MODEL FOR CALCULATION OF MUZZLE VELOCITY OF ORTHODOX GUNS

ASIM RAY

Calcutta Technical School, Calcutta

(*Received* 15 *November* 1969)

Ballistics calculated with this model gives values for muzzle velocity fairly in agreement with those given by Hunt-Hinds system. Equations in this model have also been integrated approximately and the approximate formulae also are found to give quite good values for muzzle velocity.

Hunt-Hinds System[1] (HHS), of internal ballistics is the best of all the existing solutions in this field. It assumes a shot-start pressure, i.e. it is thought that the shot begins to move only when certain pressure (shot-start pressure) has been developed in the gun. It appears that it does not take into account the bore resistance. If it does, it is done by changing the effective shot-weight which is true only when the resistance is proportional to the pressure. But so far our experimental knowledge goes, the bore resistance is seen to be quite different. Now why HHS should be so good even after neglecting or poorly representing the bore resistance, is a matter of investigation. As an answer to this we may think that the shot-start pressure actually replaces the whole resistance by a concentrated resistance—so to say—at the initial position of the shot. With this picture in mind one may ask, whether we can represent the whole resistance by a constant resistance spread over—so to say—from shot-start to all burnt and also get the muzzle velocity in fair agreement with those given by HHS.

We have found the answer in the affirmative. We have demonstrated this by calculating the muzzle velocity of orthodox gun by assuming a constant resistance, from shot-start to all burnt, in the isothermal model after neglecting the co-volume and then comparing it with the muzzle velocity given by HHS. Here we must say how the comparison between the two solutions was made. We must remember that the shot-start pressure term ζ_H in the HHS is first determined from the recorded peak pressure; then with this ζ_H the muzzle velocity and other features are calculated. Similarly we have first found the bore-resistance term ζ_0 from the recorded peak pressure and then with this ζ_0 muzzle velocity has been calculated. The two muzzle velocities are found to be in quite satisfactory agreement when we remember that our calculation is in isothermal model without the co-volume term. We have integrated our equations numerically as they are, as usual, non-linear and non-integrable. But an approximate solution has been found which compares favourably with exact numerical solution for values of M around unity (M is the central ballistic parameter). This approximate solution also gives values for muzzle velocity in good agreement with that in HHS if we first calculate the bore-resistance term ζ_0' (according to the approximate solution) to give the recorded peak pressure and then calculate the muzzle velocity with this ζ_0'. The approximate solution is easy to handle, and useful for values of M in the neighbourhood of unity. The whole discussion is confined only to tubular propellants; but can be easily extended to cover quadratic form function and co-volume.

BURNING EQUATIONS: APPROXIMATE SOLUTION

With notations used in HHS[1], the basic equations for this model are

$$FCZ = Ap\,(x + l)\ , \tag{1}$$

and

$$D\,\frac{df}{dt} = -\beta p\ , \tag{2}$$

$$\omega\, v\,\frac{dv}{dx} = Ap_s - AA_0\ , \tag{3}$$

where p_s is the pressure at the shot-base and AA_0 is the constant resistance and

$$Z = 1 - f\ .$$

The initial conditions are

$$x = 0\ ,\ v = 0\ ,\ p = p_0\ ,\ Z = Z_0 \quad \text{at} \quad f = f_0\ .$$

Considering the instant of shot-start, from (3) it follows that

$$A\,(p_s)_0 = AA_0\ .$$

So by Lagrange's correction namely

$$p_s = \frac{p}{1 + \dfrac{C}{3\omega}}$$

we have

$$AA_0 = A\,(p_s)_0 = \frac{Ap_0}{1 + \dfrac{C}{3\omega}}$$

Therefore equation (3) can now be written as

$$\left(\omega + \frac{1}{3}C\right) v\,\frac{dv}{dx} = A\,(p - p_0)\ ,$$

and modifying the shot-mass as in HHS we have

$$\omega_1\, v\,\frac{dv}{dx} = A\,(p - p_0) \tag{4}$$

where

$$\omega_1 = 1{\cdot}05\omega + \frac{1}{3}C\ .$$

Now with the following usual substitutions

$$\xi = 1 + \frac{x}{l}\ , \tag{5}$$

$$\eta = \frac{vAD}{FC\beta}\ , \tag{6}$$

$$\zeta = \frac{pAl}{FC}\ , \tag{7}$$

$$\zeta_0 = \frac{p_0\,Al}{FC}\ , \tag{8}$$

$$M = \frac{A^2 D^2}{FC\beta^2 \omega_1}, \tag{9}$$

equations (1) to (4) reduce to

$$Z = \zeta \xi \tag{10}$$

$$\eta \frac{df}{d\xi} = -\zeta \tag{11}$$

$$\eta \frac{d\eta}{d\xi} = M(\zeta - \zeta_0) \tag{12}$$

$$Z = 1 - f; \tag{13}$$

and the initial conditions are now

$$\xi = 1, \ \eta = 0, \ \zeta = \zeta_0 \text{ at } Z = Z_0 \text{ or at } f = f_0 .$$

We note further that equation (10) gives the initial values

$$Z_0 = \zeta_0 \tag{14}$$

From (10), (11) and (13) we have

$$\eta = \zeta \frac{d\xi}{dZ} = \zeta \frac{d}{dZ}\left(\frac{Z}{\zeta}\right)$$

or

$$\eta = 1 - \frac{Z}{\zeta}\frac{d\zeta}{dZ} . \tag{15}$$

From (11), (12) and (13) we have

$$\frac{d\eta}{dZ} = M\left(1 - \frac{\zeta_0}{\zeta}\right) \tag{16}$$

which by (15) gives

$$\frac{d}{dZ}\left[\frac{Z}{\zeta}\frac{d\zeta}{dZ}\right] = -M\left(1 - \frac{\zeta_0}{\zeta}\right) \tag{17}$$

This is the differential equation for pressure ζ. The equation is non-linear and can be integrated numerically which has been done. However, we first give an approximate solution of this equation by successive approximations.

Since $\zeta = \zeta_0$ initially, as a first approximation we put $\zeta = \zeta_0$ on the right hand side of (16) and get

$$\frac{d}{dZ}\left[\frac{Z}{\zeta}\frac{d\zeta}{dZ}\right] = 0 . \tag{18}$$

Integrating and using (15), we have

$$\frac{Z}{\zeta}\frac{d\zeta}{dZ} = 1 - \eta \tag{19}$$

$$= \text{Constant} = 1 \text{ (initial value)}.$$

TABLE 1

$M = 1, \zeta_0 = Z_0 = 0 \cdot 1$

	Numerical integration			Approximate solution		
Z	ξ	η	ζ	ξ	η	ζ
0·1	1·00000	0	·10000	1·00000	0	·10000
0·2	1·00546	·03782	·19888	1·00648	·03063	·19867
0·3	1·03209	·09792	·29069	1·03026	·09014	·29118
0·4	1·07208	·16769	·37311	1·06736	·16137	·37475
0·5	1·12175	·24325	·44572	1·11575	·23906	·44812
0·6	1·18080	·32228	·50812	1·17384	·32082	·51113
0·7	1·24852	·40358	·56065	1·24124	·40541	·56397
0·8	1·32488	·48641	·60383	1·31760	·49206	·60712
0·9	1·40985	·57032	·63838	1·40211	·58028	·64191
1·0	1·5037	·65499	·66503	1·4987	·66974	·66723

TABLE 2

$M = 1, \zeta_0 = Z_0 = 0 \cdot 3$

	Numerical integration			Approximate solution		
Z	ξ	η	ζ	ξ	η	ζ
0·3	1·00000	0	·30000	1·00000	0	·30000
0·4	1·00128	·01366	·39949	1·00128	·01370	·39948
0·5	1·00755	·04646	·49624	1·00765	·04675	·49620
0·6	1·02000	·09105	·58823	1·02030	·09206	·58806
0·7	1·03845	·14349	·67411	1·03894	·14581	·67380
0·8	1·06240	·20144	·75298	1·06336	·20575	·75237
0·9	1·09188	·26346	·82431	1·09359	·27042	·82296
1·0	1·1263	·32837	·88787	1·1289	·33881	·88581

TABLE 3

$M = 2, \zeta_0 = Z_0 = 0 \cdot 1$

	Numerical integration			Approximate solution		
Z	ξ	η	ζ	ξ	η	ζ
0·1	1·00000	0	·10000	1·00000	0	·10000
0·2	1·01122	·05432	·19776	1·01338	·06126	·19735
0·3	1·05576	·17085	·28415	1·06149	·18028	·28263
0·4	1·12956	·30754	·35413	1·13952	·32274	·35102
0·5	1·22890	·45508	·40687	1·24495	·47812	·40164
0·6	1·35330	·60814	·44336	1·37796	·64164	·43543
0·7	1·50374	·76423	·46549	1·54056	·81082	·45437
0·8	1·68256	·92180	·47547	1·73632	·98412	·46075
0·9	1·89279	1·0798	·47551	1·96965	1·1606	·45693
1·0	2·1381	1·2374	·46769	2·2460	1·3395	·44523

This on integration subject to the initial condition (14) gives

$$\zeta = Z .$$

For further approximation we put the first approximation Z of ζ on the right hand side of (17) and get

$$\frac{d}{dZ}\left[\frac{Z}{\zeta}\frac{d\zeta}{dZ}\right] = -M\left(1-\frac{\zeta_0}{Z}\right) . \tag{20}$$

Integrating subject to initial conditions we have

$$\frac{Z}{\zeta}\frac{d\zeta}{dZ} = 1 - M\,(Z - Z_0) + M\,\zeta_0 \log\frac{Z}{Z_0} , \tag{21}$$

since by (15)

$$\left(\frac{Z}{\zeta}\frac{d\zeta}{dZ}\right)_0 = (1-\eta)_0 = 1 .$$

Integrating (21) and remembering the initial conditions we get

$$\log\frac{\zeta}{\zeta_0} = \log\frac{Z}{Z_0} - M\,(Z - Z_0) + M\,Z_0 \log\frac{Z}{Z_0} + \frac{M\zeta_0}{2}\left(\log\frac{Z}{Z_0}\right)^2 . \tag{22}$$

No simple integration for a further approximation is possible in this manner. To this approximation we may use (22) to determine ζ as a function of Z.

Since by (15) and (21)

$$\eta = M\,(Z - Z_0) - M\,\zeta_0 \log\frac{Z}{Z_0} , \tag{23}$$

equation (23) determines η in this approximation as a function of Z. Equation (10) also gives

$$\xi = \frac{Z}{\zeta} . \tag{24}$$

So we may take (22) and (24) together to determine ξ in this approximation as a function of Z. Thus in this approximate solution we may express ξ, η, ζ as a function of Z by (22) to (24).

NUMERICAL AND APPROXIMATE SOLUTIONS

Here we discuss how (22) to (24) compare with the exact solution found by numerical integration. Equations (15) and (16) were integrated numerically with initial conditions $\eta = 0$, $\zeta = \zeta_0 \ (= Z_0)$ at $Z = Z_0$ by the method of Runge-Kutta in steps of $Z = 0 \cdot 05$ in three cases namely (a) $M = 1$, $\zeta_0 = Z_0 = 0 \cdot 1$, (b) $M = 2$, $\zeta_0 = Z_0 = 0 \cdot 1$, (c) $M = 1$, $\zeta_0 = Z_0 = 0 \cdot 3$. The values of ξ, η and ζ obtained by numerical method as also obtained by approximate solution are given in Tables 1 to 3 in steps of $Z = 0 \cdot 1$. It is evident that the approximate solution is as good as the numerical solution especially when M is in the neighbourhood of unity. For large M the difference between the two solutions is not quite negligible.

MAXIMUM PRESSURE IN APPROXIMATE SOLUTION

For maximum pressure we put

$$\frac{d\zeta}{dZ} = 0$$

which by (21) gives

$$1 - M\,(Z - Z_0) + M\,\zeta_0 \log \frac{Z}{Z_0} = 0\,. \qquad (25)$$

Further differentiating (21) we have

$$\frac{d^2\zeta}{dZ^2} = -\frac{\zeta}{Z^2}\left(1 + M\zeta_0 \log \frac{Z}{Z_0}\right)$$

under the condition $\frac{d\zeta}{dZ} = 0\,.$

Hence $\frac{d^2\zeta}{dZ^2} < 0$ when $\frac{d\zeta}{dZ} = 0\,.$

Thus if Z_1, the root of the equation (25) be less than unity, then at $Z = Z_1$ the pressure will be a maximum and since $Z = Z_1$ satisfies (25) we must also have

$$1 - M\,(Z_1 - Z_0) + M\zeta_0 \log \frac{Z_1}{Z_0} = 0\,. \qquad (26)$$

The maximum pressure ζ_1 will, by (22), (26) and (25), be given by

$$\log \zeta_1 = -1 + \log Z_1 + \frac{M\zeta_0}{2}\left(\log \frac{Z_1}{Z_0}\right)^2. \qquad (27)$$

If, however, Z_1 be greater than 1 the maximum pressure will occur at all-burnt.

MUZZLE VELOCITY

It is well known that η_3 (η at muzzle) is given by

$$\eta_3{}^2 = \eta_2{}^2 + \frac{2M}{\gamma - 1}\left[1 - \left(\frac{\xi_2}{\xi_3}\right)^{\gamma-1}\right], \qquad (28)$$

where suffixes 2 & 3 indicate values at all-burnt and at muzzle respectively.

COMPARISON WITH HHS

Comparison is shown by taking the following concrete examples: (In all the examples we have taken $\gamma = 1\cdot25$, $\theta = 0$, $B = 0\cdot25$ where B is the co-volume term in HHS).

Example 1 : $M = 1, \quad \xi_3 = 9$

Numerical integration with $\zeta_0 = 0\cdot1$ gives (Table 1)

$$\zeta_1 = \zeta_2 = 0\cdot665$$

and $\eta_2 = 0\cdot655, \qquad \xi_2 = 1\cdot504, \qquad \eta_3 = 1\cdot820.$

For same peak pressure approximate integration gives :

$$\zeta_0' = 0\cdot099, \qquad \eta_3' = 1\cdot827,$$

and HHS by H.M.S.O.[1] Tables gives :

$\zeta_H = 0\cdot213$, $\eta_{3H} = 1\cdot821$ with $dE = 0\cdot02$. (dE is the correction term for muzzle velocity in HHS). Thus the muzzle velocities as calculated by (i) numerical integration, (ii) approximate solution and (iii) HHS Tables[1], all giving the same peak pressure, are

$$\eta_3 = 1\cdot820, \qquad \eta_3' = 1\cdot827 \qquad \eta_{3H} = 1\cdot821$$

The agreement is quite good.

Example 2 : $M = 2, \qquad \xi_3 = 9$

Numerical integration with $\zeta_0 = 0\cdot1$ gives (Table 3)

$\zeta_1 = 0\cdot477, \qquad \zeta_2 = 0\cdot468, \qquad \eta_2 = 1\cdot237, \qquad \xi_2 = 2\cdot138, \qquad \eta_3 = 2\cdot523.$

Approximate solution gives

$$\zeta_0' = 0\cdot108, \qquad \eta_3' = 2\cdot555$$

HHS gives

$$\zeta_H = 0\cdot232 \qquad \eta_{3H} = 2\cdot532 \quad \text{with} \quad dE = 0\cdot01.$$

The three muzzle velocities to be compared are

$$\eta_3 = 2\cdot523, \qquad \eta_3' = 2\cdot555, \qquad \eta_{3H} = 2\cdot532$$

Example 3 : $M = 4, \qquad \xi_3 = 32\cdot25$

HHS with $\zeta_H = 0\cdot1$ gives

$$\eta_{3H} = 3\cdot670 \text{ with } dE = 0.$$

Approximate solution gives

$$\zeta_0' = 0\cdot038, \qquad \eta_3' = 4\cdot212$$

The two muzzle velocities differ considerably. This is probably due to abnormally large ξ_3 and large M which will seldom occur in practice.

Example 4 : $M = 0\cdot8, \qquad \xi_3 = 4$

HHS with $\zeta_H = 0\cdot2$ gives

$$\eta_{3H} = 1\cdot348 \text{ with } dE = 0\cdot02$$

Approximate solution gives

$$\zeta_0' = 0\cdot18, \qquad \eta_3' = 1\cdot346 .$$

We take another example to see how exact solution compares with HHS for large M in which case approximate solution is no good as seen in Example 3. For this example details of numerical integration cannot be given.

Example 5 : $M = 4, \qquad \xi_3 = 9$

Numerical integration with $\zeta_0 = 0\cdot001$ gives

$$\zeta_1 = 0\cdot113, \qquad \eta_3 = 2\cdot378$$

HHS gives

$$\zeta_H = 0\cdot022, \qquad \eta_{3H} = 2\cdot223 \text{ with } dE = 0\,.$$

The agreement between muzzle velocities is not bad.

From these examples it is permissible to conclude that except for large M the internal ballistic system with constant bore resistance, as analysed above and represented by Tables 1–3 of numerical integration, gives the muzzle velocity in satisfactory agreement (Examples 1, 2 & 5) with the values given by HHS, the basis of comparison being the same value of peak pressure. A very good correspondence thus exists between the ballistics with bore resistance developed by us and the ballistics of HHS. Further our approximate formulae are also quite usable, except possibly for large M, (Examples 1–4) in calculating the muzzle velocity which is in agreement with that of HHS.

ACKNOWLEDGEMENT

The author gratefully remembers the suggestions and criticisms of Late Prof. N. R. Sen during the preparation of this paper and acknowledges with thanks the facilities enjoyed by him at the Seminar organised under the research scheme of National Professor S. N. Bose at the Indian Association for the Cultivation of Science, Jadavpur.

REFERENCE

1. H.M.S.O. — Internal Ballistics, (1951).