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# Heat Transfer Between Two Parallel Rotating Porous Disks with Different Permeability

MOHAN SINGH & S. C. RAJVANSHI
Department of Applied Sciences, Punjab Engineering College, Chandigarh-160012

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**Abstract.** Heat transfer between two parallel porous disks, rotating in the same direction, in the presence of a fluid source on the axis, has been investigated. The energy equation has been simplified by expressing the temperature in powers of $(Re/r^2)$, where $Re$ is the source Reynolds number and $r$ is the radial co-ordinate. The resulting equations have been solved numerically by Goodman-Lance shooting method. The effects of suction and injection on the temperature profiles and Nusselt number have been obtained.

## 1. Introduction

Radial flow between porous or solid boundaries has been extensively studied owing to its applications in multiple disk pumps, multiple disk turbines, gaseous diffussion, boundary cooling and air cooling of turbines. Above mentioned applications of radial flow indicate that the thermal analysis of the fluid surrounding the rotating components in different types of machinery is also very necessary.

Khan[1] investigated the laminar source flow of a viscous incompressible fluid between two parallel coaxial porous disks rotating at the same speed, the suction at one disk being equal to injection at the other. Elkouh[2] studied the laminar source flow of a viscous incompressible fluid between two stationary porous disks, with uniform suction or injection at both the disks. Prakash and Rajvanshi[3] have studied the laminar source flow between two parallel rotating porous disks with different permeability.

The problem of heat transfer between parallel disks has also been studied by many authors. Kreith[4] investigated the heat transfer between two corotating parallel solid disks. Rajvanshi[5] studied the heat transfer between two parallel coaxial disks, rotating at different speeds, in the presence of a fluid source. Gaur and Chaudhry[6] analysed the heat transfer between parallel porous disks of different permeability. Singh and Rajvanshi[7] have studied the heat transfer between parallel porous disks, in the presence of a fluid source along the axis. The energy equation has been simplified by expressing the temperature in powers of the radial coordinate. The resulting equations have been solved numerically by the Goodman-Lance shooting method. The effects of equal suction and equal injection at the two disks on the temperature profiles and Nusselt number have been studied.

In the present paper the temperature distribution between two parallel rotating porous disks maintained at uniform different temperatures, has been obtained. A fluid source is assumed to be present on the axis. The disks have been taken to be of different permeability. The temperature has been expressed in powers of $(Re/r^2)$, where $Re$ is the source Reynolds number and $r$ is the radial coordinate. In most of the previous investigations solution has been obtained in the form of a perturbation, which is valid only for small values of the parameter. In this investigation the resulting equations have been solved numerically by Goodman-Lance shooting method. The values of the velocity components required for the numerical solution have been obtained by solving numerically the relevant equations of Prakash and Rajvanshi.[3] This has increased the validity of the results than hitherto available from previous investigations. The effects of suction and/or injection at one or both the disks on the temperature profiles and Nusselt number have been shown graphically.

## 2. Governing Equations and Boundary Conditions

Consider the axially symmetric flow of a viscous incompressible fluid between two infinite parallel porous disks, rotating in the same direction at constant equal angular velocity $\omega$. Let a fluid source of strength $Q$ be present on the axis which is taken as $z$-axis. The origin is taken midway between the disks. Let $2a$ be the distance between the disks. In polar cylindrical co-ordinate system $(\bar{r}, \bar{\theta}, \bar{z})$, the surfaces of the two disks are given by $\bar{z} = -a$ and $\bar{z} = +a$ respectively. Let $\bar{w}_1$ be the constant suction velocity at the upper disk and $\bar{w}_2$ be the constant injection velocity at the lower disk. Let $\bar{u}, \bar{v}, \bar{w}$ be the velocity components in $\bar{r}, \bar{\theta}, \bar{z}$ directions respectively, and $\bar{T}$ the temperature at any point $(\bar{r}, \bar{\theta}, \bar{z})$. The boundary conditions are

$$\left.\begin{aligned}
&\bar{u}(\bar{r}, \pm a) = 0,\ \bar{v}(\bar{r}, \pm a) = \bar{r}\omega,\ \bar{w}(\bar{r}, +a) = \bar{w}_1 \\
&\bar{w}(\bar{r}, -a) = \bar{w}_2,\ \bar{T}(\bar{r}, +a) = \bar{T}_1,\ \bar{T}(\bar{r}, -a) = \bar{T}_2, \\
&\int_{-a}^{a} 2\pi\, \bar{r}\, \bar{u}\, d\bar{z} + \pi\bar{r}^2\bar{w}_2 - \pi\bar{r}^2\bar{w}_1 = Q,
\end{aligned}\right\} \tag{1}$$

where $\bar{T}_1$ and $\bar{T}_2$ are the constant temperatures of the upper and lower disk respectively. We introduce the following dimensionless variables

$$\left.\begin{array}{l} r = \bar{r}/a,\ z = \bar{z}/a,\ \ u = \bar{u}/a\omega,\ v = \bar{v}/a\omega,\ \ w = \bar{w}/a\omega, \\ w_1 = \bar{w}_1/a\omega,\ w_2 = \bar{w}_2/a\omega,\ T = (\bar{T}-\bar{T}_2)/(\bar{T}_1 - \bar{T}_2). \end{array}\right\} \tag{2}$$

The dimensionless form of energy equation in polar cylindrical co-ordinates for the axisymmetric case is

$$(\partial^2 T/\partial r^2) + r^{-1}(\partial T/\partial r) + (\partial^2 T/\partial z^2) = Pr[u(\partial T/\partial r) + w(\partial T/\partial z)]$$
$$- Pr\ S[2\{(\partial u/\partial r)^2 + (u/r)^2 + (\partial w/\partial z)^2\} + \{(\partial w/\partial r) + (\partial u/\partial z)\}^2$$
$$+ (\partial v/\partial z)^2 + \{(\partial v/\partial r) - (v/r)\}^2]. \tag{3}$$

where

$$Pr(\text{Prandtl number}) = \mu C_v/k.$$

$$S = \nu^2/[a^2 C_v\ (T_1 - T_2)].$$

$\mu$, $C_v$, $k$ and $\nu$ are viscosity, specific heat at constant volume, thermal conductivity and kinematic viscosity respectively. The boundary conditions (1) reduce to

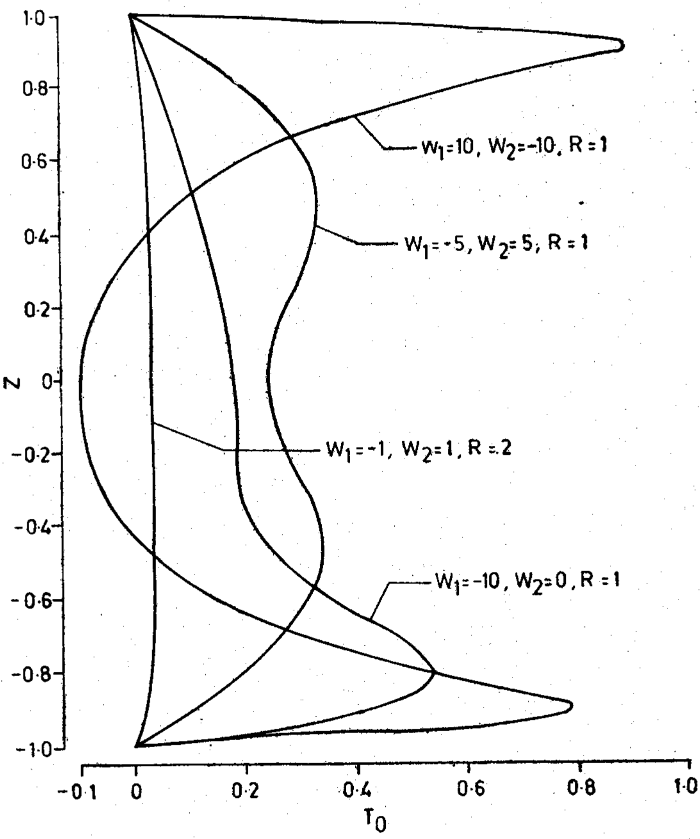


**Figure 1.** Variations of $T_0$.

$$\left.\begin{array}{l} u\,(r,\,\pm\,1) = 0,\ v\,(r,\,\pm\,1) = r, \\ w\,(r,\,+\,1) = w_1,\ w\,(r,\,-\,1) = w_2 \\ \int_{-1}^{1} u\,dz + \tfrac{1}{2}r\,(w_2 - w_1) = 2\,(Re/r), \end{array}\right\} \quad (4)$$

$$T(r,\,-\,1) = 0,\ T(r,\,+\,1) = 1, \quad (5)$$

where

$$Re = Q/(4\,\pi\,a^3\,\omega).$$

The Reynolds number for the rotation is defined as

$$R = \omega a^2/\nu. \quad (6)$$

Following Prakash and Rajvanshi[3] $u$, $v$, $w$ are taken in the following form

$$u = \tfrac{1}{2}\,r f'_{-1} + (Re/r) f'_0 + (Re^2/r^3)\ f'_1 + (Re^3/r^5)\ f'_2 + \ldots, \quad (7)$$

$$v = r g_{-1} + (Re/r) g_0 + (Re^2/r^3) g_1 + (Re^3/r^5) g_2 + \ldots, \quad (8)$$

$$w = -f_{-1} + 2(Re^2/r^4)\ f_1 + 4(Re^3/r^6)\ f_2 + \ldots, \quad (9)$$

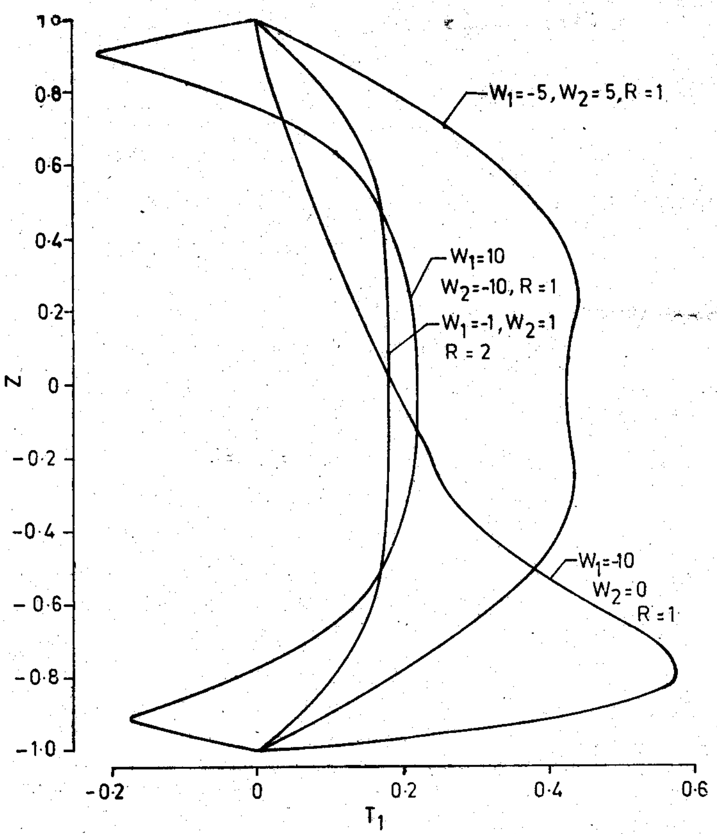


**Figure 2.** Variations of $T_1$.

where prime denotes differentiation with respect to $z$. We assume the following form for $T$

$$(T/r^2) = T_0 + (Re/r^2)T_1 + (Re/r^2)^2 T_2 + \dots \tag{10}$$

Using Eqns. (7) to (10) in Eqn. (3) and equating coefficients of like powers of $r$, we obtain a system of ordinary linear differential equations. The first three equations of the system are

$$T_0'' = Pr\,[f_{-1}'\,T_0 - f_{-1}T_0'\,] - PrS\,[g_{-1}'^2 + (1/4)f_{-1}''^2], \tag{11}$$

$$T_1'' = Pr[2f_0'\,T_0 - f_{-1}\,T_1'\,] - PrS[f_0''\,f_{-1}'' + 2g_0'\,g_{-1}']$$

$$- [4T_0 + 3Pr\,S\,f_{-1}'^2]/Re. \tag{12}$$

$$T_2'' = Pr[2f_1'\,T_0 + 2f_1\,T_0' - f_{-1}'T_2 - f_{-1}\,T_2'\,]$$

$$- Pr\;S\;[g_0'^2 + 2g_{-1}'\,g_1' + f_0''^2 + f_1''\,f_{-1}''].$$ (13

The boundary conditions (5) reduce to

$$T_0(-1) = T_0(+1) = 0, \tag{14}$$

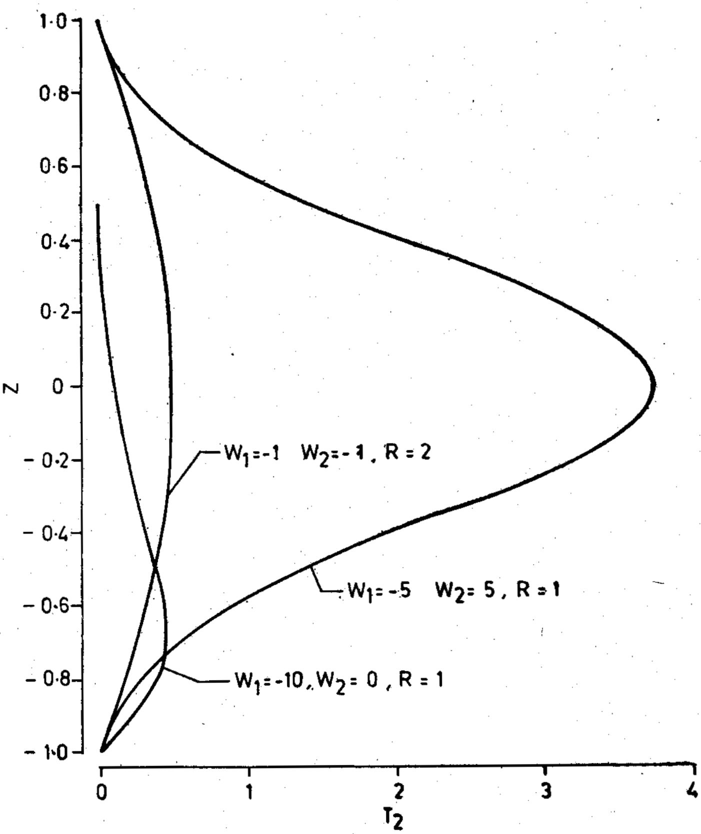


**Figure 3.** Variations of $T_2$.

$$T_1(-1) = 0, \; T_1(+1) = 1/Re, \tag{15}$$

$$T_2(-1) = T_2(+1) = 0, \tag{16}$$

## 3. Solutions of Equations

The governing equations have been solved numerically. Following Roberts and Shipman[8], Goodman-Lance shooting method has been used to solve these differential equations. The values of $f_n(z)$, $g_n(z)$ and their derivatives required in the numerical solution of these equations have been obtained by solving numerically the equations for $f_n(z)$ and $g_n(z)$ given in Prakash and Rajvanshi[3], by the same method. The system for $f_{-1}(z)$ and $g_{-1}(z)$ is non-linear. A perturbation method has been adopted to solve this non-linear system at low values of $w_1$ and $w_2$. An arbitrary parameter $\beta$ is introduced as coefficient of non-linear terms in the system. Initially we assume $\beta = 0$, and the system is solved as a linear system. Then $\beta$ is set equal to 0.05 and the initial guess for the missing initial conditions is given as the values of these got from the solution when $\beta=0$. Solution converges after very few iterations. This increment to $\beta$ is continued till $\beta$ becomes 1 and in each case the initial conditions

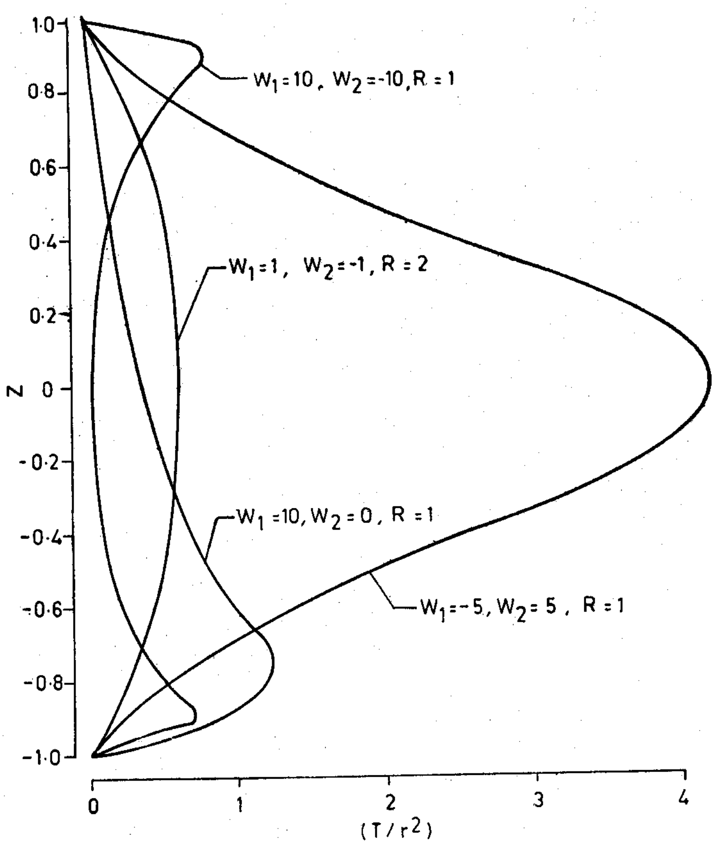


**Figure 4.** Temperature profiles for $(Re/r^2) = 0.5$.

obtained are given as initial guess for the next value of $\beta$. This process is applied to get solution for three cases. After that for higher values of $w_1$ and $w_2$ the shooting method is applied by guessing the missing initial conditions by extrapolation from the solution for three previous cases. This saved a lot of computation time as the solutions converged after very few iterations. Equations for $f_0$, $g_0$ and $f_1$, $g_1$ are linear. Solutions of these were obtained by the same shooting method. Subsequently the solutions of the temperature equations were obtained.

## 4. Discussion of Results

For numerical work we have taken

$$Pr = 1, \; S = 0.2, \; Re = 1000. \tag{17}$$

Solutions have been obtained in the following four cases

(i) $w_1 = 10$, $w_2 = -10$, $R = 1$, i.e. equal suction at both the disks,

(ii) $w_1 = -5$, $w_2 = +5$, $R = 1$, i.e. equal injection at both the disks,

(iii) $w_1 = -10$, $w_2 = 0$, $R = 1$, i.e. injection at the upper disk, the lower being non-porous,

(iv) $w_1 = -1$, $w_2 = 1$, $R = 2$, i.e. equal injection at both, but with increased rotation Reynolds number.

Variations of $T_0$, $T_1$, $T_2$ have been shown in Figs. 1, 2 and 3 respectively. In case of equal suction at both the disks $T_0$ is maximum near the disks then decreases uniformly and is minimum in the plane $z = 0$ and the curve is symmetrical about the centre. $T_1$ assumes a minima near the disks then increases uniformly to assume a maxima at $z = 0$ and the curve is symmetrical about $z=0$. $T_2$ is negligible and the curve is not shown for this case in fig. 3. In case of equal injection at both $T_0$ and $T_1$ both assume a maxima in the region between the plates and the plane $z = 0$ before assuming a minima at $z = 0$. Curves are symmetrical about $z=0$. $T_2$ increases uniformly and assumes a maxima at $z = 0$. When lower plate is non-porous and there is injection at the upper plate $T_0$, $T_1$, $T_2$ all assume a maxima near the lower plate then decrease uniformly to take the assigned value at the upper boundary. In case (iv) when $R = 2$ and injection velocity is small; $T_0$, $T_1$, $T_2$, all remain uniform in the central region. In the region near the plates the increase is sharp from the assigned boundary value to the value in the central region. Fig. 4 depicts $(T/r^2)$ against $z$ for the above four cases when $(Re/r^2) = 0.5$. It is noted that in the case of equal suction the temperature profile assumes a maxima near the disks, indicating the existence of a thermal boundary layer in that region. In case of equal injection on the two plates the momentum transfer increases in the central region. This affects the temperature profile and it possesses a maxima at $z = 0$. No thermal boundary layer effects are seen in this case.

The average Nusselt number at the upper plate is given by

$$(Nu)_{\bar{z}=+a} = [1/(\bar{r}^2-\bar{r}_0^2)] \int_{r_0}^{\bar{r}} 2\bar{r}(\partial \bar{T}/\partial \bar{z})_{\bar{z}=+a}\, d\bar{r}, \tag{18}$$

where $\bar{r}_0$ is the distance of a referenced point on the disk from the axis. Using Eqns. (2) and (10) in Eqn. (18) we get the Nusselt number in the dimensionless form as

$$(N^*u)_{z=1} = (r^{*^2} + r_0^{*^2})\ (T_0')_{z=1} + 2\,(T_1')_{z=1} + 4\,[\log (r^*/r_0^*)/(r^{*^2} - r_0^{*^2})]\ (T_2')_{z=1}, \tag{19}$$

where

$$r^* = \bar{r}/\sqrt{Re},\ \ r_0^* = \bar{r}_0/\sqrt{Re},\ N_u^* = Nu/Re,$$

The Nusselt number at the lower plate is got from Eqn. (19) by replacing every where $z = 1$ by $z = -1$. In addition to Eqn. (17) for numerical work we assume $r_0^* = 1$. Variations of $Nu$ at the upper and lower plates from $r^* = 1$ to $r^* = 3$, have been shown in figures 5 and 6 respectively.

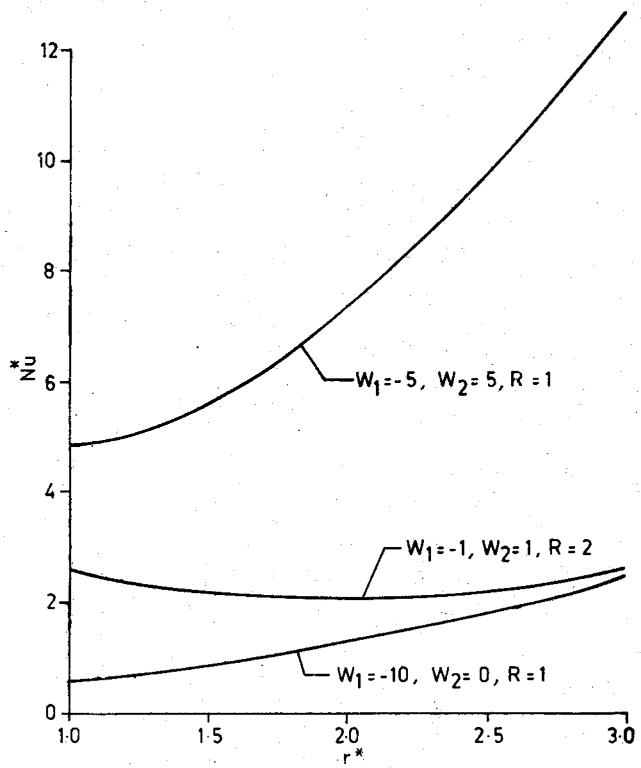


**Figure 5.** Nusselt number at the upper plate.

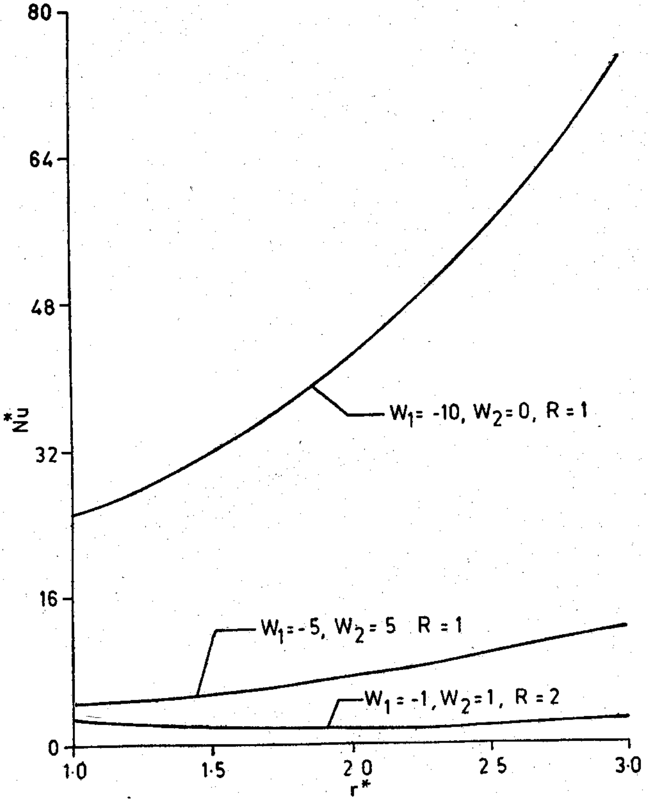


**Figure 6.** Nusselt number on the lower plate.

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