PIEZOMETRIC EFFICIENCY IN AN ORTHODOX GUN

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It has been shown that the muzzle velocity and the piezometric efficiency have greater values in case of a suitable moderated charge than in case of a single charge. The suitable moderated charge should consist of two components such that the pressure driving the shot remains absolutely constant throughout the period when the second component burns, the constant pressure being equal to the pressure at the burnt of the first component.

In a recent paper Ray[1] discussed the possibility of getting high piezometric efficiency in an orthodox gun by the use of a moderated charge. In fact in that paper it was demonstrated that if a moderated charge of two components burns in an orthodox gun the pressure during the period when the second component burns can be kept constant by a suitable choice of the size and shape of the second component of the moderated charge. Constancy of pressure gives a higher value to the piezometric efficiency but the amount of increment was not discussed in that paper. Moreover it will be of interest to know whether this increment in piezometric efficiency is associated with the increase of muzzle velocity. In the present paper it has been tried to throw some light on these points. In fact it has been shown that if firstly piezometric efficiency and muzzle velocity is calculated with a charge of amount C, a size D/β and shape θ and secondly piezometric efficiency and muzzle velocity is calculated with a moderated charge of two components having amounts C_1 and C_2 so that $C = C_1 + C_2$, the first component having size $D_1/\beta_1 = D/\beta$ and shape $\theta_1 = \theta$ (same as the first case) and giving a constant pressure in the second stage of burning (by Ray's method). Then we have observed that piezometric efficiency and muzzle velocity are both greater in the second case. For simplicity in discussion we adopt an isothermal model although Ray considered his problem in the general non-isothermal model. Also Kapur[3,4] discussed the theory of moderated charges in the general non-isothermal model.

BALLISTIC EQUATIONS FOR THE SINGLE CHARGE

The covolume term is entirely neglected and the kinetic energy term in the energy equation is neglected during the burning of the charge. The shot start pressure is taken to be zero. Then the equations are

$$FCZ = p\left(K_0 + Ax - \frac{C}{\delta}\right) \tag{1}$$

$$\omega_1 v \frac{dv}{dx} = Ap \tag{2}$$

$$Z = (1 - f)(1 + \theta f) \tag{3}$$

$$D\frac{df}{dt} = -\beta p \tag{4}$$

Making the following transformation

$$\xi = 1 + \frac{x}{l}$$

$$Al = K_0 - \frac{C}{\delta}$$

$$\eta = \frac{vAD}{FC\beta}$$

$$\zeta = \frac{pAl}{FC} \text{ and } M = \frac{A^2D^2}{FC\beta^2\omega_1}$$

The above equations can be made dimensionless and the reduced equations are,

$$Z = \zeta\xi \tag{5}$$

$$M\zeta = \eta \frac{d\eta}{d\xi} \tag{6}$$

$$Z = (1-f) \quad (1 + \theta f) \tag{7}$$

and

$$\zeta = -\eta \frac{df}{d\xi} \tag{8}$$

The above equations were solved by the method of Crow given in H.M.S.O[2]. The solution gives,

$$f = 1 - \frac{\eta}{M} \tag{9}$$

$$\xi = \left[\frac{(1+\theta)}{(1+\theta f)}\right]^N \tag{10}$$

where

$$N = \frac{M}{\theta}$$

and

$$\zeta = \frac{(1+\theta)^2 (\xi^{1/N} - 1)}{\theta\zeta^{1+2/N}} \tag{11}$$

This expression for ζ can be written as a function of η by (9) and (10). Again the maximum pressure occurs when

$$\eta = \frac{(1+\theta)}{\left(1 + \frac{2\theta}{M}\right)} \tag{12}$$

and the condition for the true maximum is $M > (1 - \theta)$ and the maximum pressure will occur before all burnt of the charge. If $M \leqslant (1 - \theta)$ the maximum pressure will occur at all burnt.

Then the expression for the maximum pressure in non-dimensional form is given by

$$\zeta_{max} = \frac{(1+\theta)^2 (N+1)^{N+1}}{\theta (N+2)^{N+2}} \tag{13}$$

From the given transformation we have

$$p_{max} = \frac{FC}{Al} \frac{(1+\theta)^2 (N+1)^{N+1}}{\theta (N+2)^{N+2}}, \quad M > (1-\theta) \tag{14}$$

Again (13) for $\theta = 0$ reduces to

$$p_{max}\Big|_{\theta=0} = \frac{FC}{Al}\,\frac{1}{e^M}\ , \quad (M > 1)$$

Also $$p_{max} = \frac{FC}{Al}\,\frac{1}{(1+\theta)^N}\ , \quad (M \cdot 1 - \theta) \qquad (15)$$

Since the value of ζ at $f = 0$, i.e. at all burnt is

$$\zeta_B = \frac{1}{(1+\theta)^N}$$

The value of η at all burnt is given by

$$\eta_B = M \text{ and } v_B = \frac{FC\beta}{AD} M \qquad (16)$$

The value of ξ at all burnt is given by

$\xi_B = (1+\theta)^N$ and consequently the distance travelled by the shot at all burnt is

$$x_B = l\left[(1+\theta)^N - 1\right] \qquad (17)$$

Now there are two possibilities :

(i) the shot may leave the muzzle before all burnt of the charge, i.e. $\frac{x_B}{d} > 1$ or

(ii) the shot may leave the muzzle after all burnt of the charge, i.e. $\frac{x_B}{d} < 1$, d being the length of the gun.

Again after all burnt the gas expands adiabatically and the corresponding equations are

$$\omega v \frac{dv}{dx} = Ap \qquad (18)$$

$$pV^\gamma = \text{constant} \qquad (19)$$

where V represents the volume.

(19) can be written as,

$$pA^\gamma (x+l)^\gamma = p_B A^\gamma (x_B + l)^\gamma$$

or $$p = p_B\left(\frac{x_B + l}{x + l}\right)^\gamma \qquad (20)$$

From (18) and (20) we have,

$$v\frac{dv}{dx} = \frac{A}{\omega}\, p_B\, (x_B + l)^\gamma \frac{1}{(x+l)^\gamma}$$

Integrating one gets

$$v^2 = v_B{}^2 + \frac{2A}{\omega} \frac{p_B}{1-\gamma} (x_B + l)^\gamma \left[(x+l)^{1-\gamma} - (x_B + l)^{1-\gamma} \right]$$

Since $x = x_B$ when $v = v_B$

Putting of the value of x_B, v_B and p_B from (15) to (17) in the above formula for v^2, we have

$$v^2 = \frac{F^2C^2\beta^2}{A^2 D^2} M^2 + \frac{2A}{\omega(1-\gamma)} \frac{FC}{Al(1+\theta)^N} \{l(1+\theta)^N\}^\gamma \times$$
$$\times \left[(x+l)^{1-\gamma} - \{l(1+\theta)^N\}^{1-\gamma} \right]$$

If V be the muzzle velocity of the shot we have

$$V^2 = \frac{F^2C^2\beta^2}{A^2D^2} M^2 + \frac{2FC}{\omega(1-\gamma)\{l(1+\theta)^N\}^{1-\gamma}} \times$$
$$\times \left[(d+l)^{1-\gamma} - \{l(1+\theta)^N\}^{1-\gamma} \right]$$

The simplified equation for the muzzle velocity is given by

$$V^2 = \frac{FC}{\omega_1} \left[M + \frac{2}{1-\gamma} \left\{ \left(\frac{1 + \frac{d}{l}}{(1+\theta)^N} \right)^{1-\gamma} - 1 \right\} \right] \tag{21}$$

If $\bar{p}$ be the mean pressure which applied to the shot base for the total shot travel would give the observed muzzle velocity, then the corresponding equation connecting $\bar{p}$ is

$$\tfrac{1}{2}\omega V^2 = Ad\bar{p}$$

or

$$\bar{p} = \frac{\omega V^2}{2Ad} \tag{22}$$

putting the value of V^2 from (21) we have

$$\bar{p} = \frac{FC}{2Ad} \left[M + \frac{2}{1-\gamma} \left\{ M + \frac{2}{1-\gamma} \quad \left\{ \left(\frac{1 + \frac{d}{l}}{(1+\theta)^N} \right)^{1-\gamma} - 1 \right\} \right] \tag{23}$$

The piezometric efficiency is defined to be the ratio of the mean pressure to the maximum pressure.

Hence from (23) and (14) when

$$M > (1 - \theta)$$

$$\text{Piezometric efficiency} = \frac{\bar{p}}{p_{max}}$$

$$= \frac{l}{2d} \frac{\theta (N+2)^{N+2}}{(1+\theta)^2 (N+1)^{N+1}} \left[M + \frac{2}{1-\gamma} \left\{ \left(\frac{1+\frac{d}{l}}{(1+\theta)^N} \right)^{1-\gamma} - 1 \right\} \right]$$

$$\text{for } \left(\frac{x_B}{d} < 1 \right) \quad (24)$$

and when $M \leqslant (1-\theta)$ from (23) and (15)

$$\text{Piezometric efficiency} = \frac{l}{2d} (1+\theta)^N \left[M + \frac{2}{1-\gamma} \left\{ \left(\frac{1+\frac{d}{l}}{(1+\theta)^N} \right)^{1-\gamma} - 1 \right\} \right] \quad (25)$$

Also the muzzle velocity in non-dimensional form is denoted by η_m which from (21) and the transformation formula reduces to

$$\eta_m^2 = M \left[M + \frac{2}{1-\gamma} \left\{ \left(\frac{1+\frac{d}{l}}{(1+\theta)^N} \right)^{1-\gamma} - 1 \right\} \right] \quad (26)$$

$$\text{for } \left(\frac{x_B}{d} < 1 \right)$$

All these formulae for piezometric efficiency and muzzle velocity will hold good if $\frac{x_B}{d} < 1$ but if $\frac{x_B}{d} > 1$, i.e. the shot leaves the muzzle before all burnt of the charge, then from (10) for $x = d$ the value of f is to be calculated and then $\eta_m = M(1-f)$ gives the corresponding muzzle velocity. The mean pressure is then determined. For the calculation of maximum pressure in this case, the cases we have considered that f corresponding to ejection of the shot is less than f corresponding to $p_{max.}$ under the condition $M > (1-\theta)$ and therefore the maximum pressure was calculated by (14). In other cases the value of maximum pressure can be obtained numerically from the ballistic equations. The piezometric efficiency can be calculated from the definition. The numerical values of the muzzle velocity and piezometric efficiency are given for $M = 1$ and $M = 2$ for different values of l/d and θ.

MODERATED CHARGE OF TWO COMPONENTS

Ballistic Equations when the First Component Burns

Under the same conditions as in the single charge, the equations become

$$F_1 C_1 Z_1 = p \left[K_0 + Ax - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} \right] \quad (27)$$

$$\omega_1 v \frac{dv}{dx} = Ap \quad (28)$$

$$D_1 \frac{df_1}{dt} = -\beta_1 p \quad (29)$$

$$Z_1 = (1-f_1)(1+\theta_1 f_1) \quad (30)$$

These equations are to be solved with the initial conditions

$$x = 0,\ v = 0, \quad p = 0 \text{ at } f_1 = 1 \text{ i.e. } Z_1 = 0$$

We make the following usual substitution

$$\xi = 1 + \frac{x}{l_1}$$

$$Al_1 = K_0 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}$$

$$\eta = v \frac{AD_1}{F_1 C_1 \beta_1}$$

$$\zeta = p \frac{Al_1}{F_1 C_1}$$

and

$$M_1 = \frac{A^2 D_1^2}{F_1 C_1 \beta_1^2 \omega_1}$$

The above equations reduce to

$$\left.\begin{aligned} Z_1 &= \zeta \xi \\ M_1 \zeta &= \eta \frac{d\eta}{d\xi} \\ \zeta &= -\eta \frac{df_1}{d\xi} \\ Z_1 &= (1 - f_1)(1 + \theta_1 f_1) \end{aligned}\right\} \quad (31)$$

Solving the above equations as in the case of single charge we find that the maximum pressure occurs when

$$\eta_1 = \frac{N_1 (1 + \theta_1)}{N_1 + 2} \qquad \text{when } N_1 = \frac{M_1}{\theta_1}$$

and the condition for true maximum is

$$M_1 > (1 - \theta_1)$$

If $M_1 \leqslant (1 - \theta_1)$, the maximum pressure occurs at all burnt of the component charge. Then the value of the maximum pressure is given by

$$p_{max} = \frac{F_1 C_1}{A l_1} \frac{(1 + \theta_1)^2 (N_1 + 1)^{N_1 + 1}}{\theta_1 (N_1 + 2)^{N_1 + 2}} \quad (32)$$

The value of ζ at $f_1 = 0$, i.e. at burnt of the first component charge is given ζ_{B1} where

$$\zeta_{B1} = \frac{1}{(1 + \theta_1)^{N_1}} \quad (33)$$

Hence the value of the pressure p at $f_1 = 0$ is p_{B1} where

$$p_{B1} = \frac{F_1 C_1}{Al_1} \frac{1}{(1 + \theta_1)^{N_1}} \quad (34)$$

Also x_{B1} the value of x where the first component charge burns out and v_{B1} the corresponding value of the velocity, then

$$v_{B1} = \frac{F_1 C_1 \beta_1}{AD_1} M_1 \quad (35)$$

$$x_{B1} = l_1 \left[(1 + \theta_1)^{N_1} - 1 \right] \quad (36)$$

Ballistic Equations when the Second Component Burns

The equations are :

$$F_1C_1 + F_2C_2Z_2 = p\,[K_0 + Ax - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}] \qquad (37)$$

$$\omega_1 \frac{dv}{dt} = Ap \qquad (38)$$

$$D_2 \frac{df_2}{dt} = -\beta_2 p \qquad (39)$$

$$Z_2 = (1 - f_2)\,(1 + \theta_2 f_2) \qquad (40)$$

We are to obtain the solution of these equations with initial conditions $x = x_{B1}, v = v_{B1}$ at $f_2 = 1$. x_{B1}, v_{B1}, p_{B1} are the value. of x, v, p when the first component just burns out.

Let us assume that the solution of the above equation is possible with

$$p = p_{B1} \qquad (41)$$

and seek conditions so that the solutions may give $x = x_{B1}$, $v = v_{B1}$ at $f_2 = 1$ and the systems of (37) to (40) may remain consistent for the solution $p = p_{B1}$.

From (38) and (39)

$$\frac{dv}{df_2} = -\frac{AD_2}{\beta_2\omega_1} \qquad (42)$$

or

$$\frac{d}{df_2}\left[\frac{df_2}{dt}\,,\ \frac{dx}{df_2}\right] = -\frac{AD_2}{\beta_2\omega_1}$$

which by (39) and (41) reduces to

$$\frac{d^2x}{df_2^2} = \frac{AD_2^2}{\beta_2^2\omega_1 p_{B1}}\,. \qquad (43)$$

From (42)

$$v = \frac{F_1C_1\beta_1}{AD_1}M_1 + \frac{AD_2}{\beta_2\omega_1}(1 - f_2) \qquad (44)$$

since $\qquad v = v_{B1}$ at $f_2 = 1$

Then the value of v at $f_2 = 0$ when the second component charge burns out

$$v_{B2} = \frac{F_1C_1\beta_1}{AD_1}M_1 + \frac{AD_2}{\beta_2\omega_1} \qquad (45)$$

From (37) and (40)

$$F_1C_1 + F_2C_2(1 - f_2)\,(1 + \theta_2 f_2) = p_{B1}\left\{K_0 + Ax - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}\right\} \qquad (46)$$

Now we impose the condition that (37) gives $x = x_{B1}$, $v = v_{B1}$ at $f_2 = 1$ and further (37) is consistent with (43).

Now $x = x_{B1}$ will satisfy (46) if

$$F_1C_1 = p_{B1} \left\{ K_0 + Ax_{B1} - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} \right\} \tag{47}$$

which is true for (47) is obtained from (27) by considering burnt values.

Now differentiating (37) and (40) w. r. t. f_2

$$F_2C_2 \frac{dZ_2}{df_2} = p_{B1} A \frac{\frac{dx}{dt}}{\frac{df_2}{dt}}$$

and

$$\frac{dZ_2}{df_2} = (1 - f_2)\,\theta_2 - (1 + \theta_2 f_2)$$

and these equation with (39) yield

$$F_2C_2 \left[(1 - f_2)\,\theta_2 - (1 + \theta_2 f_2) \right] = - \frac{AD_2}{\beta_2} \frac{dx}{dt} .$$

Now $v = v_{B1}$ and $f_2 = 1$ will satisfy the above equation if

$$F_2C_2 (1 + \theta_2) = \frac{AD_2}{\beta_2} v_{B1}$$

Putting the value of v_{B1} the above equation reduces to

$$(1 + \theta_2) = \frac{F_1C_1}{F_2C_2} \frac{D_2}{D_1} \frac{\beta_1}{\beta_2} M_1 \tag{48}$$

Now to satisfy the condition that (37) and (41) will be consistent, we differentiate (37) twice w. r. t. f_2.

We have,

$$F_2C_2 \frac{d^2Z}{df_2^2} = A\, p_{B1} \frac{d^2x}{df_2^2}$$

or

$$F_2C_2 \frac{d^2Z}{df_2^2} = p_{B1} \frac{A^2D_2^2}{\beta_2^2 \omega_1 p_{B1}} .$$

Since

$$\frac{d^2Z_2}{df_2^2} = - 2\theta_2$$

we have

$$2\theta_2 = - \frac{A^2D_2^2}{\beta_2^2 \omega_1 F_2C_2} \tag{49}$$

The simultaneous satisfaction of the equations (48) and (49) gives the condition that $p = p_B$ may be a solution of the equations (37) to (40). From (49) it is evident that the second component must have progressively increasing burning surface.

Introducing the dimensionless constant,

$$\frac{F_2C_2}{F_1C_1} = \beta_0 \quad \text{and} \quad \frac{\frac{D_2}{\beta_2}}{\frac{D_1}{\beta_1}} = \alpha_0 .$$

We have
$$\left.\begin{aligned} 1+\theta_2 &= \frac{\alpha_0}{\beta_0} M_1 \\ 2\,\theta_2 &= -\frac{M_1\alpha_0^2}{\beta_0} \end{aligned}\right\} \tag{50}$$
and

Eliminating θ_2 we have,

$$M_1\alpha_0^2 + 2\,M_1\alpha_0 - 2\beta_0 = 0\,.$$

giving
$$\alpha_0 = -1 + \sqrt{1 + \frac{2\beta_0}{M_1}} \tag{51}$$

Since α_0 is always positive.

Now after all burnt the gas expands adiabatically and the corresponding equations are

$$\omega_1 v \frac{dv}{dx} = Ap\,. \tag{52}$$

$$pV^{\gamma} = \text{constant}\,. \tag{53}$$

Equation (53) gives,

$$pA^{\gamma}\,(x+l_1)^{\gamma} = p_{B1}\,A^{\gamma}\,(x_{B2}+l_1)^{\gamma}$$

or
$$p = p_{B1}\left(\frac{x_{B2}+l_1}{x+l_1}\right)^{\gamma} \tag{54}$$

where x_{B2} is the distance traversed by the shot at all burnt of the second component, which is to be calculated.

Now from (52) to (54)

$$vdv = \frac{A}{\omega_1}\,p_{B1}\,(x_{B2}+l_1)^{\gamma}\,\frac{dx}{(x+l_1)^{\gamma}}$$

Integrating with the condition that $x = x_{B2}$, $v = v_{B2}$ one gets

$$v^2 = v_{B2}^2 + \frac{2Ap_{B1}}{\omega_1\,(1-\gamma)}\,(x_{B2}+l_1)^{\gamma}\left[\,(x+l_1)^{1-\gamma} - (x_{B2}+l_1)^{1-\gamma}\right] \tag{55}$$

Let us calculate the shot travel when the second component burns
From (44), (39) and (41) we have

$$\frac{dx}{df_2}\,\frac{\beta_2 p_{B1}}{D_2} = \frac{F_1C_1\beta_1}{AD_1}\,M_1 + \frac{AD_2}{\beta_2\omega_1}\,(1-f_2)$$

Integrating with the condition that $x = x_{B1}$ at $f_2 = 1$,

$$\frac{\beta_2\,p_{B1}}{D_2}\left[\,x - l_1\left\{(1+\theta_1)^{N_1} - 1\right\}\right] = M_1\,\frac{F_1C_1\beta_1}{AD_1}\,(1-f_2) + \frac{AD_2}{2\beta_2\omega_1}\,(1-f_2)^2$$

Let $x = x_{B2}$ when the second component burns out, i.e. at $f_2 = 0$, then

$$x_{B2} = l_1\,(1+\theta_1)^{N_1}\left[\,1 + M_1\alpha_0 + \frac{M_1\alpha_0^2}{2}\,\right] - l_1$$

Let $$1 + M_0\alpha_0 + \frac{M_1\alpha_0^2}{2} = M_0 \tag{56}$$

Then $$x_{B2} = l_1 (1+\theta_1)^{N_1} M_0 - l_1 \tag{57}$$

From (45), (55) and (57) by putting the value of x_{B2} and v_{B2} we have

$$v^2 = \left(\frac{F_1 C_1 \beta_1}{AD_1} M_1 + \frac{AD_2}{\beta_2 \omega_1} \right)^2 + \frac{2A}{(1-\gamma)\,\omega_1} \frac{F_1 C_1}{Al_1} \frac{1}{(1+\theta_1)^{N_1}} l_1 (1+\theta_1)^{N_1} M_0 \times$$
$$\times \left[(x+l_1)^{1-\gamma} - \left\{ l_1 (1+\theta_1)^{N_1} M_0 \right\}^{1-\gamma} \right]$$

If V be the muzzle velocity of the shot,

$$V^2 = \frac{A^2 D_1^2}{\beta_1^2 \omega_1^2} \left[(1+\alpha_0)^2 + \frac{2 M_0}{(1-\gamma) M_1} \left\{ \left(\frac{1+\frac{d}{l_1}}{M_0 (1+\theta_1)^{N_1}} \right)^{1-\gamma} - 1 \right\} \right] \tag{58}$$

The mean pressure $\overline{p}$ is given by $\overline{p} = \frac{\omega_1 V^2}{2\,Ad}$

$$= \frac{\omega_1}{2Ad} \frac{A^2 D_1^2}{\beta_1^2 \omega_1^2} \left[(1+\alpha_0)^2 + \frac{2 M_0}{(1-\gamma) M_1)} \left\{ \left(\frac{(1+\frac{d}{l_1}}{(1+\theta_1)^{N_1} M_0} \right)^{1-\gamma} - 1 \right\} \right]$$

Considering the expression for the maximum pressure for the two cases

(i) $M_1 > (1-\theta_1)$

and (ii) $M_1 \leqslant (1-\theta_1)$

we have the expression for the piezometric efficiency of the gun.

When $M_1 > (1-\theta_1)$

Piezometric efficiency =

$$\frac{l_1}{2\,d} \frac{\theta_1 (N_1+2)^{N_1+2}}{(1+\theta_1)^2 (N_1+1)^{N_1+2}} \left[M_1 (1+\alpha_0)^2 + \frac{2M_0}{1-\gamma} \cdot \right.$$
$$\left. \cdot \left\{ \left(\frac{1+\frac{d}{l_1}}{M_0 (1+\theta_1)^{N_1}} \right)^{1-\gamma} - 1 \right\} \right] \tag{59}$$

for $$\left(\frac{x_{B1}}{d},\ \frac{x_{B2}}{d} < 1 \right)$$

and when $M_1 \leqslant (1-\theta_1)$

Piezometric efficiency =

$$\frac{l_1}{2d} (1+\theta_1)^{N_1} \left[M_1 (1+\alpha_0)^2 + \frac{2 M_0}{1-\gamma} \left\{ \left(\frac{1+\frac{d}{l_1}}{M_0 (1+\theta_1)^{N_1}} \right)^{1-\gamma} - 1 \right\} \right]$$

for $$\left(\frac{x_{B1}}{d},\ \frac{x_{B2}}{d} < 1 \right) \tag{60}$$

The corresponding expressions for the muzzle velocity in non dimensional form can be written as,

$$\eta_{m3}{}^2 = M_1 \left[M_1 (1+\alpha_0)^2 + \frac{2M_0}{1-\gamma} \left\{ \left(\frac{1+\frac{d}{l_1}}{M_0 (1+\theta_1)^{N_1}} \right)^{1-\gamma} - 1 \right\} \right] \quad (61)$$

Now for the moderated charge there are three possibilities:

(i) $\frac{x_{B1}}{d} > 1$, i.e. shot leaves the gun before all burnt of the first component such that the constant pressure phase has not been reached. We are not concerned with this because we are to consider the cases under constant pressure phase with the second component.

(ii) $\frac{x_{B1}}{d} < 1$ but $\frac{x_{B2}}{d} > 1$, i.e. the shot leaves the gun when the second component is burning.

(iii) $\frac{x_{B1}}{d} < 1$ and $\frac{x_{B2}}{d} < 1$, i.e. the shot leaves the gun after all burnt of the second component. We are concerned with the last two possibilities. In this case the muzzle velocity and piezometric efficiency are given by (59) to (61). For the second possibility the calculations of the muzzle velocity and piezometric efficiency are as follows. From (46) and (50) for $x = d, f_2$ have been calculated and with this value of f_2 the muzzle velocity can be calculated from (44) and in non-dimensional form it reduces to

$$\eta_{m2} = M_1 \left[1 + \alpha_0 (1 - f_2) \right] \quad (62)$$

The maximum pressure has been calculated on the same principle as in the case of single charge and hence the piezometric efficiency can also be calculated by deducing the mean pressure from the muzzle velocity and taking ratio of the mean pressure to the maximum pressure.

NUMERICAL CALCULATION

Now we are going to establish numerically that both the muzzle velocity and piezometric efficiency have greater values for the moderated charges. For the simplicity of calculation we have considered that the propellant for the single charge and moderated charges have the same chemical composition, i.e. $F = F_1 = F_2$, $\delta = \delta_1 = \delta_2$ and $\gamma = \gamma_1 = \gamma_2$. Also we have stated that the ballistic of the single charge is equal to that of the first component, i.e. $D/\beta = D_1/\beta_1$ and $C = C_1 + C_2$. Under these condition the ratio M/M_1 reduces to C_1/C and we choose the value of M and C_1/C, the value of M_1 has been determined. l/d and l_1/d are considered to be equal as we considered same gun in two cases. Also $\beta_0 = F_2C_2/F_1C_1 = (C/C_1 - 1)$. Hence when we choose C_1/C, β can be found out and α_0 is then determined by (51).

$\frac{l}{d} = 0\cdot5, \theta = 0, M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·093	0·8114	90	1·12	0·8222	97	1·098	0·8163
		92	1·105	0·8200	98	1·095	0·8157
		95	1·1	0·8190			

$\frac{l}{d} = 0\cdot5, \theta = 1, M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·44	0·7522	70	1·552	0·8422	88	1·437	0·8035
		72	1·531	0·8405	90	1·413	0·7966
		78	1·511	0·8235	95	1·375	1·7747
		80	1·488	0·8224	98	1·353	0·7630
		85	1·438	0·8136			

$\frac{l}{d} = 0\cdot2, \theta = 0, M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·562	0·6632	55	1·862	0·9157	80	1·766	0·8480
		60	1·840	0·9106	85	1·727	0·8100
		65	1·824	0·8975	90	1·665	0·7529
		70	1·815	0·8950	95	1·616	0·7095
		75	1·796	0·8778	98	1·584	0·6827

$\frac{l}{d} = 0\cdot2, \theta = 1, M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·709	0·4927	45	2·532	0·8354	75	2·135	0·6776
		50	2·503	0·8224	80	2·049	0·6500
		55	2·501	0·8156	85	1·958	0·6015
		58	2·443	0·8114	90	1·851	0·5472
		60	2·395	0·7829	95	1·784	0·5182
		65	2·301	0·7429	98	1·740	0·5051
		70	2·236	0·7100			

$\frac{l}{d} = 0 \cdot 1,\ \theta = 0,\ M = 1$

Single charge		Moderated charges					
M.C.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·833	0·3207	60	2·488	0·8414	90	2·068	0·5378
		80	2·181	0·6457			

$\frac{l}{d} = 0 \cdot 1,\ \theta = 0,\ M = 2$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
2·348	0·7496	90	2·635	0·9295	98	2·610	0·9173

$\frac{l}{d} = 0 \cdot 1,\ \theta = 1,\ M = 2$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
2·751	0·4483	85	2·870	0·7293	90	2·792	0·6224
		87	2·815	0·7025	98	2·770	0·6223

$\frac{l}{d} = 0 \cdot 1,\ \theta = -0 \cdot 1,\ M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·812	0·4709	45	2·613	0·8309	70	2·304	0·7986
		50	2·592	0·8135	80	2·144	0·6785
		60	2·510	0·8094	90	2·000	0·6385
		65	2·402	0·8054	95	1·886	0·6345

$\frac{l}{d} = 0 \cdot 1,\ \theta = -0 \cdot 2,\ M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·787	0·4800	80	1·932	0·6925	95	1·859	0·5315
		90	1·872	0·5430			

$\frac{l}{d} = 0 \cdot 1, \theta = -0 \cdot 6, M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·611	0·5905	98	1·627	0·6115	90	1·692	0·6735
		95	1·644	0·6400	87	1·712	0·6852

$\frac{l}{d} = 0 \cdot 1, \theta = -0 \cdot 8, M = 1$

Single charge		Moderated charges					
M.V.	P.E.	C_1/C (%)	M.V.	P.E.	C_1/C (%)	M.V.	P.E.
1·345	0·6656	98	1·316	0·6610	90	1·282	0·6820
		95	1·311	0·6750	87	1·123	0·6937

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REFERENCES

1. RAY, A., *Proc. Nat. Inst. Sci.*, India, **A30** (1964), 445.
2. H.M.S.O., "Internal Ballistics", 1951.
3. KAPUR, J. N., *Proc. Nat. Inst. Sci.*, India, **A23** (1957).
4. KAPUR, J. N., *Proc. Nat. Inst. Sci.*, India, **A24** (1958).