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# Mathematical Modelling of Underwater Reverberation

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**Abstract.** A comprehensive treatment of underwater reverberation mentioning the causes for reverberation has been discussed. The different approaches adopted by earlier workers for evolving a mathematical model of reverberation is also discussed. The implementation and validation of a model for volume reverberation on a general purpose computer is presented. The model is a simulation of volume reverberation produced by a constant carrier transmit pulse of arbitrary shape and duration, for use by advanced signal processors.

## 1. Introduction

The performance of an active sonar system to detect underwater targets in the ocean is usually limited by the masking influence of the sound scattered from the inhomogeneties in the ocean and irregularities of the ocean surface and bottom. This back-scattered sound which is due to the propagation of signals transmitted in a statistically homogenous medium, is commonly called reverberation. Sea reverberation relates to the time variation of the total scattered sound field observed at the point of reception following the transmission of a sound signal. It is analogous to radar clutter and scattering of light in turbid media in optics.

A comprehensive treatment of sonar reverberation summarising the results of the research conducted during the World War II was published in 1946[1]. Since then, this subject has assumed an increasingly important place in the fields of underwater acoustics and in sonar design. The theoretical formulae for the average reverberation intensity were derived as in conventional ray theory[2,3]. The expected intensities of reverberation from the volume, surface and bottom are examined separately for their theoretical dependence on many variables. Some of the variables considered are time, the directivity characteristics of the transducer, the transmission loss between the projector and the scatterers, the intensity and duration of the projected signal and the scattering power of the portion of the ocean under consideration.

This paper outlines some of the experimental studies carried out and the problems faced in evolving a mathematical model of reverberation.

## 2. Scatterers and Scattering Strength

As sound propagates in the ocean, it is partially scattered by various inhomogeneties of the medium and irregularities of the boundaries. The scattering of sound in the ocean may be caused by one or more of the following :

(a) air bubbles, (b) fish, marine creatures, micro-organisms, (c) solid particles, (d) temperature inhomogeneties (e) irregularities of the ocean surface, (f) irregularities of the bottom, and (g) inhomogeneties of the bottom soil composition.

The above scatterers can be classified into three basically different classes depending on the reverberation they produce :

(a) scatterers occuring in the volume or body of the sea, which produce 'volume reverberation'. They are caused by micro-organisms, fish, thermal irregularities, etc.

(b) scatterers on or near the surface which cause surface reverberation.

(c) scatterers on or near the sea bottom producing bottom reverberation.

The reverberation is dependent on the transmitted wave-form[4]. Although knowledge of scattering cross section as well as range and velocity structure is far from complete, differences in range and velocity distribution between real target and reverberation do exist although the separation is not as pronounced as one would wish. The sonar design problem is to select the particular signal wave-form which will exploit these differences.

## 3. General Analysis

Two approaches were adopted by earlier workers namely the 'physical approach' first proposed by Carl Eckart[5], and 'discrete scatterer approach' first suggested by Faure[6].

The physical approach analyses the dependence of the scattering acoustic field on the properties of the scattering surface which is described in geometrical terms. The results are generally based on the assumption of a plane monochromatic incident wave.

The discrete scatterer approach assumes that the scattering surface can be treated as if it were a planer surface in which was imbedded a collection of (random) point scatterers. Faure was the first to publish theoretical model for reverberation noise following the discrete scatterer approach. The desirable feature of this approach is that it allows an inclusion of the effect of important sonar system parameters such as the transmitter and receiver directivities and system geometry.

Modern techniques of signal processing of target echoes against reverberation require a knowledge of the second order statistics of the signal and the noise[7]. These data are difficult to acquire under controlled laboratory conditions and almost impossible at sea. The difficulty of this situation is reflected in the literature since there are a very large number of papers and yet a few firm conclusions[8]. The problem of getting reliable statistics for reverberation is complicated by the fact that it is not a stationary process so that one must compute ensemble averages.

According to Shotskii[9] the reduction in the effective span of the reverberation layer in the transmission of tone pulses is equivalent to the reduction in the duration of the transmitted signal, so that the reverberation correlation interval is decreased. The effective width of reverberation spectrum broadens in the transmission of tone pulses and narrows in the transmission of FM pulses. In the latter case, this effect is associated with the skewing of the spectrum, because the spectral components of the

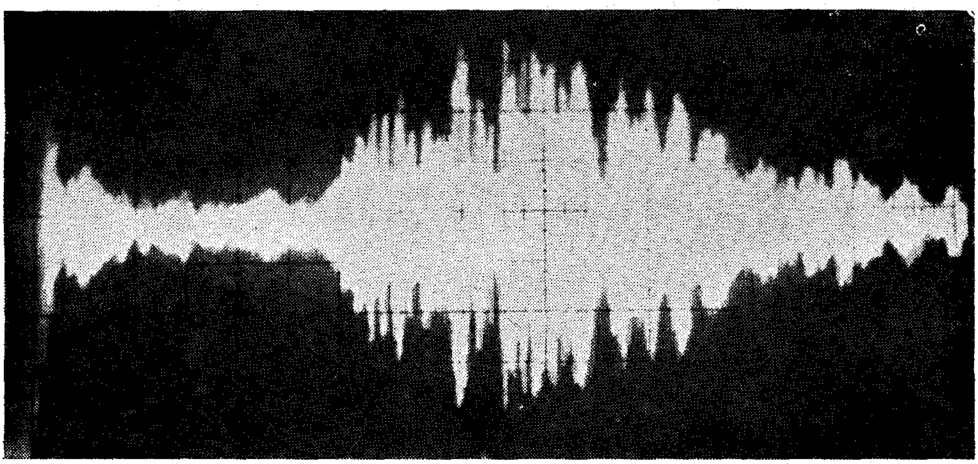

**Figure 1.** Reverberation returns from the rough bottom of the sea insonified by CW pulse width = 10 ms, carrier frequency $f_0$ = 9.5 KHz.

FM signal that are transmitted at the end of transmission period are more strongly emphasised in reverberation. Figs. 1 and 2 give a typical example of reverberation after transmission of CW tone pulse and FM pulse with the same centre frequency.

## 4. Mathematical Modelling

Mathematical modelling of the different types of reverberations can be made by assuming the discrete scatterer approach.

Ol' Shevskii[10] and Middleton[11-14] have modelled acoustic reverberation as the superposition of echoes from Poisson distributed scatterers. Ol' Shevskii's analysis of the statistical characteristic of sea reverberation is based on a discrete model of sound scattering by inhomogeneties in the medium and irregularities at its boundaries· His presentation also takes into account the nature of the transmitted signal, the band pass characteristics of the receiver and motions of the source and the inhomogeneties.

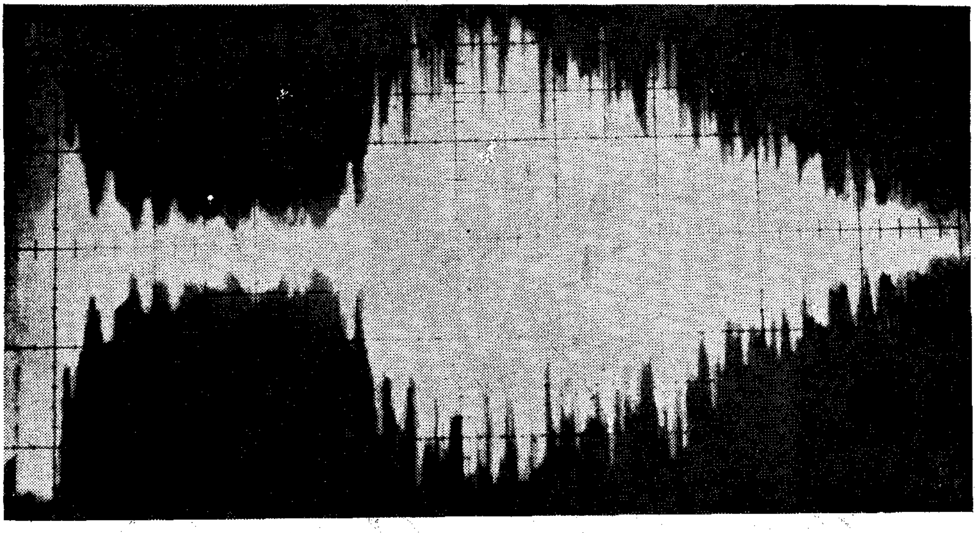

Figure 2. Reverberation returns from the rough bottom of the sea insonified by F M pulse width = 10 ms, band width = 9.3–9.7 KHz.

Middleton in his modelling has included the effects of the transducer beam pattern, variable ocean sound velocity profile, sound attenuation scatterer motion and non-uniformly distributed scatterers. Middleton has shown that if the average number of scatterers per unit volume (scatterer density) and probability distribution of the scatterer reflectivity are known, then the mean variance and the higher moments of the scattered sound field can be calculated.

William & Talivaldis[15] discusses the parameters to be measured for Faure-Ol' Sheveskii-Middleton model for volume reverberation. The details of the experimental set up and measurement methods are given. The preliminary estimate of the scatterer reflectivity and density obtained are used to initiate a computer simulation of the scattering process.

## 5. Scatterer Reflectivity

The reverberation experiments were carried out[16] at various locations in Arabian Sea on board RV Gaveshini of National Institute of Oceanography. Frequencies used were 9.5 and 19.1 KHz and pulse duration was kept at 10 ms in each case. Back-Scatterer reflectivities per unit area were calculated from the measurements and averaged. For their estimation, the reverberation data recorded were fed to a level recorder.

The RMS sound pressure and the corresponding reverberation levels (RL) were then calculated. Using the reverberation level values, scat·erer reflectivities were estimated.

## 6. Scatterer Density

To estimate the scatterer density the recorded data on time domain is utilised. From the geometry of the experiment (Fig. 3) the time taken to get the first surface echo and the first bottom echo were computed. For the above experiment, surface echo will be received after 128 msec and bottom echo after 10.71 seconds.

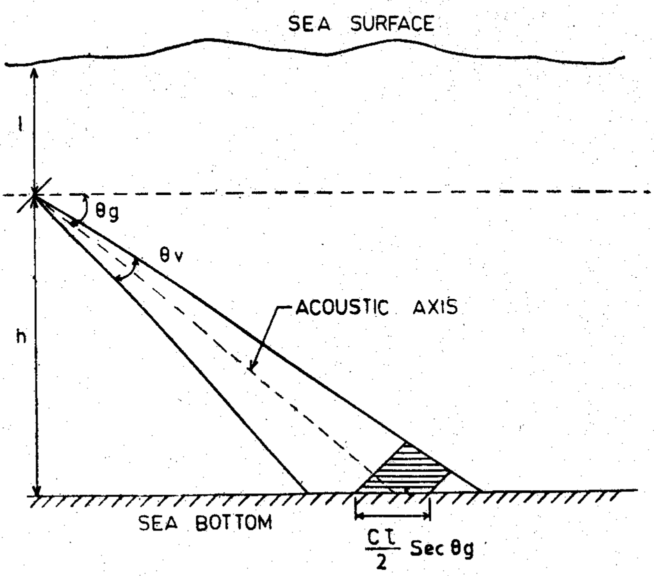


**Figure 3.** Geometry of the reverberation experiment.

In order to estimate the scatterer density in a given volume of the ocean the total number of reflections in a given portion of the time recorded corresponding to that volume was counted and a one to one correspondence between echo observed and scatterer was assumed. The contribution due to the side lobes of the transducer were neglected since their levels were 13 to 14 dB below the main lobe. The echoes received between 50 and 100 ms after transmission were considered in the present case.

The geometry used to estimate the approximate volume is given in Fig. 4. The shape at the wave-front can be approximated as a section of a spherical surface bounded by the horizontal and vertical beam pattern.

The reflections between 50 and 100 ms were counted for consecutive $x$ records. This is done by storing the required portion of the record on a storage oscilloscope

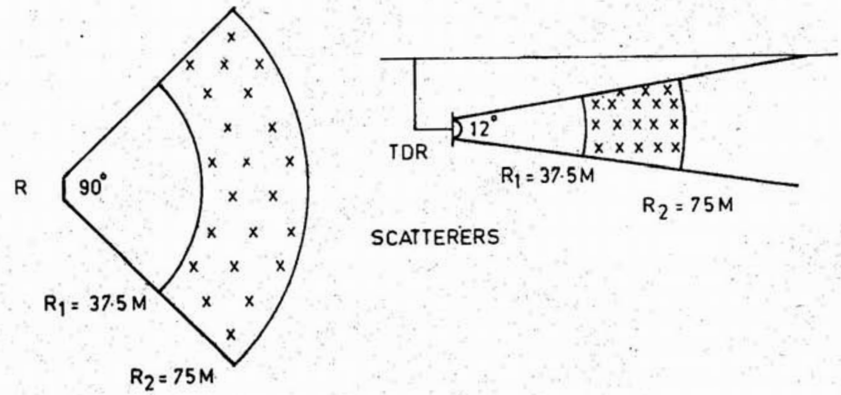


**Figure 4.**

and counting them. The number in each record carried the total number obtained for $x$ records is $n$. The scatterer density per unit volume is thus obtained as

$$\frac{\text{Total scatterers}}{\text{No. of records} \times \text{volume}}$$

$$= \text{scatterer density } P$$

$$= \frac{n}{x \times \text{Area}} = P = 0.0012 \text{ scatterer/m}^3$$

## 7. Volume Reverberation Simulation

The insonified volume was divided into smaller incremental volumes $\Delta\, v$ and a Poisson distributed random number of scatterers were assigned to each (Fig. 5). The scatterers in each volume $\Delta\, v$ were then assigned three coordinates from uniformly distributed random numbers between 0 and $(\Delta\, v)^{1/3}$. To each of these scatterers, scatterer reflectivity values were assigned, from an experimental distribution. The range coordinates, and reflectivities of all the scatterers thus distributed in the insonified volume were stored in a disk file. This file constituted the simulated ocean volume for reverberation.

The reverberation simulation programme was written to accept the disk file input containing the range, bearing and reflectivity of each scatterer in the simulated ocean volume. For each scatterer a digital replica of the transmitted signal was constructed and multiplied by the scatterer reflectivity and transducer beam pattern gain at the scatterer's bearing. The replica of the signal, with the appropriate amplitude and phase delay, was then added to a reverberation output array. After all scatterers in

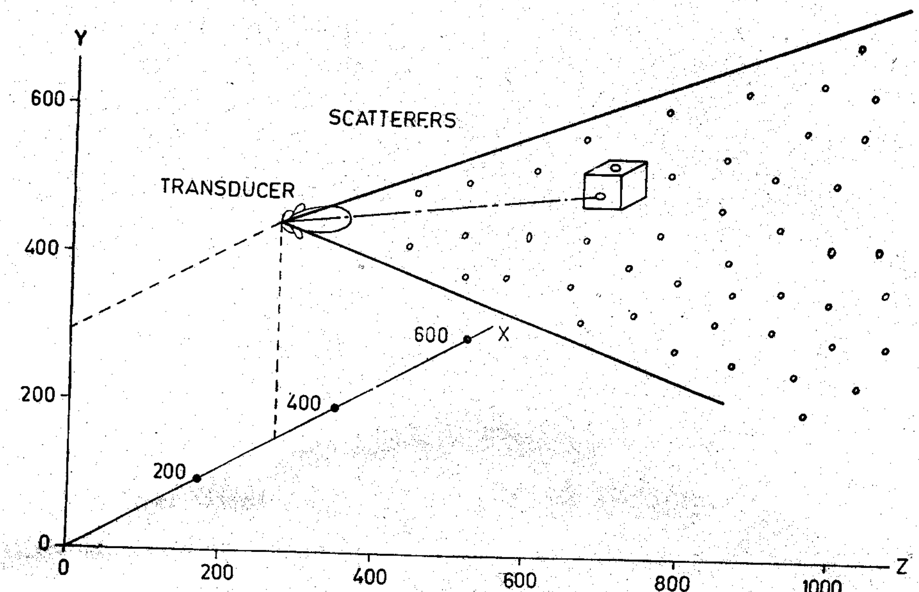


**Figure 5.** Simulation coordination.

the input file has been processed, the reverberation output array was envelope detected, printed and plotted.

The preliminary estimate of the scatterer reflectivity and density are used to initiate the programme. The initial estimate of the scatterer density and reflectivity are then adjusted until the envelope of the simulated reverberation is statistically similar to the envelope of the reverberation measured.

## 8. Conclusion

A critical review of the problems in evolving a mathematical model of reverberation has been presented. This will enable a sonar designer for designing an optimum processor and for evaluating sonar performance under simulated reverberation conditions.

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