CIRCULAR NOMOGRAM

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The paper describes constructional details of a circular nomogram designed for solving equations of the forms :

$$\phi(p) . \psi(q) = \zeta(s) . \mu(t)$$

$$\prod_{i=1}^{n} \phi_i(x_i) = \prod_{j=1}^{n \text{ or } n-1} \psi_j(y_j)$$

Nomogram[1—5] is an easily understood graphical aid which provides convenient means of solving equations. For technologists it is a very useful tool because it helps avoiding repeated calculations and facilitates rapid computation[6]. A very interesting nomogram was introduced by Whittaker for solving the quadratic equation[1]. He had constructed the three scales with two along two lines at right angles and the third along the semi-circle having one of the straight scales as diameter. It is an instance marked by the absence of parallel scales. The use of curves[1] in a nomogram is traced to the works of D' Ocagne (1884).

A new alignment chart termed 'circular nomogram' has been designed in this paper for solving equations of the form :

$$\phi(p) . \psi(q) = \zeta(s) . \mu(t) \tag{1}$$

Arcs of the same circle are used for all the four scales and an elegant method of construction has been developed. A special slide rule has been designed for a multiple circular nomogram for solving a number of equations of the type (1). Also a generalised circular nomogram has been designed for solving equations of the form

$$\prod_{i=1}^{n} \phi_i(x_i) = \prod_{j=1}^{n \text{ or } n-1} \psi_j(y_j) \tag{2}$$

CIRCULAR NOMOGRAM

Method of Construction

Consider the equation

$$\phi(p) . \psi(q) = \zeta(s) . \mu(t)$$

Construct the scale equations[3]

$$\tan \frac{\alpha}{2} = m_p \, . \, \phi(p) \tag{3}$$

$$\tan \frac{\beta}{2} = m_q \, . \, \psi(q) \tag{4}$$

$$\tan \frac{\gamma}{2} = m_s \, . \, \zeta(s) \tag{5}$$

and

$$\tan \frac{\delta}{2} = m_t \, . \, \mu(t) \tag{6}$$

where

$$m_p \, . \, m_q = m_s \, . \, m_t \tag{7}$$

Draw a circle of convenient radius. Let O be the centre and AB, a diameter. Mark on the circle the points P, Q on one side of AB and S, T on the other such that

$$\angle \underline{AOP} = \alpha, \ \angle \underline{BOQ} = \beta, \ \angle \underline{BOS} = \gamma \text{ and } \angle \underline{AOT} = \delta.$$

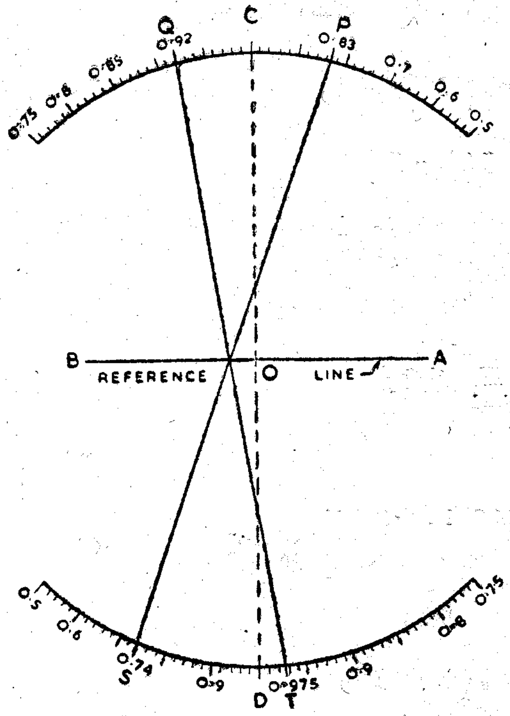

Fig. 1—Circular nomogram.

Now graduate the scales p, q, s and t on the circle using the scale equations (3) to (6), respectively. Choose the scale[2] moduli m_p, m_q, m_s and m_t such that they satisfy (7) and that each scale is restricted to one quadrant of the circle. This restriction can be relaxed if greater accuracy is required.

METHOD OF SOLUTION

Given any three of the variables, say p, q and s, to find the fourth one, t, we join graduations p and s by a straight line and from q draw the straight line through the point of intersection of the line (p, q) with the reference[3] line AB (Fig. 1) to meet the t-scale at the graduation which indicates the desired solution.

Proof—Let the line joining the graduations p and s and the line joining q and t intersect on the reference line AB at X.

It is easy to prove that

$$OX = \frac{a \sin \dfrac{\gamma - \alpha}{2}}{\sin \dfrac{\gamma + \alpha}{2}} = \frac{a \sin \dfrac{\beta - \delta}{2}}{\sin \dfrac{\beta + \delta}{2}}$$

It follows that

$$\tan \frac{\alpha}{2} \cdot \tan \frac{\beta}{2} = \tan \frac{\gamma}{2} \cdot \tan \frac{\delta}{2} \tag{8}$$

From (8), the scale equations (3) to (6) and the modulus relation (7), we have

$$\phi(p) \, . \, \psi(q) = \zeta(s) \, . \, \mu(t).$$

Corollary 1—For solving equations of the form

$$\phi(p) = \zeta(s) \, . \, \mu(t),$$

we set $\psi(q) = 1$. Here the q-scale degenerates into a fixed point.

Corollary 2—If (1) is modified to the form

$$\phi(p) = \psi(q) \, . \, \zeta(s) \, . \, \mu(t)$$

the scale equation (4) changes to

$$\cot \frac{\beta}{2} = \frac{1}{m_q} \cdot \psi(q)$$

Illustrative Example

Construct a circular nomogram for solving the equation $\dfrac{t}{q} = \left(\dfrac{p}{s}\right)^{\frac{1}{2}}$, where p ranges from $0 \cdot 5$ to $1 \cdot 0$, q from $0 \cdot 75$ to $1 \cdot 0$, s from $0 \cdot 5$ to $1 \cdot 0$ and t from $0 \cdot 75$ to $1 \cdot 0$.

Choose moduli $m_p = m_q = m_s = m_t = 1$. This choice facilitates restriction of the scales to half a quadrant each and thereby reduces the width of the chart (Fig. 1).

Given $p = 0 \cdot 83$, $q = 0 \cdot 92$ and $s = 0 \cdot 74$, the chart reads $t = 0 \cdot 975$.

Special Slide Rule for a Multiple Circular Nomogram

A model of a special slide rule designed for solving a number of equationsof the type (1) is illustrated in Fig. 2. It consists of a thin transparent circular celluloid disc with a number of concentric grooves. The four scales for each relation of the type (1) are constructed along the inner edge of a groove. Two fine, slightly stretched, elastic strings (see x, Fig. 2) have their ends fixed at the edge of the disc such that they lie just beneath the disc along two perpendicular diameters which are equally inclined to the reference line AB. The strings can be fixed along any two chords of the nomogram of our choice by means of four tiny smooth forks (rounded) which can slide along the groove around the nomogram, one for each scale, and can be clamped from below (see y, Fig. 2). We obtain a solution when the chords intersect on the reference line.

GENERALISED CIRCULAR NOMOGRAM

We now generalise the above method for solving equations of the form

$$\prod_{i=1}^{n} \phi_i(x_i) = \prod_{i=1}^{n} \psi_i(y_i) \tag{9}$$

Define $\zeta_i\,(z_i)$ by the recurrence relation

$$\zeta_i(z_i)\,.\,\phi_{i+2}\,(x_{i+2}) = \zeta_{i+1}(z_{i+1})\,.\,\psi_{i+1}(y_{i+1})\,, \quad i = 1, 2, 3 \ldots. (n-2),$$

with

$$\zeta_1\,(z_1).\,\psi_1\,(y_1) = \phi_1\,(x_1).\,\phi_2\,(x_2)$$

and

$$\zeta_{n-1}\,(z_{n-1}) = \psi_n\,(y_n)$$

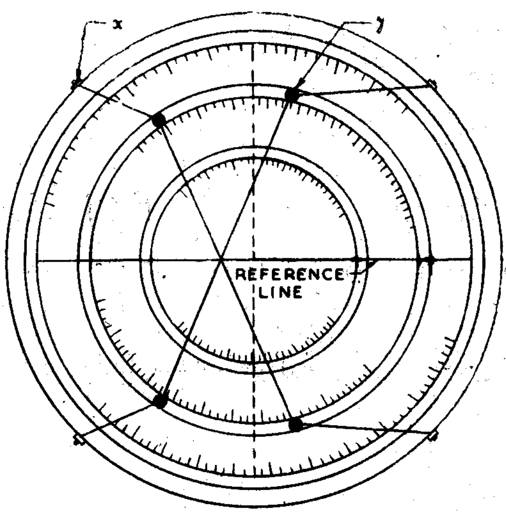

Fig. 2—Special slide rule for a multiple circular nomogram.

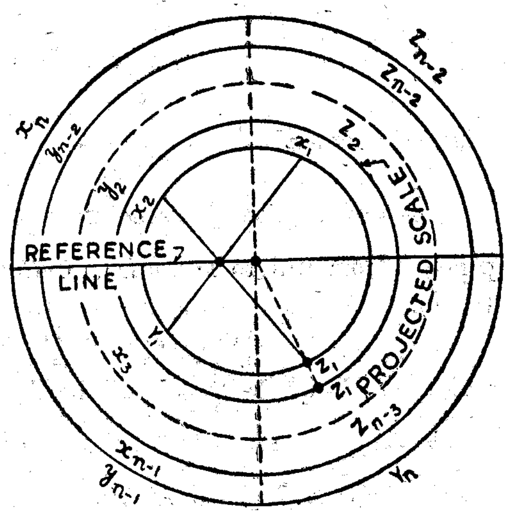

Fig. 3—Generalised circular nomogram showing projection of z_1 from the first circle to the second circle.

Method of Construction

Construct scale equations

$$\tan \frac{\alpha_i}{2} = l_i \ \phi_i(x_i) \, , \quad i = 1, 2, \ldots n,$$

$$\tan \frac{\beta_i}{2} = m_i \ \psi_i(y_i) \, ; \quad i = 1, 2, \ldots n,$$

and

$$\tan \frac{\gamma_i}{2} = k_i \ \zeta_i(z_i) \, , \quad i = 1, 2, \ldots (n-1),$$

where the scale moduli l_i , m_i , k_i are governed by the relation

$$l_{i+2} \, . \, k_i = m_{i+1} \, . \, k_{i+1} \, , \quad i = 1, 2, \ldots (n-2)$$

with

$$k_{n-1} = m_n$$

Construct the scales z_i , x_{i+2} , z_{i+1} , y_{i+1} on a circular nomogram. Repeat this in succession for $i = 0, 1, 2, \ldots\ldots\ldots, (n-2)$ with the scale $z_0 \equiv x_1$ on concentric circles. It has to be noted that each of the scales z_i , $i = 1, 2, \ldots\ldots\ldots\ldots, (n-2)$ has to appear on two of the $(n-1)$ circular nomograms. This can be done by projecting from the centre the scale z_i on the i th nomogram on to the $(i + 1)$th circle. The projection of z_1 from the first circle to the second circle is illustrated in Fig. 3. The number of circles used by admitting adjacent scales, one on each side of an arc can be reduced considerably.

It may be noted that the number of variables in (9) is even. When it is odd, say, $(2n-1)$, we set $\psi_n(y_n) = 1$.

ADVANTAGES

The circular nomogram has the following advantages :

(*i*) All the four scales are constructed on the same circle.

(*ii*) The usual logarithmic scales are completely eliminated.

(*iii*) Only slight modifications are necessary in the circular nomogram when the form of the equation is changed from multiplication to division (vide cor. 2), by virtue of the reciprocal relation between the tangent and the cotangent.

(*iv*) The circular nomogram is a convenient choice for small ranges of the variables.

(*v*) By the method of radial projection on a concentric circle of proportional radius the scale length can be changed to any desired number.

(*vi*) Using adjacent scales the same circle can be used for multiplication or division involving five or six variables.

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