# A NOTE ON ATTAINMENT OF CONSTANT DRIVING PRESSURE IN A TAPERED BORE GUN WITH MODERATED CHARGES

R. N. BHATTACHARYYA

Maharaja Manindra Chandra College, Calcutta

(*Received* 4 *August* 1969; *revised* 31 *March* 1970)

It has been shown that suitable moderated charge with two components can be found such that in a tapered bore gun the pressure driving the shot remains absolutely constant throughout the period when the second component burns, the constant pressure being the pressure at the 'burnt' of the first component.

In a recent paper Ray[1] demonstrated that if a moderated charge of two components (of which first component is known and the second component is also known except for the size and shape) burns in an orthodox gun of known constant cross sectional area, the pressure, during the period the second burns, can be kept constant by suitable choice of the size and shape of the second component. In the present paper we have demonstrated that a constant pressure phase can be attained in the second stage of burning by a suitable choice of the size of the second component (all other physical properties regarding the two components being known) and a suitable choice of cross sectional area.

## BALLISTIC EQUATIONS WHEN THE FIRST COMPONENT BURNS

Assuming (i) linear rate of burning and (ii) neglecting co-volume terms, the fundamental equations of internal ballistics are:

$$F_1C_1Z_1 = P\left[\int_0^x A dx + A_0 l - C_1/\delta_1 - C_2/\delta_2\right] + \frac{1}{2}\omega_1(\gamma - 1)v^2 \quad (1)$$

$$\omega_1 \frac{dv}{dt} = AP \quad (2)$$

$$D_1 \frac{df_1}{dt} = -\beta_1 P \quad (3)$$

$$Z_1 = (1 - f_1)(1 + \theta_1 f_1) \quad (4)$$

where the suffix 1 represents the first component.

The area of the cross section $A$ of the bore is assumed to be a continuous and differentiable function of the shot travel, i.e. $x$; $A_0$ being the area of the cross section at $x = 0$ and we take

$$A = A(x) \quad (5)$$

To rewrite the equations (1) to (5) in terms of non-dimensional variables, we make the following transformations.

$$\xi = \frac{1}{A_0 l}\left[\int_0^x A dx - C_1/\delta_1 - C_2/\delta_2\right] \quad (6)$$

$$\zeta = \frac{A_0 l}{F_1 C_1} P \tag{7}$$

$$\eta = \frac{A_0 D_1}{F_1 C_1 \beta_1} v \tag{8}$$

$$M_1 = \frac{A_0^2 D_1^2}{F_1 C_1 \beta_1^2 \omega_1} \tag{9}$$

The transformed equations are

$$Z = \zeta\xi + \frac{1}{2} (\gamma - 1) \eta^2 / M_1 \tag{10}$$

$$\eta \frac{d\eta}{d\xi} = M_1 \zeta \tag{11}$$

$$\eta \frac{df}{d\xi} = -\left( \frac{A_0}{A} \right) \zeta \tag{12}$$

$$Z = (1 - f_1)(1 + \theta_1 f_1) \tag{13}$$

and

$$A = A(\xi) \tag{14}$$

Equations (10) to (14) are to be integrated with the initial conditions $\xi = \xi_0$, $\eta = 0$, $\zeta = \zeta_0$ and $A = A_0$. Kapur[2] explained method of solving the ballistic equations for tapered bore gun. Now suppose $\xi_{B1}$, $\eta_{B1}$, $\zeta_{B1}$, the values of $\xi$, $\eta$, and $\zeta$ at the instant when the first component burns out, are known.

## BALLISTIC EQUATIONS WHEN THE SECOND COMPONENT BURNS

The equations are

$$F_1 C_1 + F_2 C_2 Z_2 = P \left\{ \int_0^x A dx + A_0 l - C_1/\delta_1 - C_2/\delta_2 \right\} + \frac{1}{2} \omega_1 (\gamma - 1) v^2 \tag{15}$$

$$\omega_1 \frac{dv}{dt} = \omega_1 v \frac{dv}{dx} = AP \tag{16}$$

$$D_2 \frac{df_2}{dt} = - \beta_2 P \tag{17}$$

$$Z_2 = (1 - f_2)(1 + \theta_2 f_2) \tag{18}$$

Here we shall assume a solution $P = P_{B1}$ (if possible) and suppose that these equations determine the unknown functions $A$, $x$, $Z_2$ and $f_2$ subject to the conditions

$$x = x_{B1}, \; v = v_{B1}, \text{ and } A = A_{B1} \text{ at } f_2 = 1$$

Following Ray's method we get,

$$F_2 C_2 \frac{dZ_2}{df_2} = - \gamma v \frac{A D_2}{\beta_2} \tag{19}$$

and

$$v = v_{B1} - \frac{D_2}{\beta_2 \omega_1} \int_1^{f_2} A df_2 \tag{20}$$

From (18) and (19) we get

$$1+\theta_2 = \frac{\gamma D_2 v_{B1}}{F_2 C_2 \beta_2} A_{B1} \tag{21}$$

Now if the shape ( $\theta_2$ ) of the second component be known, the size $(D_2/\beta_2)$ can be determined from the relation (21).

In order to get area in the second stage of burning, we proceed as follows :

From (18), (19) and from the relation $\frac{dv}{df_2} = -\frac{AD_2}{\beta_2\omega_1}$ we have the first order differential equation satisfied by area $A$ during the second stage of burning as follows :

$$\frac{dA}{df_2}(\theta_2 - 2\theta_2 f_2 - 1) = -\frac{\gamma D_2^2}{F_2 C_2 \beta_2^2 \omega_1} A^3 - 2\theta_2 A \tag{22}$$

Solving (22) we have for $\theta_2 = 0$

$$\left(\frac{A_{B1}}{A}\right)^2 = 1 + \frac{2\beta_0 M_1}{\gamma \eta_{B1}^2}(1-f_2) \tag{23}$$

and for $\theta_2 \neq 0$ and $-1$

$$\left(\frac{A}{A_{B1}}\right)^2 \times \frac{2\theta_2 + \dfrac{\beta_0 M_1 (1+\theta_2)^2}{\gamma\eta_{B1}^2}}{2\theta_2 + \dfrac{\beta_0 M_1 (1+\theta_2)^2}{\gamma\eta_{B1}^2}\left(\dfrac{A}{A_{B1}}\right)^2} = \frac{(1+2\theta_2 f_2 - \theta_2)^2}{(1+\theta_2)^2} \tag{24}$$

where

$$\beta_0 = \frac{F_2 C_2}{F_1 C_1} .$$

The simultaneous satisfaction of equation (21) and (23) or (24) gives the condition that $P = P_{B1}$, may be a solution of the equations (15) to (18) if the shape of the second component be considered known.

*Determination of x and v*

*Case I* : $\theta_2 = 0$

From (23) we have

$$\left(\frac{A_{B1}}{A}\right)^2 = 1 + M_0(1-f_2)$$

where

$$M_0 = \frac{2\beta_0 M_1}{\gamma\eta_{B1}^2}$$

Then from (20) we get,

$$v = v_{B1} + \frac{2D_2}{\beta_2\omega_1}\frac{A_{B1}}{M_0}\left[\sqrt{1+M_0(1-f_2)} - 1\right] \tag{25}$$

and

$$\frac{x - x_{B1}}{l} = \frac{\beta_0}{\gamma}\frac{A_0}{A_{B1}}\frac{1}{\zeta_{B1}}(1-f_2) - \frac{\beta_0}{\zeta_{B1}}\frac{A_0}{A_{B1}}\left[(1-f_2) - \frac{2}{3M_0}\left\{\left(1+M_0(1-f_2)\right)^{\frac{3}{2}} - 1\right\}\right]. \tag{26}$$

*Case II :* $\theta_2 \neq 0$ and $-1$

From (24) we have

$$\left(\frac{A}{A_{B1}}\right)^2 \times \frac{2\theta_2 + KA_{B1}^2}{2\theta_2 + KA^2} = \frac{(f_2')^2}{(1+\theta_2)^2}$$

where $1 + 2\theta_2 f_2 - \theta_2 = f_2'$

and $$KA_{B1}^2 = \frac{\beta_0 M_1 (1 + \theta_2)^2}{\gamma \eta_{B1}^2}$$

We shall consider two cases:

*Subcase I :* $\theta_2 > 0$

Let $$(K_1)^2 = \frac{(1+\theta_2)^2}{4\theta_2^2} + \frac{(1+\theta_2)^2}{2\theta_2 K A_{B1}^2}$$

Then

$$v = v_{B1} + \frac{D_2}{\beta_2 \omega_1 \sqrt{2\theta_2 K}} \left[ (4\theta_2^2 K_1^2 - f_2'^2)^{\frac{1}{2}} - \left\{ 4\theta_2^2 K_1^2 - (1+\theta_2)^2 \right\}^{\frac{1}{2}} \right] \tag{27}$$

and

$$x - x_{B1} = K_2 \Bigg[ \theta_2 K_1^2 \Bigg\{ \sin^{-1} \frac{1+\theta_2}{2\theta_2 K_1} - \sin^{-1} \frac{f_2'}{2\theta_2 K_1} + \frac{1+\theta_2}{4\theta_2^2 K_1^2} \times$$
$$\times \sqrt{4\theta_2^2 K_1^2 - (1+\theta_2)^2} - \frac{f_2'}{4\theta_2^2 K_1^2} \sqrt{4\theta_2^2 K_1^2 - f_2'^2} \Bigg\}$$
$$+ \sqrt{4\theta_2^2 K_1^2 - (1+\theta_2)^2} \times \left( \frac{f_2' - 1 - \theta_2}{2\theta_2} \right) \Bigg] +$$
$$+ \frac{1+\theta_2}{2\theta_2} \frac{\beta_0}{\gamma} \frac{A_0}{A_{B1}} \frac{1}{\zeta_{B1}} (1 + \theta_2 - f_2') \tag{28}$$

where $$K_2 = \frac{(1+\theta_2)\, l \sqrt{M_1}\, \beta_0^{3/2}}{\gamma^{3/2}\, \eta_{B1}\, \zeta_{B1} \sqrt{2\theta_2}} \frac{A_0}{A_{B1}}$$

*Subcase II :* $\theta_2 < 0$

$$v = v_{B1} - \frac{2D_2\theta_2}{\beta_2 \omega_1 \sqrt{-2\theta_2 K}} \left[ \left( \frac{f_2'^2}{4\theta_2^2} - K_3^2 \right)^{\frac{1}{2}} - \left( \frac{(1+\theta_2)^2}{4\theta_2^2} - K_3^2 \right)^{\frac{1}{2}} \right] \tag{29}$$

and

$$\frac{x - x_{B1}}{l} = K_2' \Bigg[ 2\theta_2 \Bigg\{ \frac{f_2'}{4\theta_2} \left( \frac{f_2'^2}{4\theta_2^2} - K_3^2 \right)^{\frac{1}{2}} - \frac{K_3^2}{2} \log \left| \frac{f_2'}{2\theta_2} + \sqrt{\frac{f_2'^2}{4\theta_2^2} - K_3^2} \right| -$$
$$- \frac{1+\theta_2}{4\theta_2} \left( \frac{(1+\theta_2)^2}{4\theta_2^2} - K_3^2 \right)^{\frac{1}{2}} + \frac{K_3^2}{2} \log \Bigg| \frac{1+\theta_2}{2\theta_2} +$$
$$+ \sqrt{\frac{\overline{1+\theta_2}^2}{4\theta_2^2} - K_3^2} \Bigg| \Bigg\} + \left( \frac{(1+\theta_2)^2}{4\theta_2^2} - K_3^2 \right)^{\frac{1}{2}} (1 + \theta_2 - f_2') \Bigg] +$$
$$+ \frac{1+\theta_2}{2\theta_2} \frac{\beta_0}{\gamma} \frac{A_0}{A_{B1}} \frac{1}{\zeta_{B1}} (1 + \theta_2 - f_2') \tag{30}$$

where $$K_2' = \frac{(1+\theta_2) \sqrt{M_1}\, \beta_0^{3/2}}{\gamma^{3/2}\, \eta_{B1}\, \zeta_{B1} \sqrt{-2\theta_2}} \frac{A_0}{A_{B1}}$$

Hence the equations (24), (26), (28) and (30) give the area $A$ for any shot travel for all values of $\theta_2$ except at $\theta_2 = -1$. The numerical values of $x$ and $f_2$ and $A$ have been calculated for negative values of $\theta_2$.

In order to check whether area thus obtained is decreasing, the numerical values have been obtained and for the simplicity of calculations we consider that the area in the first stage of burning is constant. The solution of the equations during the first stage of burning having constant bore area are obtainable from tables given in H.M.S.O. (1951).

Let $$\beta_0 = 1 \,,\ M_1 = 1\cdot 254 \,,\ \zeta_0 = 0\cdot 2 \,,\ \gamma_1 = 1\cdot 4 \,,\ \theta_1 = 0$$

then $$\frac{\eta_{B1}}{M_1} = 0\cdot 800.$$

Further we take $$\theta_2 = -0\cdot 2$$

then $$\frac{D_2/\beta_2}{D_1/\beta_1} = 0\cdot 5696 \,, \qquad \zeta_{B1} = 0\cdot 404$$

| $[A/A_{B1}]$ | $[f_2]$ | $[(x-x_{B1})/l]$ |
|---|---|---|
| 0·98 | 0·8975 | 0·0354 |
| 0·97 | 0·8369 | 0·0652 |
| 0.96 | 0·7728 | 0·1254 |
| 0·95 | 0·6934 | 0·1864 |
| 0·94 | 0·5936 | 0·2559 |
| 0·93 | 0·4839 | 0·4041 |
| 0·92 | 0·3872 | 0·6226 |
| 0·90 | 0·0300 | 0·8221 |

Let $$\beta_0 = 1 \,,\ M_1 = 1 \,,\ \zeta_0 = 0\cdot 1 \,,\ \gamma_1 = 1\cdot 4 \,,\ \theta_1 = 0$$

then $$\frac{\eta_{B1}}{M_1} = 0\cdot 900 \,,\quad \theta_2 = -0\cdot 1 \,,\quad \frac{D_2/\beta_2}{D_1/\beta_1} = 0\cdot 4296$$

and $$\zeta_{B1} = 0\cdot 404$$

| $[A/A_{B1}]$ | $[f_2]$ | $[(x-x_{B1})/l]$ |
|---|---|---|
| 0·98 | 0·8632 | 0·0526 |
| 0·97 | 0·8229 | 0·1125 |
| 0·96 | 0·7257 | 0·1659 |
| 0·95 | 0·6225 | 0·2354 |
| 0·94 | 0·5539 | 0·3125 |
| 0·93 | 0·4727 | 0·4495 |
| 0·92 | 0·2295 | 0·7229 |
| 0·90 | 0·0612 | 0·9121 |

ACKNOWLEDGEMENT

The author is extremely grateful to Dr. A. Ray for suggesting the problem and for his guidance and helpful suggestions in course of preparation of this paper.

## REFERENCES


1. RAY, A., *Proc. Nat. Inst. Sci.* (India), 30(A), (1964).
2. KAPUR, J. N., *Proc. Nat. Inst. Sci.* (India), 23(A), (1957), 438.