PRESSURE-RATIO IN H/L GUN DURING BURNING

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In the present paper conditions have been obtained so that the pressure-ratio in an H/L gun may remain less than critical during burning. Furthermore, conditions have been obtained to the effect that the pressure-ratio may be constant at a value less than or equal to critical during burning.

In a high-low pressure gun the propellant burns at a higher pressure while the shot is exposed to a comparatively low pressure. So the ratio ω of the pressure in the shot chamber (P_2) to the pressure in the propellant chamber (P_1) is always less than unity. Now it is well known that the calculation of internal ballistics during burning becomes very simplified if the pressure-ratio $\omega = \frac{P_2}{P_1}$ is less than the critical pressure-ratio $\omega^* = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$ the value of which according to Corner[1] is $0 \cdot 555$ with $\gamma = 1 \cdot 25$, after correcting for friction. In this paper we have deduced a condition to that effect after neglecting shot-start pressure and assuming the equality of covolume with specific volume. The condition determines a lower limit to the volume-ratio of the two chambers of the H/L gun when the loading condition and the throat-area of the nozzle are given. We have also given conditions to the effect the pressure-ratio may remain constant at a value less than or equal to the critical value during the period of burning. In this case also the condition similarly determines a lower limit to the volume-ratio of the two chambers in addition to the determination of a specific value of the leakage parameter (dependent on propellant property).

FUNDAMENTAL EQUATIONS DURING BURNING

Assuming that the pressure-ratio is less than critical, Corner[1] discussed the internal ballistics of H/L gun using tubular propellant. The following equations were given by him :

$$P_1 = \frac{C\lambda\,(1 - \psi)\,\phi}{\left(U_1 - \frac{C}{\delta} \right)(1 + b\phi)} \tag{1}$$

where

$$b = \frac{C\left[\frac{1}{\delta} - \eta\,(1 - \psi) \right]}{U_1 - \frac{C}{\delta}}. \tag{2}$$

Also

$$P_2 = \frac{\mu\,(W_2 C \psi \lambda)^{\frac{1}{2}}\,\phi}{A\,(X - \nu\phi)} \tag{3}$$

where
$$\mu = \frac{\beta C\lambda\,(1-\psi)}{D\left(U_1 - \dfrac{C}{\delta}\right)}, \tag{4}$$

$$X = \mu\left(\frac{W_2}{\lambda C\psi}\right)^{\frac{1}{2}}\left(\frac{U_2}{A} + x\right), \tag{5}$$

$$\nu = \frac{\mu\eta}{A}\left(\frac{W_2 C\psi}{\lambda}\right)^{\frac{1}{2}}, \tag{6}$$

and X satisfies the equation

$$\frac{d}{d\phi}\left[\frac{\phi}{1+b\phi}\;\frac{dX}{d\phi}\right] = \frac{1+b\phi}{X-\nu\phi} \tag{7}$$

with initial conditions $X = X_0 \; (x = 0)$

and $\phi \dfrac{dX}{d\phi} = 0 \; (V = 0)$.

So we can write

$$\omega = \frac{P_2}{P_1} = B\;\frac{1+b\phi}{X-\nu\phi} \tag{8}$$

with
$$B = \frac{\mu\,(W_2 C\psi\lambda)^{\frac{1}{2}}\left(U_1 - \dfrac{C}{\delta}\right)}{AC\lambda\,(1-\psi)} \tag{9}$$

CONDITION FOR ω TO BE LESS THAN ω^*

If ω is less than critical during the period of burning then (1) to (9) are true. From (7) and (8), we get

$$\frac{d}{d\phi}\left[\frac{\phi}{1+b\phi}\;\frac{dX}{d\phi}\right] = \frac{\omega}{B} < \frac{\omega^*}{B}$$

Integrating twice subject to initial conditions

$$X = X_0\;(x=0),\; \phi\frac{dX}{d\phi} = 0\;(V = 0)$$

we obtain
$$X \leqslant X_0 + \frac{\omega^*}{B}\left(\phi + \frac{1}{2}\,b\phi^2\right) \tag{10}$$

In our subsequent discussion b will be shown to be positive. Thus using suffix B to denote burnt values of variables from (7) we get

$$\frac{d}{d\phi}\left[\frac{\phi}{1+b\phi}\cdot\frac{dX}{d\phi}\right]=\frac{1+b\phi}{X-\nu\phi}>\frac{1}{X_B}. \tag{11}$$

Integrating inequality (11) subject to initial conditions, one has

$$\frac{dX}{d\phi}>\frac{1+b\phi}{X_B}>\frac{1}{X_B},$$

which on integration, subject to initial conditions, yields

$$X>X_0+\frac{\phi}{X_B} \tag{12}$$

From (10), considering burnt values we get

$$X_B\leqslant X_0+\frac{\omega^*}{B}\left(1+\frac{b}{2}\right)$$

and with this from (12) one has

$$X>X_0+\frac{\phi}{X_0+\frac{\omega^*}{B}\left(1+\frac{b}{2}\right)} \tag{13}$$

Thus from (8) and (13) we have

$$\omega=B\,\frac{1+b\phi}{X-\nu\phi}<B\,\frac{1+b\phi}{X_0+\phi\left[\frac{1}{X_0+\frac{\omega^*}{B}\left(1+\frac{b}{2}\right)}-\nu\right]} \tag{14}$$

or

$$\omega<\omega_1(\phi)$$

where

$$\omega_1(\phi)=B\,\frac{1+b\phi}{X_0+d\phi} \tag{15}$$

and

$$d=\frac{1}{X_0+\frac{\omega^*}{B}\left(1+\frac{b}{2}\right)}-\nu. \tag{16}$$

It is easy to show that the greatest value of $\omega_1(\phi)$ occurs at $\phi=0$ or at $\phi=1$ according as $bX_0\leqslant$ or $>d$ and the greatest value of $\omega_1(\phi)$ is accordingly

$$B/X_0 \text{ or } B(1+b)/(X_0+d).$$

So if we have

$$\left.\begin{aligned}&\frac{B}{X_0}<\omega^* &&\text{when } bX_0\leqslant d\\ \text{or}\quad&\frac{B(1+b)}{X_0+d}<\omega^* &&\text{when } bX_0>d\end{aligned}\right\} \tag{17}$$

pressure-ratio ω will remain less than critical during burning. We will now write the inequalities in (17) in different forms. For this we take the following dimensionless quantities :

$$\alpha = \frac{U_1 - \frac{C}{\delta}}{\frac{C}{\delta}} \tag{18}$$

$$K = \frac{U_2}{U_1} \tag{19}$$

$$\frac{U_2}{\frac{C}{\delta}} = K(\alpha + 1) \tag{20}$$

$$M = \frac{A^2 D^2}{\beta^2 C \lambda W_2} \tag{21}$$

To simplify the analysis we assume $\eta = \frac{1}{\delta}$. In that case from (2) and (18) we get

$$b = \frac{\psi}{\alpha}; \tag{22}$$

and this indicates that b is positive as mentioned earlier. Further we have the following identities :

from (5), (9), (18) and (20)

$$\frac{B}{X_0} = \frac{\psi}{1-\psi} \cdot \frac{\alpha}{K(\alpha+1)} \tag{23}$$

from (5), (6) and (20)

$$\frac{X_0}{\nu} = \frac{K(\alpha+1)}{\psi} \tag{24}$$

from (6), (9) and (19) [or from (23) and (24)]

$$\frac{B}{\nu} = \frac{\alpha}{1-\psi} \tag{25}$$

from (4) to (6), (18), (20) and (21)

$$\nu X_0 = \frac{K(\alpha+1)(1-\psi)^2}{M\alpha^2} \tag{26}$$

From

$$\frac{B(1+b)}{X_0 + d} < \omega^*$$

we have $$X_0 + d > \frac{B}{\omega^*} (1 + b)$$

or by (16)

$$X_0 + \frac{1}{X_0 + \frac{\omega^*}{B}\left(1 + \frac{b}{2}\right)} - \nu > \frac{B}{\omega^*} (1 + b)$$

or by (22) and (24) to (26)

$$\frac{K(\alpha + 1)}{\psi} + \frac{1}{\frac{K(\alpha + 1)(1 - \psi)^2}{M\alpha^2} + \frac{\omega^*(1 - \psi)\left(1 + \frac{\psi}{2\alpha}\right)}{\alpha}} > 1 + \frac{\alpha\left(1 + \frac{\psi}{\alpha}\right)}{\omega^*(1 - \psi)}$$

or $$K + \frac{\alpha_0}{K + \delta_0} > \gamma_0 \tag{27}$$

where $$\alpha_0 = \frac{M\alpha^2\psi}{(\alpha + 1)^2 (1 - \psi)^2} \tag{28}$$

$$\delta_0 = \frac{M\alpha\omega^*\left(1 + \frac{\psi}{2\alpha}\right)}{(\alpha + 1)(1 - \psi)} \tag{29}$$

$$\gamma_0 = \frac{\psi}{(\alpha + 1)}\left[1 + \frac{\alpha + \psi}{\omega^*(1 - \psi)}\right] \tag{30}$$

From $$bX_0 \leqslant d$$

by (16) we have $$bX_0 \leqslant \frac{1}{X_0 + \frac{\omega^*}{B}\left(1 + \frac{b}{2}\right)} - \nu$$

or by (22) and (24) to (26)

$$\frac{K(\alpha + 1)}{\alpha} + 1 \leqslant \frac{1}{\frac{K(\alpha + 1)(1 - \psi)^2}{M\alpha^2} + \frac{\omega^*\left(1 + \frac{\psi}{2\alpha}\right)(1 - \psi)}{\alpha}}$$

or by (29)
$$K + \rho_0 \leqslant \frac{\tau_0}{K + \delta_0} \tag{31}$$

where
$$\rho_0 = \frac{\alpha}{\alpha + 1} \tag{32}$$

$$\tau_0 = \frac{M\alpha^3}{(\alpha + 1)^2 \ (1 - \psi)^2} \tag{33}$$

From
$$\frac{B}{X_0} < \omega^*$$

by (23) we get
$$K > \frac{\psi \alpha}{(1 - \psi) \ (\alpha + 1) \ \omega^*} \tag{34}$$

So with help of (27), (31) and (34) from (17) we have the result that pressure-ratio will be less than critical during burning if we have

$$\left.\begin{aligned} & K > \frac{\psi\alpha}{(1 - \psi) \ (\alpha + 1) \ \omega^*} \text{ when } K + \rho_0 \leqslant \frac{\tau_0}{K + \delta_0} \\ \text{or} \quad & K + \frac{\alpha_0}{K + \delta_0} > \gamma_0 \text{ when } K + \rho_0 > \frac{\tau_0}{K + \delta_0} \end{aligned}\right\} \tag{35}$$

These conditions give a lower limit to the volume-ratio of the two chambers of an H/L gun when M, α and ψ are known. It may be noted that the conditions suffer from the following defects. Firstly there may be cases of subcritical pressure-ratio ($\omega < \omega^*$) in which neither of the conditions in (35) holds. Secondly the conditions may be too strong to be useful in practical ballistics. To discuss the second point, we consider the following examples in which we take $\alpha = 1$, $\psi = 0 \cdot 5$, $\omega^* = 0 \cdot 555$. With these values from (35) we see that for $M = 1, 2, 4, \infty$ the lower limits of K are respectively $1 \cdot 166$, $1 \cdot 156$, $1 \cdot 022$ and $0 \cdot 900$. With these lower limits and corresponding values of M, α, ψ we calculate b, ν and X_0 by (22), (24) and (26) and then we calculate the value of ω at burnt in the first case by the tables given by Corner[1] and the value of ω as $\phi \to 0$ in the last three cases. In the examples considered these values are respectively ·499, ·433, ·489 and ·555. These indicate that in each of the examples there is at least one stage during burning when the pressure-ratio is not much less than its critical value and so we may conclude that the conditions are not too strong, and may be used as a guide in practical ballistics. In passing we note the following points :

(1) Limit of K decreases when ψ decreases, i.e. for lesser leakage

(2) Limit of K decreases with increase of M, i.e. for lighter shot.

CASE OF CONSTANT PRESSURE-RATIO

If possible let the pressure-ratio ω be constant at a value less than or equal to the critical during burning. Therefore from (8) we get

$$X = \frac{B}{\omega_0} (1 + b\phi) + \nu\phi$$

where ω_0 is the value of the constant pressure-ratio. This value should be the initial value of ω i.e. $\omega_0 = \frac{B}{X_0}$ [from (8) with $\phi \to 0$, $X_0 \neq 0$]. So the above equation leads to

$$X = X_0 (1 + b\phi) + \nu\phi \tag{36}$$

Again (7) and (8) give

$$\frac{d}{d\phi} \left[\frac{\phi}{1 + b\phi} \frac{dX}{d\phi} \right] = \frac{\omega_0}{B} = \frac{1}{X_0}$$

which on integration subject to initial conditions yields

$$X = X_0 + \frac{1}{X_0} \left(\phi + \frac{1}{2} b\phi^2 \right) \tag{37}$$

Now (36) and (37) are to be consistent for which we require

$$b = 0 \tag{38}$$

$$\nu X_0 = 1 \tag{39}$$

Furthermore $\omega_0 = \frac{B}{X_0}$ is less than or equal to critical for which we require

$$\frac{B}{X_0} \leqslant \omega^*$$

or, by (23)

$$K \geqslant \frac{\psi\alpha}{(1 - \psi)(\alpha + 1)\omega^*} . \tag{40}$$

Thus (38), (39) and (40) are to be true simultaneously for pressure-ratio to be constant at a sub-critical or the critical value. We want to see whether this is possible.

From (38) and (2) we get

$$\psi = 1 - \frac{1}{\eta\delta} \tag{41}$$

which shows, as mentioned earlier, that in this case of constant pressure-ratio ψ is determined by propellant property. For service propellants generally $\eta = 26{\cdot}5$ lbs/cu. inch and $\frac{1}{\delta} = 17{\cdot}5$ lbs/cu. inch, so ψ is about $\frac{1}{3}$ which is a bit less than the practical value of ψ which is about $\frac{1}{2}$. Usually α is about unity; so with $\alpha = 1$, $\psi = \frac{1}{3}$ and $\omega^* = 0{\cdot}555$, from (40) we get

$$K \geqslant 0{\cdot}45.$$

With (26), an alternative form of (39) is

$$K (\alpha + 1) (1 - \psi)^2 = M\alpha^2 \tag{42}$$

from which with $K = 0 \cdot 45$ we have $M = 0 \cdot 4$ in this example. Thus (38), (39) and (40) can hold simultaneously and so can be considered as conditions for a constant pressure-ratio less than or equal to critical.

REFERENCE

1. CORNER, J., "Theory of Internal Ballistics of Guns" (John Wiley & Sons, New York), 1950.