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Cold Stress at High Altitudes

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Abstract. The problem of cold at high altitudes has been analysed from a purely physical standpoint. It has been shown that Siple's Wind-Chill Index is not reliable because (i) it does not make use of the well established principles governing the physical processes of heat transfer by convection and radiation, and (ii) it assumes that the mean radiant temperature of the surroundings is the same as the ambient dry bulb temperature. A Cold Stress Index has been proposed which is likely to be a more reliable guide for assessing the climatic hazards of high altitude environments. The Index can be quickly estimated with the help of two nomograms devised for the purpose.

1. Introduction

While there are a large number of indices of heat stress that have been developed by various workers from time to time with varying degrees of success, very few attempts seem to have been made for the assessment of cold stress under outdoor conditions, particularly at high altitudes. The Wind-Chill Index of Siple & Passel[1] is perhaps the most popular index for estimating the cooling effect of the environment on the exposed parts of the body, such as the face and the hands. It is expressed in k cal/m^2 hr and is supposed to give the cooling rate of the exposed body at a neutral skin temperature of 33°C. Computations are avoided by using charts available for the purpose.

While the Wind-Chill Index appears to be a useful guide in deciding outdoor activities as well as protective requirements for the extremities, the formula used for its computation restricts its application only to near sea level conditions. Further, it is entirely empirical, being based on the observed time of freezing of water contained in a cylinder.

The importance of devising a more reliable index of cold stress applicable to high altitudes was highlighted by reports of cold injury during 1971 operations, occurring under conditions which give Wind-Chill Index of about 900 k cal/m^2 hr, i. e., between cold and very cold. Near sea level, one would hardly expect any cold injury, particularly among tough, well acclimatised soldiers, since danger level of Wind-Chill Index is $\geqslant$ 1400.

Apparently, the failure of Wind-Chill Index at high altitudes may be ascribed to its following shortcomings :

(1) Failure to make use of the well-established principles governing the physical processes of heat transfer by convection and radiation.

(2) Tacit assumption that the mean radiant temperature (m. r. t. or t_r) of the surroundings is the same as the ambient dry bulb temperature.

An attempt has been made in the present paper to study the problem of cold at high altitudes from a purely physical standpoint and to estimate, on the basis of available data, the total cooling power of the outdoor environment during winter, of two high altitude stations representative of the eastern and western Himalaya.

2. Cooling by Convection

Under outdoor conditions, we may safely ignore natural convection, and restrict ourselves to forced convection determined by the shape and size of the exposed body, its mean surface temperature and a number of parameters characteristics of the ambient air, namely, wind speed, temperature, density, dynamic viscosity and thermal conductivity. The most satisfactory way of studying the problem is in terms of two dimensionless numbers, the Reynold's number, N_{Re} and Nusselt's number, N_{Nu}, expressed as

$$N_{Re} = D \rho V/\mu \tag{1}$$

and

$$N_{Nu} = h_c D/K \tag{2}$$

where

D = A characteristic linear dimension of the body, such as diameter of a cylinder or a sphere

ρ = Density of air

V = Wind speed

μ = Dynamic viscosity of air

K = Thermal conductivity of air

h_c = Convective heat transfer coefficient.

Let us now consider the practical range of conditions prevailing at heights vary from about 3 to 5.5 km, with barometric pressure from 0.7 to 0.5 atmos. The lower limit of air temperature may be reasonably taken to be -40°C. Wind speed may vary from about 1 km/hr (calm) to above 50 km/hr. Average values for air density, viscosity and thermal conductivity under such conditions may be approximately taken to be 0.7 kg/m³, 0.06 kg/hr m and 0.02 k cal/hr m c respectively. The value of D for the human body may vary from about 0.02 m (finger) to about 0.4 m (trunk). With these values substituted in Eqn. (1), we find that Reynold's number may vary from about 200 to 2,00,000.

On the basis of controlled experiments carried out by a number of workers[2-7], the best fitting curves for cylinders and spheres were recommended[8]. However, there is no single equation available which could cover the entire range of Reynold's numbers met in practice under outdoor conditions at high altitudes.

Our analysis of the recommended curves has resulted in the following equations : for cylinders with air flow perpendicular to their axes

$$N_{Nu} = 2.11 + 0.1738\,(N_{Re})^{0.6268} \tag{4}$$

and for spheres

$$N_{Nu} = 2.11 + 0.2775\,(N_{Re})^{0.6268} \tag{5}$$

These two equations have been found to be accurate within $\pm$ 2.5% over the range of N_{Re} from 10^2 to 10^5. The constant 2.11 is explained by the fact that in absolutely still air, N_{Re} cannot fall below the theoretical minimum value of 2 due to conduction through the stagnant layers of air.

Our analysis of the reported values of density, viscosity and thermal conductivity of air[9], yield the following equations :

$$\rho = 1.2944\,(T_0/T)\,(P/P_0)\ \text{kg/m}^3 \tag{6}$$

$$\mu = 0.06187\,(T/T_0)^{0.7843}\ \text{kg/hr m} \tag{7}$$

$$K = 0.02074\,(T/T_0)^{0.8440}\ \text{k cal/hr m c} \tag{8}$$

where T is the absolute temperature and P, the pressure of the air, T_0 and P_0 being the corresponding values at N.T.P. Within the practical range, μ and K are independent of pressure.

With the above expressions substituted in Eqns. (1) and (2), Eqns. (4) and (5) lead to thefollowing expressions for the convective heat transfer coefficient :

$$h_c = C_1\,(T/T_0)^{0.8440} + C_2\,[(P/P_0)^{0.6268}/(T/T_0)^{0.2744}]\ V^{0.6268} \tag{9}$$

where $C_1 = 4.376/D$ and $C_2 = 10.26/D^{0.3732}$ for cylinders in transverse air flow, and $16.39/D^{0.3732}$ for spheres, D being the diameter in cm and V in km/hr.

It will be seen from Eqn. (9) that within the practical range of air temperature and wind speed, h_c varies very little with air temperature, so that we can write without significant error :

$$h_c = C_1 + C_2\,(P/P_0)^{0.6268}\ V^{0.6268}\ \text{k cal/m}^2\ \text{hr c} \tag{10}$$

Table 1 gives the values of C_1 and C_2 for cylinders and spheres having diameters varying from 2 to 50 cm.

Table 1. Values of C_1 and C_2 for cylinders and spheres of different diameters

D	C_1	C_2 (with V in km/hr)	
(cm)		Cylinder	Sphere
2	2.19	7.92	12.65
5	0.88	5.63	8.99
10	0.44	4.35	6.94
15	0.29	3.73	5.96
20	0.22	3.35	5.36
30	0.15	2.88	4.61
50	0.09	2.38	3.81

It will be of interest to note that for the same diameter, convective heat loss is about 60% more for a sphere than for a cylinder. The human head, about the size of the standard black globe (15 cm), therefore, needs better protection in a cold wind than the arm (cylinder about 10 cm dia), while it will be difficult to provide adequate protection to the fingers (cylinder about 2 cm dia) without adversely affecting their operational efficiency. Fig. 1 shows the convective heat transfer coefficient at N.T.P. as a function of wind speed for different cylinder diameters, and also for a 15 cm sphere. It will be seen that a 15 cm sphere is practically equivalent to a cylinder of 5 cm dia. The coefficient of Siple's Wind-Chill Index, also shown in the figure, does not correspond to any dimension.

Let us now see how the above results compare with actual experimental data on convective heat transfer for a nude man in transverse air flow. The revised formula of Hatch[10] gives

$$h_c = 6.27\ V^{0.6}\ \text{k cal/m}^2\ \text{hr c} \tag{11}$$

where V is wind speed in m/sec.

The work of Mitchell[11] *et al.* based on direct calorimetry in a climatic chamber, has led to the following expression :

$$h_c = 6.23\ (P/P_0)^{0.6}\ V^{0.6}\ \text{k cal/m}^2\ \text{hr c} \tag{12}$$

V being in m/sec. It will be seen that both the formulae give practically identical values at sea level pressure. The range of wind speeds is from 2.4 to 17.8 km/hr. The same is shown in Fig. 1, and is found to compare very well with a cylinder of about 35 cm dia.

We are now in a position to determine the convective cooling power of the wind at any altitude. Fig. 1 gives the convection coefficient for any wind speed at sea level.

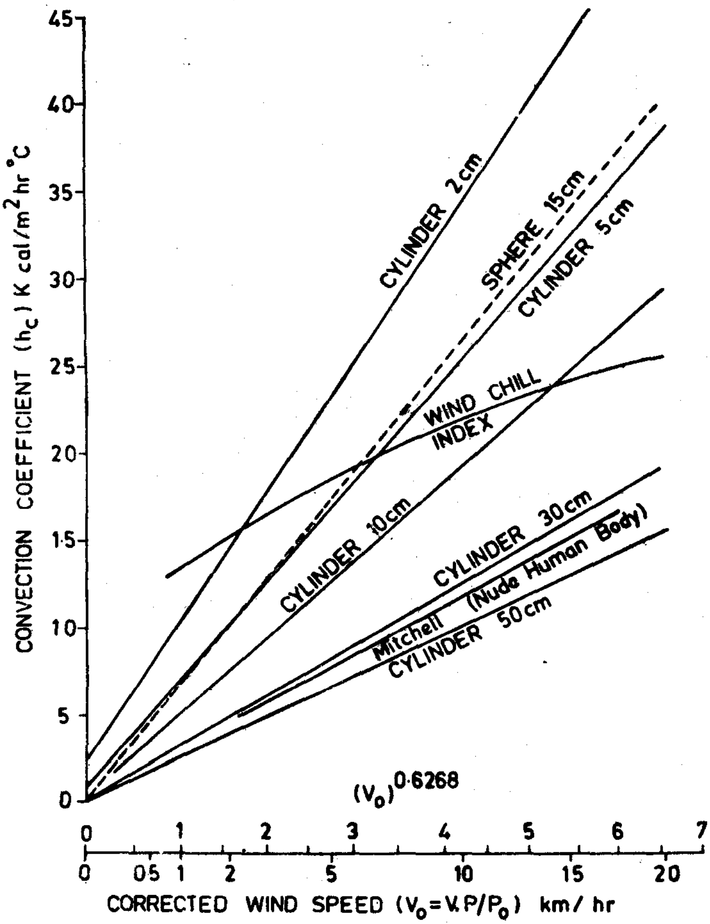

Figure 1. Convection coefficients of cylinders of diameters 2.50 and 15 cm sphere as related to wind speed and barometric pressure.

It will be evident from Eqn. (10) that wind speed V at any altitude multiplied by the fraction P/P_0, reduces to the equivalent sea level wind speed, V_0. This value is used to determine h_c from Fig. 1. The value of h_c multiplied by $(33 - t_a)$ will yield the convective cooling power of the environment in k cal/m² hr. It is assumed that the initial mean skin temperature is 33°C, i. e., comfortable.

3. Cooling by Radiation

Computation of radiation exchange between nude man and the environment is complicated by the fourth power law of Stefan-Boltzmann, according to which

$$R = \sigma \; \epsilon_s \; (T_s^4 - T_r^4) \; A_r/A \text{ k cal/m}^2 \text{ hr} \tag{13}$$

where

R = Radiant heat loss from the body

σ = Stefan-Boltzmann constant (4.88×10^{-8} k cal/m² hr K^4)

ϵ_s = Emissivity of the skin

T_r = Mean radiant temperature (m.r.t) of the surroundings (absolute $K = t_r + 273$)

T_s = Mean skin temperature, absolute $K = t_s + 273$

A_r/A = Effective fraction of total body surface exchanging radiation with the surroundings.

The linear radiant heat transfer coefficient h_r, is defined by

$$h_r = \frac{R}{T_s - T_r} \text{ k cal/m}^2\text{ hr } K \quad (14)$$

whence from Eqn. (13), it follows that

$$h_r = \sigma\epsilon_s \ (T_s^2 + T_r^2)(T_s + T_r)\, A_r/A \quad (15)$$

For radiation exchange with the atmosphere, the emissivity, ϵ_s of human skin may be assumed to be unity; and in a cold environment, A_r/A is usually taken to be 0.75 for the erect posture. T_s may be assumed to be 33 + 273 K at the start of exposure. The radiation coefficient, h_r, calculated in this way from Eqn. (15) is plotted against t_r (m. r. t.) in Fig. 2. It will be seen that a straight line nicely fits the data over the entire range + 10 to − 50°C. The equation of the line is

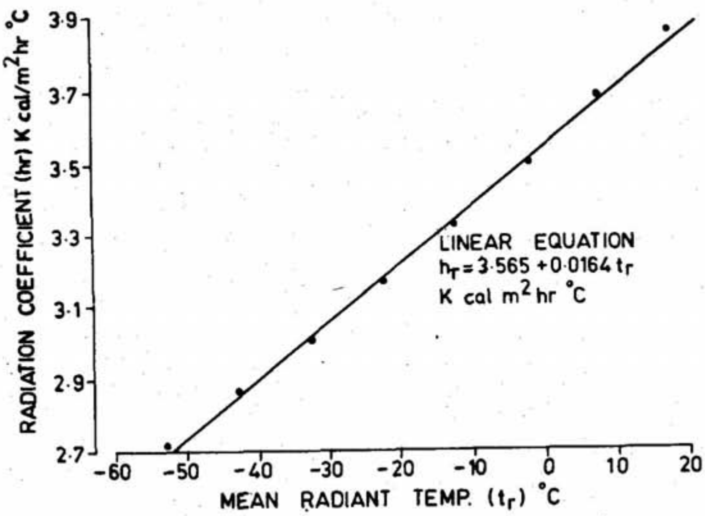

Figure 2. Linear relationship between radiation coefficient, (h_r) and mean radiant temperature of surroundings (t_r), in the range − 50°C to + 10°C (plotted points give values rigorously derived from Stefan-Boltzman's forth power law assuming mean skin temperature of 33°C

$$h_r = 3.565 + 0.0164\, t_r \text{ k cal/m}^2 \text{ hr c} \tag{16}$$

where t_r is the mean radiant temperature (m. r. t.) in °C. The value of h_r read from Fig. 2 multiplied by $(33-t_r)$ gives the radiant cooling power of the environment in k cal/m² hr.

4. Estimation of the Mean Radiant Temperature (m. r. t. or t_r) of the Surroundings

As pointed out earlier, Siple's Wind-Chill Index assumes equality of ambient dry bulb temperature (t_a) and mean radiant temperature (t_r) of the surroundings. The experimental study of Breckenridge and Goldman has revealed that t_r may be as much as 22.8°C below t_a under clear sky conditions. If this be the case near sea level, one should expect much higher differences at high altitudes with low atmospheric turbidity, particularly in the relatively dry regions of our north-western frontier (Ladakh), where ambient vapour pressure during winter months falls below 1 mb.

A body exposed to the environment, receives long infra-red radiations from the surroundings having two distinct components, namely, atmospheric radiation from the upper hemisphere, and ground radiation from the lower hemisphere, assuming an open field terrain. The mean radiant temperature, t_r, of the surroundings is defined as that uniform temperature of a black enclosure in wnich the body will exchange long wave radiation to the same extent as in the given environment. If ϵ be the apparent emissivity of the atmosphere, and t_a, the dry bulb temperature, then the above definition leads to

$$T_r^4 = \frac{1}{2}\,(\epsilon\, T_a^4 + T_{gr}^4)$$

or

$$T_r = \left[\frac{1}{2}\left(\epsilon\, T_a^4 + T_{gr}^4\right)\right]^{1/4} \tag{17}$$

where $T_r = t_r + 273$, $T_a = t_a + 273$, and T_{gr} = ground temperature in °K.

The apparent emissivity of the atmosphere can be computed from a knowledge of the vertical moisture profile, since water vapour has very strong absorption bands in the far infra-red. This, in turn, is strongly correlated with the total precipitable water vapour in the air from the ground upwards. Our earlier study[13] of radio-sonde data of 14 Indian stations at different pressure levels, has led to a simple method of estimation of ϵ from surface humidity (vapour pressure in mb). Making use of the same, the nomogram in Fig. 3 has been devised, from which t_r as well as the radiant cooling power of the environment can be readily estimated from the knowledge of t_a and vapour pressure, the results being applicable for a cloudless sky. The ground temperature has been ignored in the nomogram for certain practical reasons. From our earlier observations on sub-soil temperature profiles during January and February, at an altitude of 3.5 km, it was seen that the morning (just before sunrise) ground surface temperature was well correlated with ambient dry bulb temperature and humidity on

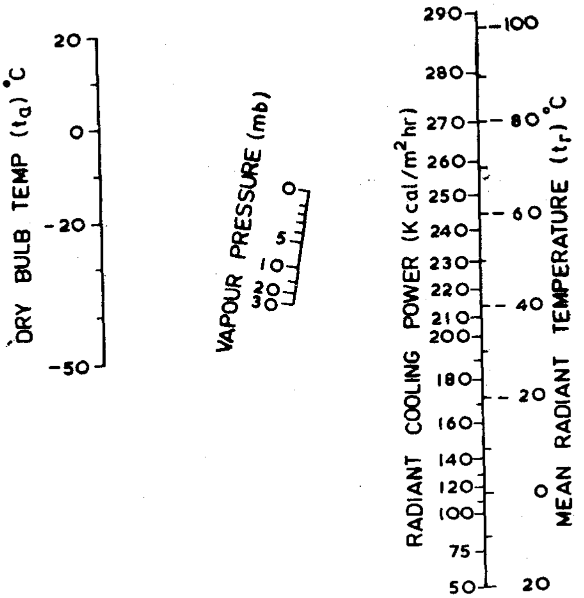

Figure 3. Nomogram for estimating mean radiant temperature (t_r) and radiant cooling power of surroundings for cloudless sky

cloudless days. Based on this finding, a rough correction has been applied to the apparent emissivity, so that Eqn. (17) could be reasonably simplified to

$$T_r = \epsilon'^{1/4}\, T_a \qquad (18)$$

where ϵ' is the apparent emissivity of the atmosphere corrected for ground radiation. However, it can be shown from Eqn. (17) that even an error of $\pm$ 5°C in ground temperature is not likely to cause an error of more than $\pm$ 2°C in t_r. This means that within the usual range of high altitude conditions, the error in the estimation of radiant cooling power is not likely to exceed $\pm$ 3 or 4 k cal/m^2 hr at the most. Hence it was considered worthwhile to retain the simplicity of the nomogram in Fig. 3, in order to enhance its practical utility.

5. Cold Stress Index

We have defined a Cold Stress Index (CSI), as the total cooling power of the environment (convective + radiant), which is the sum of the convective and radiant cooling powers. Its quantitative estimation demands a knowledge of ambient dry bulb temperature (t_a), wind speed (V) station altitude/barometric pressure (P) and m. r. t. (t_r).

Computation of Cold Stress Index

Since the CSI is the sum of radiant cooling power (Fig. 3) and convective cooling power (to be computed with the help of Fig. 1) it is to be decided which of the convection lines in Fig. 1 should be used for the purpose. For the present purpose, we are interested in the exposed parts or inadequately protected parts of the body such as the hands and the head, which actually determine tolerance to cold stress of an otherwise well-protected individual. The head may be approximated by the standard globe (15 cm sphere). Although an isolated finger is practically equivalent to a 2 cm dia cylinder, the hand as a whole, including the fingers should correspond more closely to a 5 cm dia cylinder. The very close agreement between a 15 cm dia sphere and a 5 cm dia cylinder (Fig. 1) has led us to accept the mean line as applicable to both head and hands. Nomogram in Fig. 4, readily yields the convective cooling power of the environemnt. The corrected wind speed V_0 is simply the observed speed V multiplied by the ratio P/P_0, i. e., the station barometric pressure expressed as a fraction of sea level pressure.

Practical Use of Cold Stress Index

Let us now see, in the light of the foregoing, why cases of cold injury occurred as mentioned in the beginning, under conditions corresponding to a Wind-Chill Index of only about 900 k cal/m^2 hr. The particulars are as follows :

Station altitude	: 3 km
Barometric pressure (P/P_0)	: 0.7 atmos
Vapour pressure	: 1.2 mb
Dry bulb temperature (t_a)	: + 5°C
Observed wind speed (V)	: 32 km/hr

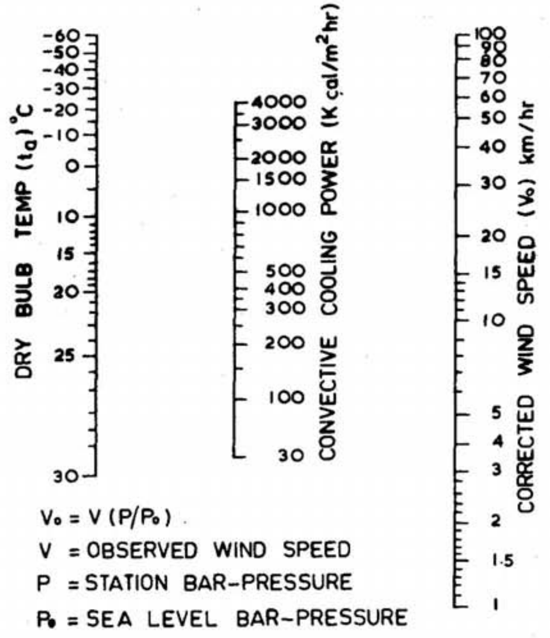

Figure 4. Nomogram for estimating convective cooling power of environment

Wind-Chill Index (read from the chart) : 880 k cal/m² hr

From Fig. 3 we see that

Radiant cooling power = 223 k cal/m² hr

The corrected wind speed

$$V_0 = V x \frac{P}{P_0} = 32 \times 0.7 = 22.4 \text{ km/hr}$$

With this value of V_0 together with t_a = 5°C, Fig. 4 yields convective cooling power of 1120 k cal/m² hr

Hence

CSI = Radiant cooling power + convective cooling power

= 223 + 1120 = 1343 k cal/m² hr.

This value is 50% more than the Wind-Chill Index, representing dangerous exposure, since it is stated that exposed flesh freezes at Wind-Chill Index of 1400 k cal/m² hr. It is little surprising, therefore, that cold injuries should have occurred under such conditions.

Comparative Values of CSI in the Eastern and Western Himalaya

We select two stations A and B of comparable altitudes in the eastern and western Himalaya. The data (Report DLJ/MET/70/4, Defence Laboratory, Jodhpur, 1970) together with computed values of CSI are presented in Table 2. January, the coldest month for both the regions, has been chosen for the purpose :

It would thus be seen that the two stations at almost the same altitude, widely differ in CSI values. Although ambient temperature is 5.6°C higher in the eastern

Table 2. Meteorological parameters (monthly average) and computed CSI for two high altitude stations in January 1969 at 0830 hrs IST

Station	Altitude (km)	Barometric pressure (mb)	t_a (°C)	V (km/hr)	Vapour pressure (mb)	Radiant cooling power (k cal/m² hr)	Convec. cooling power (k cal/m² hr)	CSI (tk-cal/m² hr)
A Eastern sector	4.41	583	−13.2	54.7	4.0	227	2350	2577
B Western sector	4.27	595	−18.8	13.2	0.8	253	1150	1403

sector, the CSI is about 84% higher, chiefly because of the large difference in wind speeds.

6. Discussion

It will be pertinent to pose the question as to why for the same ambient air temperature and wind speed, the environment at high altitudes is more hazardous than at near sea level. The situation is rather paradoxical, because convective loss of heat is actually reduced due to reduced air density. However, answer to the problem is provided by the component of radiant heat transfer, which increases with altitude under clear sky conditions, as will be evident from Fig. 3. We have ignored the factor of atmospheric turbidity which in any case, is very low at high altitude.

From Fig. 3, one can easily see that the importance of mean radiant temperature of the surroundings cannot be under estimated at high altitudes, because t_r may be more than 40°C below the ambient air temperature, under clear sky conditions. This is because the ambient vapour pressure is usually very low, less than 1 mb even in a saturated environment as observed in the eastern sector of the Himalaya.

7. Conclusions

Evidently at high altitudes, the Wind-Chill Index of Siple cannot be relied upon. The proposed CSI appears to be a better guide for assessing the climatic hazards of such environments.

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