B_6) \left\{ \frac{J_1(v)}{v} - \epsilon_1 \frac{J_1(v\epsilon_1)}{v} \right\} \times \left\{ \epsilon_2 \frac{J_1(v\epsilon_2)}{v} \right\} \\ & \left. \times \cos(\theta - \theta_3) \cos(\theta - \theta_1) \right], \end{aligned}$$

where $A_1, A_2, A_3, A_4, A_5, A_6, B_4, B_5$ and B_6 are already defined.

3. Results and Discussion

We have computed the image intensity distributions of an optical system masked by three polarisation masks P_1, P_2 and P_3 placed at azimuths θ_1, θ_2 and θ_3 . The effect of degree of polarisation P' , the azimuth θ' and ellipticity angle ϵ' of the incident beam on image intensity distribution has been investigated. The computations have also been made for different orientation of the analyser.

The effect of degree of polarisation P' in case of elliptically polarised light (EPL) on the image intensity distribution has been shown in Fig. 2. It is observed that as

the value of P' is increased, the value of half-width at half-height of the central maxima is reduced and energy diffracted into the secondary maxima remains almost unchanged. Therefore, improvement in the images is expected as regards two point resolution of points having nearly equal brightness. Fig. 3 shows the results with

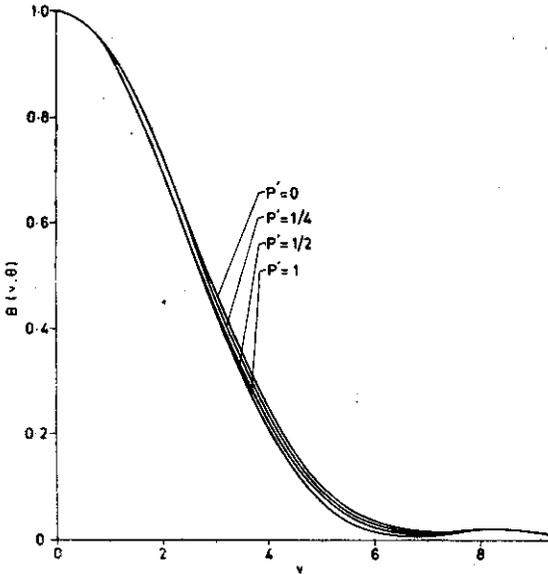


Figure 2. Intensity distribution at $\theta = \pi/4$ for different degrees of polarisation P' in case of EPL beam in which $\epsilon' = \pm \pi/6$ and $\theta' = 0.0$.

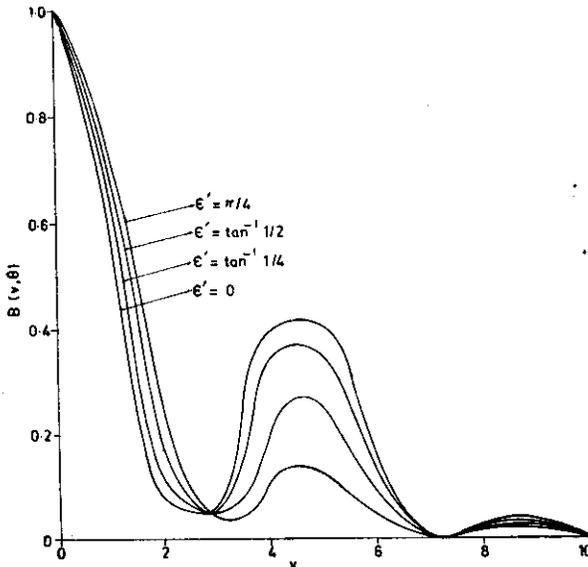


Figure 3. Intensity distribution at $\epsilon_1 = 0.60$, $\epsilon_2 = 0.40$ and $\theta = \pi/2$ for different values of ϵ' in case of EPL.

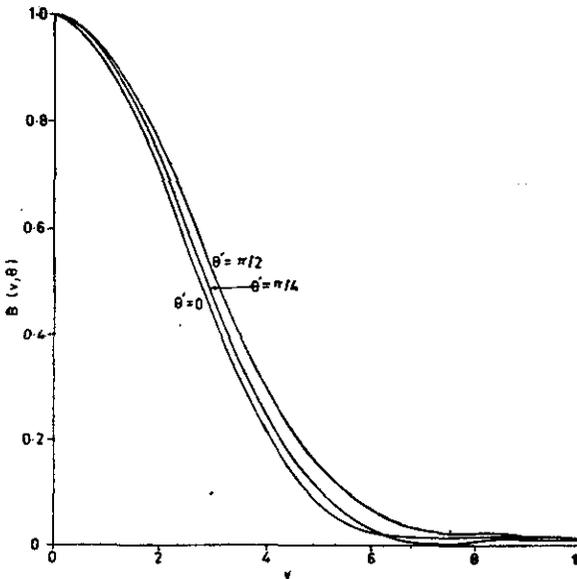


Figure 4. Intensity distribution at $\theta = \pi/4$ for different values of azimuth θ' in case of EPL in which $P' = 1.0$ and $\epsilon' = \pm \pi/6$.

variation of ellipticity angle ϵ' of the ellipse of polarisation of the incident illumination at $\theta = \pi/4$ in case of fully polarised beam. It is noticed that as the value of ϵ' is decreased in steps, the width of the central maxima is reduced and energy is diffracted into secondary maxima which result in an increase in two point resolution. Fig. 4 represents the results of the image intensity distributions for different values of azimuth angle θ' in case of EPL beam. The system would give better two point resolution at $\theta = 0$. On the other hand, the system would give better contrast in the resolved details at $\theta' = \pi/4$. It means that the azimuth angle also plays an important role in modifying the diffraction pattern.

Fig. 5 represents the results of image intensity distributions at θ_1, θ_2 and $\theta_3 = \pi/4, 0$ and $3\pi/4$ respectively for different values of the analyser θ in case of EPL beam. By varying the value of θ , the diffraction pattern is effectively changed. The central maxima is broadened and energy diffracted into secondary maxima is reduced at $\theta = \pi/4$. However at $\theta = 3\pi/4$, the central maxima is minimised. On comparison of the results shown in Figs. 5 and 6, it becomes clear that the performance of the optical system depends too much upon the orientation of the polarisation masks.

Besides the image intensity distributions, the polarisation form θ_0 and ϵ_0' and degree of polarisation P_0 represent the physically significant information of the out-put beam that characterises a beam of partially polarised light. The value of θ_0 is the same as the orientation of the analyser θ . Further the value of ϵ_0' comes out zero which means out-coming emergent beam is linearly polarised light. The degree of polarisation of the out-put beam is unity which means totally polarised light. It is due to ideal linear nature of polarisation masks P_1, P_2 and P_3 .

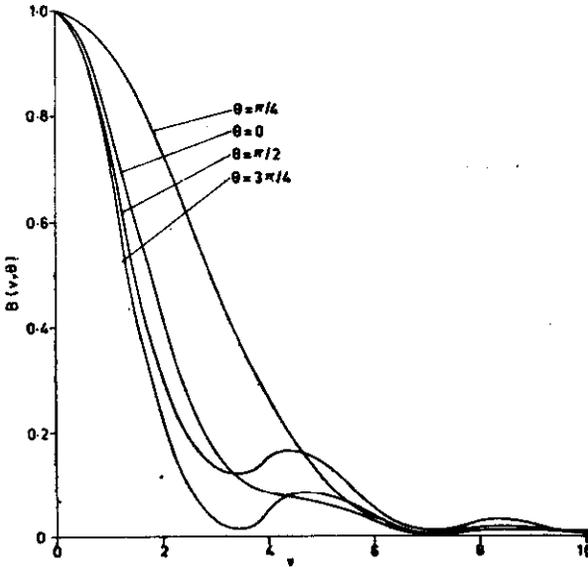


Figure 5. Intensity distribution for different orientation of analyser θ in case of EPL beam in which $P' = 1.0$, $\epsilon' = \pm \pi/6$ and $\theta' = \pi/4$.

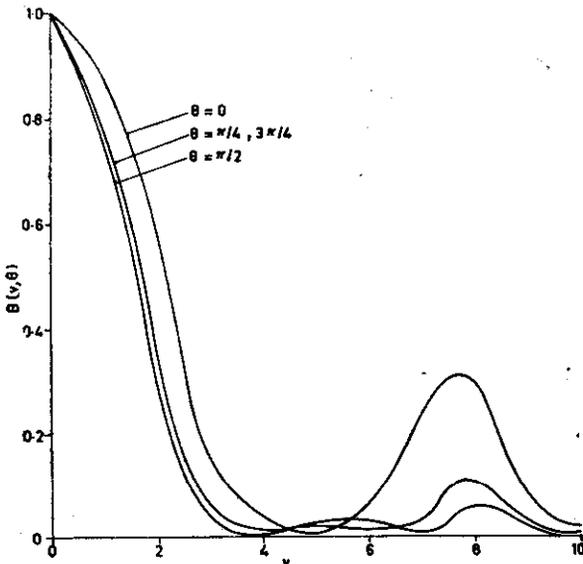


Figure 6. Intensity distribution at $\theta_1 = \pi/2$, $\theta_2 = 0$, $\theta_3 = \pi/2$, $\epsilon_1 = 0.60$, $\epsilon_2 = 0.40$, $\epsilon' = \tan^{-1} \frac{1}{2}$ for different values of θ in case of EPL beam.

4. Conclusion

It may be concluded that by suitable choice of the state of polarisation of the incident beam and with proper orientation of the polarisation masks. One can modify

the performance of the optical system in a desired way. Further, by changing the orientation of the analyser, the performance of the optical system can be altered continuously. This technique is very effective for achieving apodisation for improvement of images required in various situations. These results may find application in optical data processing systems and in various other fields.

Acknowledgements

The author is grateful to Dr. R. Hradaynath, Director IRDE Dehradun and Shri N. N. Seth, Assistant Director IRDE for their interest in this work. The author is also thankful to Shri A. K. Musla for computation and for rendering assistance from time to time.

References

1. Jacquinet, P. & Roizen-Doissier, B., 'Progress in Optics', Wolf, E., Ed. (North Holland, Amsterdam, 1964), Vol. III, Chap. 2.
2. Gupta, S. D., *Proc. Ind. Nat. Sci. Acad.*, A39 (1973), 314.
3. Gupta, S. D., *Ind. J. Pure and Appl. Phys.*, 12 (1974), 133.
4. Gupta, S. D., *J. Opt.*, 4 (1976), 37.
5. Gupta, S. D., *Jap. J. Appl. Phys.*, 15 (1976), 1709.
6. Gupta, S. D., *J. Opt.*, 4 (1975), 32.
7. Gupta, S. D., *Optik*, 41 (1974), 25.
8. Gupta, S. D., *Ind. J. Pure & Appl. Phys.*, 14 (1976), 372.
9. Gupta, S. D., *Ind. J. Pure & Appl. Phys.*, 15 (1977), 188.
10. Gupta, S. D. & Singh, M., *Optica Acta*, 21 (1974), 737.
11. Gupta, S. D., *Ind. J. Pure & Appl. Phys.*, 14 (1975), 983.
12. Gupta, S. D., *Jap. J. Appl. Phys*, 14 (1975), 983.
13. Wiersma, P., *Opt. Acta*, 25 (1978), 917.
14. Jones, R. C., *J. Opt. Soc. Am.*, 46 (1956), 126.
15. Wolf, E., *Nuovo Cimento*, 12 (1954), 884.
16. Parrent, G. & Roman, P., *Nuovo Cimento*, 15 (1960), 370.
17. Wolf, E., *Nuovo Cimento*, 13 (1959), 1165-81.
18. O'Neill, E. L., 'Introduction to Statistical Optics' (Addison Wesley Publishing Co., Inc., London), Chaps. 5, 6 and 8, 1963.
19. Bennet, J. M., & Bennet, H. E., 'Hand Book of Optics', Driscoll, W. G., & Vahugan, W., Eds. (Maggaw-Hill, New York), Chap. 10.
20. Shurcliff, W. A., 'Polarised Light' (Harvard University, Cambridge, Massachusetts), 1962.
21. Azzam, R. M. A. & Bashara, N. M., 'Ellipsometry and Polarised Light', (North Holland Publishing Co., Amsterdam, New York), 1977.