IMPULSIVE DEBOOST FOR MAXIMUM AND MINIMUM ATMOSPHERIC ENTRY ANGLES

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Deboost of rocket vehicles initially moving in elliptic orbits for maximum and minimum atmospheric entry angles is analysed. It is shown that in general with specified retrofire velocity impulse and entry altitude, maximum entry angle is obtained when retrofire impulse is applied tangentially at the apogee whereas minimum entry angle occurs when retrofire impulse is applied tangentially at the perigee. Existence conditions for only maximum or minimum or both or neither maximum nor minimum entry angles have also been investigated.

The problem of maximizing the atmospheric entry angle for specified retrofire impulse in case of a rocket vehicle moving initially in elliptic orbit is analysed in reference 1. The results of the two numerical examples worked out therein lead to the conclusion that maximum entry angle at a given entry altitude occurs for tangential retrofire impulse at the apogees. But the conclusion derived there in from numerical cases lacks generality unless an analytical proof is given.

The object of this paper is to show that (i) the aforementioned conclusion is a general one, (ii) to analyse the case when the mission is impulsive deboost for minimum atmospheric entry angle with given retrofire impulse at an assigned entry altitude. In case (ii) it is shown that the retrofire impulse should be applied tangentially to the rocket at the perigee of the elliptic orbit in which the rocket was moving prior to deboost. Conditions under which either maximum or minimum or no extreme entry angle or both maximum and minimum entry angles will occur have also been derived.

ANALYSIS

Let the equation of the elliptic orbit in which the rocket is moving initially be

$$l = r\,(1 + e\cos\theta) \tag{1}$$

where l is semi-latus rectum, e is eccentricity of the orbit and r,θ represent radius vector and true anomaly respectively. If γ is angle between velocity and the local horizontal and suffixes 1 and E represent conditions just after the retrofire impulse and at the entry point respectively (Fig. 1), it is shown in reference 1 that the atmospheric entry angle γ_E at a radial distance r_E from the force centre is given by

$$\cos\gamma_E = \frac{\left[(\mu l)^{\frac{1}{2}} - \triangle V\, r_0 \cos(\gamma_0 + \beta)\right]}{r\left[\mu\left(\dfrac{2}{r_E} - \dfrac{1-e^2}{l}\right) - 2\triangle V.\, V_0\cos\beta + (\triangle V)^2\right]^{\frac{1}{2}}} \tag{2}$$

where μ is gravitational parameter, $\triangle V$ is the retrofire impulsive velocity decrement applied to the rocket at the point (r_0, θ_0) of the initial orbit at an alignment angle β and γ_0, V_0 are the values of γ and velocity of the rocket vehicle at the point (r_0, θ_0) prior to the application of the velocity impulse.

γ_0 and V_0 can be expressed as

$$\tan \gamma_0 = \frac{e \sin \theta_0}{1 + e \cos \theta_0} \qquad (3)$$

$$V_0 = \left[\frac{\mu}{l} \; (1 + e^2 + 2e \cos \theta_0) \right]^{\frac{1}{2}} \qquad (4)$$

For specified $\triangle V$ and r_E, by virtue of (1), (3) and (4), γ_E is a function of two variables β and θ_0 and the equations giving the values of β and θ_0 which after substituting in (2) will give extreme values of γ_E, are[1]

$$r_0 \sin (\gamma_0 + \beta) \; [(\mu l)^{\frac{1}{2}} - \triangle V \, r_0 \cos (\gamma_0 + \beta)] = V_0 r_E^2 \cos^2 \gamma_E \sin \beta \qquad (5)$$

$$r_0 \; [(\mu l)^{\frac{1}{2}} - \triangle V \, r_0 \cos (\gamma_0 + \beta)] \, [\mu (e + \cos \theta_0) \sin (\gamma_0 + \beta) - r_0 V^2_0 / \sin \theta_0 \cos (\gamma_0 + \beta)]$$
$$= \mu V_0 r^2_E \cos^2 \gamma_E \cos \beta \sin \theta_0 \qquad (6)$$

Equations (5) and (6) give

$$\tan \beta = \frac{e \sin \theta_0 \left(\dfrac{l}{r_0} \right)^{\frac{1}{2}}}{\dfrac{l}{r_0} \left[2 \mp \left(\dfrac{l}{r_0} \right)^{\frac{1}{2}} \right] - (1 - e^2)} \qquad (7)$$

Using (1), (3) and (4), (7) can be written as

$$\tan \beta = \frac{\sin \gamma_0}{V_0 \left(\dfrac{r_0}{\mu} \right)^{\frac{1}{2}} \mp \cos \gamma_0} \qquad (8)$$

Equation (8) gives

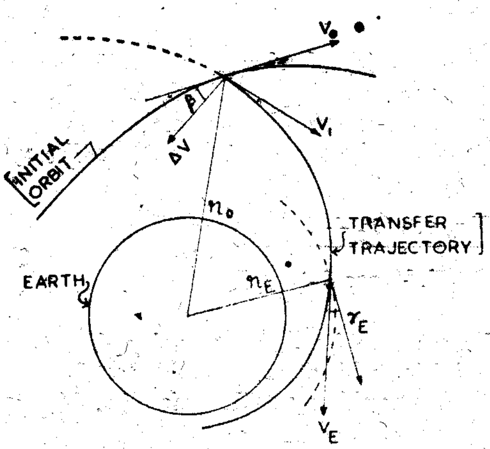

Fig. 1—Transfer geometry.

$$r_0 \sin (\gamma_0 + \beta) = V_0 \sin \beta \left[r_0 \left(\frac{r_0}{\mu} \right)^{\frac{1}{2}} + \right.$$
$$\left. + \frac{(\mu l)^{\frac{1}{2}} \mp (\mu l)^{\frac{1}{2}}}{V^2_0} \right] \qquad (9)$$

Substituting from (9) in (5), we obtain

$$V_0 \sin \beta \left[\left\{ r_0 \left(\frac{r_0}{\mu} \right)^{\frac{1}{2}} + \frac{(\mu l)^{\frac{1}{2}} \mp (\mu l)^{\frac{1}{2}}}{V^2_0} \right\} \times \right.$$
$$\left. \times \left((\mu l)^{\frac{1}{2}} - \triangle V \, r_0 \cos (\gamma_0 + \beta) \right) - r_E^2 \cos^2 \gamma_E \right] = 0 \qquad (10)$$

Combining (2) and (10), we get

$$V_0 \sin\beta \left[\left\{ r_0 \left(\frac{r_0}{\mu}\right)^{\frac{1}{2}} + \frac{(\mu l)^{\frac{1}{2}} \mp (\mu l)^{\frac{1}{2}}}{V_0^2} \right\} \left\{ \mu \left(\frac{2}{r_E} - \frac{1-e^2}{l} \right) + (\triangle V)^2 - 2\triangle V . V_0 \cos\beta \right\} - (\mu l)^{\frac{1}{2}} - \triangle V\, r_0 \cos(\gamma_0 + \beta) \right] = 0 \quad (11)$$

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Equation (11) gives

either

$$(i) \quad V_0 \sin\beta = 0 \quad (12)$$

or

$$(ii) \quad (\mu l)^{\frac{1}{2}} - \triangle V . r_0 \cos(\gamma_0 + \beta)$$

$$= \left\{ r_0 \left(\frac{r_0}{\mu}\right)^{\frac{1}{2}} + \frac{(\mu l)^{\frac{1}{2}} \mp (\mu l)^{\frac{1}{2}}}{V_0^2} \right\} . \left\{ \mu \left(\frac{2}{r_E} - \frac{1-e^2}{l} \right) + (\triangle V)^2 - 2\triangle V . V_0 \cos\beta \right\} \quad (13)$$

Equation (13) gives relation between β and θ_0 which differs from (7). Therefore β and θ_0 satisfying (5) and (6) cannot satisfy (13) for arbitrary choice of $\triangle V$ and r_E and hence (13) is invalid and therefore (12) holds good which gives $\beta = 0$. Substitution of $\beta = 0$ in (7) gives $\theta_0 = 0$ or π. Since γ_E is a continuous and derivable function of β, θ_0 in the domain $-\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2}$ and $0 \leqslant \theta_0 \leqslant 2\pi$, therefore at $(\beta = 0, \theta_0 = 0)$ and $(\beta = 0, \theta_0 = \pi)$, γ_E attains its extreme values. To decide which of the two points corresponds to the maximum and which to the minimum value of γ_E, we get from (2)

$$\cos \gamma_{E\,(\beta=0,\,\theta_0=0)} = \frac{(\mu l)^{\frac{1}{2}} - \triangle V . r_p}{r_E \left[\xi - 2\triangle V . V_p \right]^{\frac{1}{2}}} \quad (14)$$

and

$$\cos \gamma_{E\,(\beta=0,\,\theta_0=\pi)} = \frac{(\mu l)^{\frac{1}{2}} - \triangle V . r_a}{r_E \left[\xi - 2\triangle V . V_a \right]^{\frac{1}{2}}} \quad (15)$$

where

$$\xi = \mu \left(\frac{2}{r_E} - \frac{1-e^2}{l} \right) + (\triangle V)^2$$

and subscripts p, a represent values at the perigee and apogee respectively of the initial orbit.

Now since $r_p < r_a$ and $V_p > V_a$, therefore from (14) and (15) for specified $\triangle V$ and r_E, we have

$$\cos \gamma_{E\,(\beta=0,\,\theta_0=0)} > \cos \gamma_{E\,(\beta=0,\,\theta_0=\pi)}$$

which gives

$$\gamma_{E\,(\beta=0,\,\theta_0=0)} < \gamma_{E\,(\beta=0,\,\theta_0=\pi)}$$

Thus γ_E attains its maximum value at the point $(\beta = 0, \theta_0 = \pi)$ and minimum value at $(\beta = 0, \theta_0 = 0)$. The minimum and maximum value of γ_E are given by

(14) and (15) respectively. Therefore we arrive at the conclusion that for specified ΔV and r_E, atmospheric entry angle is maximum when ΔV is applied tangentially at the apogee and minimum when ΔV is applied tangentially at the perigee of the initial orbit.

EXISTENCE CONDITIONS FOR EXTREME ENTRY ANGLES

Equations (14) and (15) can be transformed into

$$\cos \gamma_{E\,(\beta=0,\ \theta_0=0)} = \frac{l}{r_E \left\{ (1+e)^2 + \dfrac{2\mu \left(\dfrac{1}{r_E} - \dfrac{1+e}{l} \right)}{\left[\left(\dfrac{\mu}{l} \right)^{\frac{1}{2}} - \dfrac{\Delta V}{1+e} \right]^2} \right\}^{\frac{1}{2}}} \tag{16}$$

and

$$\cos \gamma_{E\,(\beta=0,\ \theta_0=\pi)} = \frac{l}{r_E \left\{ (1-e)^2 + \dfrac{2\mu \left(\dfrac{1}{r_E} - \dfrac{1-e}{l} \right)}{\left[\left(\dfrac{\mu}{l} \right)^{\frac{1}{2}} - \dfrac{\Delta V}{1-e} \right]^2} \right\}^{\frac{1}{2}}} \tag{17}$$

For existence of minimum γ_E, (16) gives

$$(1+e)\left(\frac{\mu}{l}\right)^{\frac{1}{2}}\left[1 - \left(\frac{2r_E}{l + r_E\,(1+e)} \right)^{\frac{1}{2}} \right] \leqslant \Delta V \leqslant (1+e)\left(\frac{\mu}{l}\right)^{\frac{1}{2}}\left[1 + \left(\frac{2r_E}{l + r_E\,(1+e)} \right)^{\frac{1}{2}} \right] \tag{18}$$

Similarly for existence of maximum γ_E, (17) gives

$$\left(\frac{\mu}{l}\right)^{\frac{1}{2}}(1-e)\left[1 - \left(\frac{2r_E}{l + r_E\,(1-e)} \right)^{\frac{1}{2}} \right] \leqslant \Delta V \leqslant \left(\frac{\mu}{l}\right)^{\frac{1}{2}}(1-e)\left[1 + \left(\frac{2r_E}{l + r_E\,(1-e)} \right)^{\frac{1}{2}} \right] \tag{19}$$

Thus if (18) is satisfied and (19) is not satisfied, only minimum γ_E will exist and vice versa. But if (18) and (19) both are satisfied, both maximum and minimum γ_E will exist and if none of them is satisfied, no extreme will exist.

CONCLUSIONS

For a rocket vehicle moving initially in an elliptic orbit,

(i) Maximum atmospheric entry angle at a given entry altitude occurs by tangentia retrofire impulse at the apogee.

(ii) Minimum atmospheric entry angle at a given entry altitude occurs by tangentia retrofire impulse at the perigee.

(iii) Existence of only maximum or minimum or both or neither maximum nor minimum atmospheric angles depend upon (18) and (19).

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REFERENCE

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