EXPANSION FORMULAE FOR THE KAMPÉ DE FÉRIET FUNCTION INVOLVING BESSEL FUNCTION

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In this paper some integrals involving a Kampé de Fériet; function have been evaluated. These have been used to establish some expansion formulae for the Kampé de Fériet function involving Bessel function.

Some integrals involving a Kampé de Fériet function have been evaluated. These have been used to obtain some expansion formulae for the Kampé de Fériet function involving Bassel functions. Some particular cases have also been discussed.

For the sake of simplicity and brevity, the notation for the generalised hypergeometric function of two variables given by Kampé de Fériet[1] have been modified.

$$F^{g:h;H}_{i:\,j;\,J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} x, y\right] = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{\Pi(a_g)_{p+q}\,\Pi(b_h)_p\,\Pi(B_H)_q\,x^p y^q}{\Pi(\alpha_i)_{p+q}\,\Pi(c_j)_p\,\Pi(C_J)_q\,p\,!\,q\,!} \tag{1}$$

where

$$g + h < i + j + 1, \quad g + H < i + J + 1,$$

$(\mu)_p$ stands for $\Gamma(\mu + p)/\Gamma(\mu)$ $(p = 1, 2\ldots)$, $(\mu)_0 = 1$. $\Pi(\mu_i)_s$ denotes the product $(\mu_1)_s\,(\mu_2)_s\ldots\,(\mu_i)_s$. The colon (:) and semicolon (;) separate the terms of the typeproduct and $(b_h)_p$, $(B_H)_q$ on the left of (1). Here and in what follows a_m denotes the set of parameters $a_1, a_2, \ldots, a_m$ and $\triangle(\delta, a)$ will represent the set of parameters a/δ, $(a+1)/\delta, \ldots,$ $(a+\delta-1)/\delta$, where δ is a positive integer.

The following formulae will be required in the proof :

(*a*) *The formulae* :

$$\Gamma(\alpha - \delta n) = \frac{\Gamma(\alpha)\,(-1)^{\delta n}}{\prod\limits_{i=0}^{\delta-1}\left(\frac{1-\alpha+i}{\delta}\right)_n}\,\delta^{-\delta n} \tag{2}$$

where δ and n are positive integers.

$$\Gamma(\alpha + \delta n) = \Gamma(\alpha)\cdot\delta^{\delta n}\prod_{i=0}^{\delta-1}\left(\frac{\alpha+i}{\delta}\right)_n \tag{3}$$

where δ and n are positive integers.

$$\Gamma(z)\,\Gamma(1-z) = \pi/\sin \pi z \tag{4}$$

(*b*) *The integral*[2] :

$$\int_0^\infty x^{\lambda-1} e^{-a^2 x} J_\mu(bx)\,dx = \frac{\Gamma(\lambda+\mu)\,b^\mu}{a^{2(\lambda+\mu)}\,\Gamma(1+\mu)2^\mu}\ {}_2F_1\left(\frac{\lambda+\mu}{2},\frac{\lambda+\mu+1}{2};\mu+1;-\frac{b^2}{a^4}\right)$$

$$= \sum_{r=0}^{\infty} \frac{\Gamma(\lambda+\mu)\,b^\mu \left(\frac{\lambda+\mu}{2}\right)_r \left(\frac{\lambda+\mu+1}{2}\right)_r (-b^2/a^4)^r}{r!\,2^\mu\,a^{2(\lambda+\mu)}\,\Gamma(\mu+1+r)}$$

$$= \sum_{r=0}^{\infty} A(\mu,r) \qquad \text{(say)} \tag{5}$$

$$Re(\lambda+\mu) > 0.$$

(*c*) *The orthogonality property of Bessel functions*[3]:

$$\int_0^\infty x^{-1} J_{\alpha+2m+1}(x)\, J_{\alpha+2n+1}(x)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ (4m+2\alpha+2)^{-1} & \text{if } m = n \\ & Re(\alpha+m+n) > -1 \end{cases} \tag{6}$$

INTEGRALS

If $g+h < i+j+1$, $g+H < i+J+1$; $Re(\lambda+\mu) > 0$, $Re\,\lambda < 1/2$;

then

$$\int_0^\infty t^{\lambda-1} e^{-a^2 t} J_\mu(bt)\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\ xt^{2\delta}, yt^{2\delta}\right] dt = \sum_{r=0}^{\infty} A(\mu,r)\,F(\mu,r) \tag{7}$$

$$\int_0^\infty t^{\lambda-1} e^{-a^2 t} J_\mu(bt)\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\ xt^{2\delta}, y\right] dt = \sum_{r=0}^{\infty} A(\mu,r)\,F_1(\mu,r) \tag{8}$$

$$\int_0 t^{\lambda-1} e^{\pm it} J_\mu(t)\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\ xt^{2\delta}, yt^{2\delta}\right] dt = (\pm i)^{\lambda+\mu}\,B(\mu)\,F(\mu) \tag{9}$$

$$\int_0^\infty t^{\lambda-1} e^{\pm it} J_\mu(t)\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\ xt^{2\delta}, y\right] dt = (\pm i)^{\lambda+\mu}\,B(\mu)\,F_1(\mu) \tag{10}$$

$$\int_0^\infty t^{\lambda-1} \cos t\, J_\mu(t)\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\ xt^{2\delta}, yt^{2\delta}\right] dt = C(\mu)\,F(\mu) \tag{11}$$

$$\int_0^\infty t^{\lambda-1} \cos t\, J_\mu(t)\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\ xt^{2\delta}, y\right] dt = C(\mu)\,F_1(\mu) \tag{12}$$

$$\int_0^\infty t^{\lambda-1} \sin t \, J_\mu(t) \; F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} \; xt^{2\delta}, \; yt^{2\delta}\right] dt = D(\mu) \, F(\mu) \tag{13}$$

$$\int_0^\infty t^{\lambda-1} \sin t \, J_\mu(t) \; F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} \; xt^{2\delta}, \; y\right] dt = D(\mu) \, F_1(\mu) \tag{14}$$

where

$$\left.\begin{aligned} & A(\mu, r) \text{ is given by (5)}, \\ & B(\mu) = \frac{\Gamma(\lambda+\mu)\,\Gamma\{(1-2\lambda)/2\}}{\Gamma\{1+(\mu-\lambda)/2\}\,\Gamma\{(1+\mu-\lambda)/2\}2^\mu}, \\ & C(\mu) = \frac{2^{\lambda-1}\sqrt{\pi}\,\Gamma\{(\lambda+\mu)/2\}\,\Gamma\{(1-2\lambda)/2\}}{\Gamma\{1+(\mu-\lambda)/2\}\,\Gamma\{(1-\mu-\lambda)/2\}\,\Gamma\{1+\mu-\lambda)/2\}} \\ & D(\mu) = \frac{2^{\lambda-1}\sqrt{\pi}\,\Gamma\{(1+\lambda+\mu)/2\}\,\Gamma\{(1-2\lambda)/2\}}{\Gamma\{1+(\mu-\lambda)/2\}\,\Gamma\{1-(\lambda+\mu)/2\}\,\Gamma\{(1+\mu-\lambda)/2\}} \end{aligned}\right\} \tag{15}$$

$$\left.\begin{aligned} & F(\mu, r) = F^{g+2\delta:h;H}_{i:j;J}\left[\begin{matrix} a_g, \triangle(2\delta, \lambda+\mu+2r) : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} \; (2\delta/a^2)^{2\delta} x, (2\delta/a^2)^{2\delta} y\right] \\ & F_1(\mu, r) = F^{g:h+2\delta;H}_{i:j;J}\left[\begin{matrix} a_g : b_h, \triangle(2\delta, \lambda+\mu+2r) ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} \; (2\delta/a^2)^{2\delta} x, y\right] \\ & F(\mu) = F^{g+4\delta:h;H}_{i+2\delta:j;J}\left[\begin{matrix} a_g, \triangle(2\delta, \lambda+\mu), \triangle(2\delta, \lambda-\mu) : b_h ; B_H ; \\ \alpha_i, \triangle(2\delta, \lambda+1/2) : c_j ; C_J ; \end{matrix} \; (-\delta^2)^\delta x, (-\delta^2)^\delta y\right]. \\ & F_1(\mu) = F^{g:h+4\delta;H}_{i:j+2\delta;J}\left[\begin{matrix} a_g : b_h ; \triangle(2\delta, \lambda+\mu), \triangle(2\delta, \lambda-\mu) ; B_H ; \\ \alpha_i : c_j, \triangle(2\delta, \lambda+1/2) ; C_J ; \end{matrix} \; (-\delta^2)^\delta x, y\right] \end{aligned}\right\} \tag{16}$$

Proof:—To establish (7), expressing the Kampé de Fériet as in (1) and interchanging the order of integration and summation, we have

$$\sum_{p=0}^\infty \sum_{q=0}^\infty \frac{\Pi(a_g)_{p+q}\,\Pi(b_h)_p\,\Pi(B_H)_q\,x^p y^q}{\Pi(\alpha_i)_{p+q}\,\Pi(c_j)_p\,\Pi(C_J)_q\,p!\,q!} \int_0^\infty t^{\lambda+2\delta p+2\delta q-1}, e^{-a^2t} J_\mu(bt)\,dt$$

By virtue of (5), this becomes

$$\sum_{p=0}^\infty \sum_{q=0}^\infty \frac{\Pi(a_g)_{p+q}\;\Pi(b_h)_p\;\Pi(B_H)_q\,x^p y^q\,\Gamma(\lambda+\mu+2\delta p+2\delta q)\,b^\mu}{\Pi(\alpha_i)_{p+q}\,\Pi(c_j)_p\,\Pi(C_J)_q\,p!\,q!\;2^\mu a^{2(\lambda+\mu+2\delta p+2\delta q)}\,\Gamma(1+\mu)} \times$$

$$\times\, {}_2F_1\left(\frac{\lambda+\mu}{2}+\delta p+\delta q, \; \frac{\lambda+\mu+1}{2}+\delta p+\delta q ; \; \mu+1 ; \; -b^2/a^4\right) \tag{17}$$

Now making use of (3), simplifying, and applying (1), we have (7). Regarding the change of order of integration and summation, we must have

$$(i) \text{ the series } \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{\Pi(a_g)_{p+q}\,\Pi(b_h)_p\,\Pi(B_H)_q}{\Pi(\alpha_i)_{p+q}\,\Pi(c_j)_p\,\Pi(C_J)_q\,p!\,q!}\,x^p\,y^q\,t^{2\delta p+2\delta q}$$

uniformly convergent in $0 \leqslant t \leqslant c$, c being arbitrary. It is so if $g+h < i+j+1$ and $g+H < i+J+1$,

(ii) $e^{-a^2 t} J_\mu(bt)$ a continuous function for all $t > 0$ and

$$(iii) \text{ the integral } \int_0^\infty t^{\lambda-1} e^{-a^2 t} J_\mu(bt)\,dt$$

convergent. It is so by virtue of (5) if $Re(\lambda+\mu) > 0$. Hence the change of the order of integration and summation[4] is justified.

Equation (8) can similarly be obtained. Equations (9) and (10) are particular cases of (7) and (8) respectively. If we take $b=1$, $a^4 = -1$ and use Gauss Theorem in (17), we have (9). Similarly we have the result (10).

Results (11) and (13) are obtained by adding and subtracting the results (9) and using (2), (3), (4) and the duplication formula. Results (12) and (14) are similarly obtained from (10).

EXPANSION FORMULAE

Expansion formulae to be obtained are

$$t^{\lambda} e^{-a^2 t} f^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\; xt^{2\delta}, yt^{2\delta}\right]$$

$$= \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} 2(\alpha+2n+1)\,A(\alpha+2n+1,r)\,F(\alpha+2n+1,r)\,J_{\alpha+2n+1}(bt) \tag{18}$$

$$t^{\lambda} e^{-a^2 t} F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix}\; xt^{2\delta}, y\right]$$

$$= \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} 2(\alpha+2n+1)\,A(\alpha+2n+1,r)\,F_1(\alpha+2n+1,r)\,J_{\alpha+2n+1}(bt) \tag{19}$$

$$t^{\lambda} e^{\pm it} F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ a_i : c_j ; C_J ; \end{matrix}\; xt^{2\delta}, yt^{2\delta}\right]$$

$$= \sum_{n=0}^{\infty} 2(\alpha+2n+1)\,(\pm i)^{\lambda+a+2n+1}\,B(\alpha+2n+1)\,F(\alpha+2n+1)\,J_{a+2n+1}(t) \tag{20}$$

$$t^{\lambda} e^{\pm it} F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} xt^{2\delta}, y\right]$$

$$= \sum_{n=0}^{\infty} 2(\alpha+2n+1)\,(\pm i)^{\lambda+\alpha+2n+1}\, B(\alpha+2n+1)\;\; F_1(\alpha+2n+1)\, J_{\alpha+2n+1}(t) \tag{21}$$

$$t \cos t\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} xt^{2\delta}, yt^{2\delta}\right]$$

$$= \sum_{n=0}^{\infty} 2(\alpha+2n+1)\; C(\alpha+2n+1)\;\; F(\alpha+2n+1)\;\; J_{\alpha+2n+1}(t) \tag{22}$$

$$t^{\lambda} \cos t\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} xt^{2\delta}, y\right]$$

$$= \sum_{n=0}^{\infty} 2(\alpha+2n+1)\; C(\alpha+2n+1)\;\; F_1(\alpha+2n+1)\, J_{\alpha+2n+1}(t) \tag{23}$$

$$t^{\lambda} \sin t\;\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} xt^{2\delta}, yt^{2\delta}\right]$$

$$= \sum_{n=0}^{\infty} 2(\alpha+2n+1)\; D(\alpha+2n+1)\;\; F(\alpha+2n+1)\;\; J_{\alpha+2n+1}(t) \tag{24}$$

$$t^{\lambda} \sin t\;\, F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} xt^{2\delta}, y\right]$$

$$= \sum_{n=0}^{\infty} 2\,(\alpha+2n+1)\;\; D\,(\alpha+2n+1)\;\; F_1\,(\alpha+2n+1)\, J_{\alpha+2n+1}(t) \tag{25}$$

where the various symbols have been defined in (15) and (16) and

(*i*) $g+h < i+j+1,\; g+H < i+J+1$;

(*ii*) $Re\,(\alpha+1) > 0$

(*iii*) $1/2 > Re\,\lambda > 0$

(*iv*) $Re\,(\Sigma\alpha_i + \Sigma c_j - \Sigma a_g - \Sigma b_h) > 0,\; Re\,(\Sigma\alpha_i + \Sigma C_J - \Sigma a_g - \Sigma B_H) > 0$;

Proof:—To prove (18) let

$$f(t) = t^{\lambda} e^{-a^2 t} F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} \; xt^{2\delta}, \; yt^{2\delta}\right] = \sum_{n=0}^{\infty} K_n J_{\alpha+2n+1}(t) \qquad (36)$$

Equation (26) is valid since $f(t)$ is continuous and of bounded variation in the interval $0 \leqslant t < \infty$. Multiplying both sides of (26) by $t^{-1} J_{\alpha+2m+1}(t)$ and integrating with respect to t from 0 to ∞, we have

$$\int_0^{\infty} t^{\lambda-1} e^{-a^2 t} J_{\alpha+2m+1}(bt) \; F^{g:h;H}_{i:j;J}\left[\begin{matrix} a_g : b_h ; B_H ; \\ \alpha_i : c_j ; C_J ; \end{matrix} \; xt^{2\delta}, \; yt^{2\delta}\right]$$

$$= \sum_{n=0}^{\infty} K_n \int_0^{\infty} t^{-1} J_{\alpha+2m+1}(bt) \; J_{\alpha+2n+1}(bt) \, dt$$

Now using (7) and the orthogonality property (6) of Bessel functions, we get

$$K_n = \sum_{r=0}^{\infty} 2(\alpha + 2n + 1) \; A(\alpha + 2n + 1, \; r) \; F(\alpha + 2n + 1, r) \qquad (27)$$

(18) now follows from (26) and (27). The results (19) to (25) can similarly be obtained with the help of the results (8) to (14).

The necessary condition to ensure the convergence and meaning of the Kampé de Fériet function and Bessel function are covered in the conditions (*i*) and (*ii*). The remaining conditions are sufficient to ensure the convergence of the expansions. Some of these conditions also ensure that the Gamma functions which appear here are finite.

PARTICULAR CASES

Although the results (9) to (14) are the particular cases of the results (7) and (8), the results (20) to (25) are the special cases of the results (18) and (19), yet in view of the fact that the Kampé de Fériet functions in the left hand side of the results reduce to the products of pairs of hypergeometric functions, in the absence of the parameters a_g and α_i, we can obtain several results as particular cases. We give below, as example, the results obtained from (11) to (14) and (22) to (25).

$$\int_0^{\infty} t^{\lambda-1} \cos t \, J_{\mu}(t) \; {}_hF_j(b_h ; c_j ; xt^{2\delta}) \; {}_HF_J(B_H ; C_J ; yt^{2\delta}) \, dt = C(\mu) F^*(\mu) \qquad (28)$$

$$h < j + 1, \; H < J + 1 \;\; Re(\mu) > -1/2, \; Re\,\lambda < 1/2$$

$$\int_0^{\infty} t^{\lambda-1} \cos t \, J_{\mu}(t) \; {}_hF_j(b_h ; \; c_j ; xt^{2\delta}) \, dt = C(\mu) \, F^*_1(\mu) \qquad (29)$$

$$h < j + 1, \; Re(\mu) > -1/2, \; Re\,\lambda < 1/2,$$

$$\int_0^\infty t^{\lambda-1} \sin t\, J_\mu(t)\; {}_hF_j(b_h; c_j; xt^{2\delta})\; {}_HF_J(B_H; C_J; yt^{2\delta})\, dt = D(\mu)\, F^*(\mu) \tag{30}$$

$$h < j+1,\ H < J+1,\ Re(\mu) > -1/2,\ Re\,\lambda < 1/2,$$

$$\int_0^\infty t^{\lambda-1} \sin t\, J_\mu(t)\; {}_hF_j(b_h; c_j; xt^{2\delta})\, dt = D(\mu) F^*_1(\mu) \tag{31}$$

$$h < j+1,\ Re(\mu) > -1/2,\ R\,\lambda < 1/2.$$

$$t^\lambda \cos t\; {}_hF_j(b_h; c_j; xt^{2\delta})\; {}_HF_J(B_H; C_J; yt^{2\delta})$$

$$= \sum_{n=0}^{\infty} 2(\alpha+2n+1)\, C(\alpha+2n+1)\, F^*(\alpha+2n+1)\, J_{\alpha+2n+1}(t), \tag{32}$$

$$h < j+1,\ H < J+1,\ Re(\alpha+1) > 0,\ 1/2 > Re\,\lambda > 0;$$
$$Re[\Sigma c_j - \Sigma b_h] > 0,\ Re(\Sigma C_J - \Sigma B_H) > 0$$

$$t^\lambda \cos t\; {}_hF_j(b_h; c_j; xt^{2\delta}) = \sum_{n=0}^{\infty} 2(\alpha+2n+1)\, C(\alpha+2n+1)\, F^*_1(\alpha+2n+1) J_{\alpha+2n+1}(t) \tag{33}$$

$$h < j+1,\ Re(\alpha+1) > 0,\ 1/2 > Re\,\lambda > 0,\ Re[\Sigma c_j - \Sigma b_h] > 0,$$

$$t^\lambda \sin t\; {}_hF_j(b_h; c_j; xt^{2\delta})\; {}_HF_J(B_H; C_J; yt^{2\delta})$$

$$= \sum_{n=0}^{\infty} 2(\alpha+2n+1)\, D(\alpha+2n+1)\, F^*(\alpha+2n+1)\, J_{\alpha+2n+1}(t), \tag{34}$$

$$h < j+1,\ H < J+1;\ Re\,\alpha > -1,\ Re[\Sigma c_j - \Sigma b_h] > 0,\ Re[\Sigma C_J - \Sigma B_H] > 0$$

$$t^\lambda \sin t\; {}_hF_j(b_h; c_j; xt^{2\delta}) = \sum_{n=0}^{\infty} 2(\alpha+2n+1)\, D(\alpha+2n+1)\, F^*_1(\alpha+2n+1)\, J_{\alpha+2n+1}(t) \tag{35}$$

$$h < j+1,\ Re\,\alpha > -1,\ Re[\Sigma c_j - \Sigma b_h] > 0$$

where $C(\mu)$ $D(\mu)$ have been defined as in (15) and $F^*(\mu)$ is a Kampé de Fériet function which is obtained from $F(\mu)$, defined in (16), after considering the parameters a_g and α_s absent, and

$$F^*_1(\mu) = {}_{h+4\delta}F_{j+2\delta}\,[b_h, \triangle(2\delta, \lambda+\mu), \triangle(2\delta, \lambda-\mu); c_j, \triangle(2\delta, \lambda+1/2); (-\delta^2)^\delta x]$$

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