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Hydraulic Analogy for Isentropic Flow Through a Nozzle

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Abstract. Modelling aspects of isentropic compressible gas flow using hydraulic analogy are discussed. Subsonic and supersonic flows through a typical nozzle are simulated as free surface incompressible water flow in an equivalent 2-*D* model on a water table. The results are first compared for the well known classical analogy in order to estimate experimental errors. Correction factors for pressure and temperature, to account for non-ideal compressible gas flow are presented and the results obtained on the water table are modified and compared with gas dynamic solution. Within the experimental errors, it is shown that the hydraulic analogy can be used as an effective tool for the study of two dimensional isentropic flows of gases.

Nomenclature

A	Area of cross section
A^*	Critical area of cross section
C_M	Correction factor for Froude number
C_P	Correction factor for pressure distribution
C_T	Correction factor for temperature distribution
F	Froude number on water table
h	Height of water flow
M	Mach number in the gas flow
M_c	Corrected value of Froude number
P	Pressure at any point
T	Temperature at any point
V	Velocity of water
x, y	Coordinates for nozzle geometry
x_e, y_e	Coordinates for equivalent 2-D nozzle geometry
k	Specific heat ratio

1. Introduction

In a jet engine, the hot gases exhausted from the compressor turbine are expanded in a nozzle (a flow passage especially shaped to produce kinetic energy at the expense of other forms of energy) and the nozzle cross sectional area must diminish as expansion proceeds forming what is known as a convergent nozzle. However, if the total head pressure at the turbine exhaust be sufficiently high to give a supersonic velocity, where the expansion of the gas is carried to ambient pressure, the propelling nozzle should be of convergent-divergent section. Convergent nozzles are used in jet engines designed for subsonic flight conditions. In supersonic flight, however, the available expansion pressure ratio across the propelling nozzle can reach high values, and it is then of considerable benefit to employ a convergent-divergent nozzle. Experimental studies of compressible gas flows through such actual convergent, or convergent-divergent nozzles, are difficult in view of the local flow conditions which involve high pressures, high temperatures and high velocities. The region in which the experiments have to be conducted will also be inaccessible. Further, the experimentation in such actual nozzles will be time consuming and prohibitive due to the costs involved in view of the sophisticated instrumentation that is required. It is also difficult to visualise the flow phenomenon, and the only way of assessing the flow situation is by means of some indirect measurements that are made.

Certain experiments in water through analog methods permit in many cases the study pf complex gas flow phenomenon and will be useful for qualitative and quantitative investigations of high speed gas flows by means of relatively simple water table installations, in which the flows are simulated. Such an analog method, namely, hydraulic analogy is considered here to simulate the two dimensional compressible isentropic gas flow through an axisymmetric convergent-divergent nozzle on a water table and to obtain the gas dynamic solutions through water table data.

The classical hydraulic analogy[1] to isentropic compressible gas flow has been considered hitherto as a tool only for qualitative analysis of the problem. The main reason for this restriction stems from the fact that the analysis is limited to a hypothetical gas, known as hydraulic gas, or analog gas with a specific heat ratio $k = 2$, which is never encountered in the real gas flow situation. This has been one of the serious limitations for the application cf hydraulic analogy. However, to circumvent this difficulty correction factors on measured depths in a water table have been used to determine the pressure distribution in a gas flow, e.g., Harlemann & Ippen[2] have derived their correction factors using transonic similarity laws of Von Karman for a family of airfoils, which are useful to model flow over an isolated airfoil.

Adams[3] suggested analytical methods to relate the hydraulic analogy to axisymmetric compressible gas systems in which the specific heat ratio varies from the analogous value of 2. In his analysis the local Mach number of the water flow has been corrected based on the fact that the area of the flow cross section bears the same relation with the area of the cross section for initiation of choked flow for any specific heat ratio. Bryant[4] discussed the possibility of using non-rectangular channels for simulation of gas flows on a water table.

Besides the fact that the hydraulic analogy is valid for gases with specific heat ratio of the value 2 only, there are certain non-analogous effects, which occur during modelling of a two dimensional compressible gas flow on a water table. These include the growth of boundary layers, capillary waves and discontinuities in the flow i.e., shocks[5]. The vertical accelerations in the two dimensional water flow are also not negligible[6].

To circumvent the difficulty of modelling a real gas on a water table, analytical studies were undertaken by Rao[7,8] to derive correction factors for Froude number, depth ratio, and depth ratio square quantities to obtain Mach number, temperature ratio and pressure ratio respectively of the corresponding gas flow. Analytically he has shown that there is an excellent agreement between the two dimensional compressible gas flow and modified free surface incompressible water flow results for nozzle flows. The present work also establishes the validity of this modified analogy.

2. Experimental Model

Rao[8] considered (in his analytical studies) an axisymmetric, converging-diverging nozzle of Back et al.[9] with wall shape made of linear and circular segments. The nozzle inlet and throat sections are made of two circles with exit wall as a cone with 15° angle. The inlet and throat circles meet at a point with a common tangent forming the inlet wall of the nozzle and the exhaust wall is tangent to the throat circle. The nozzle wall shape is (Fig. 1).

Inlet

$$(x + 5.08)^2 + y^2 = 3.937^2, \qquad -5.08 < x < -2.54$$

Throat

$$x^2 + (y - 6.0160)^2 = 3.937^2, \qquad -2.54 < x < 1.0190$$

Exhaust

$$y = 1.9404 + 0.267949\, x, \qquad 1.0190 < x < 7.1120$$

For a given point x, y on the nozzle wall defined by $y = f(x)$, as above, the corresponding points, x_e, y_e in the equivalent 2-D geometry are obtained based on

(i) The cross sectional areas of flow boundaries being equal

$$y_e = \frac{\pi y^2}{4y_0}$$

where y_0 is the throat radius

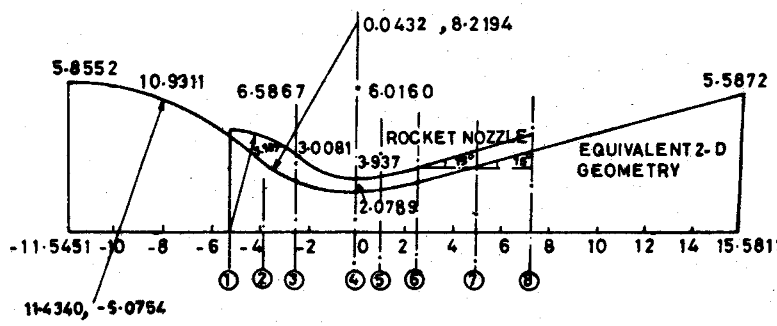

Figure 1. Axisymmetric rocket nozzle with equivalent 2-*D* geometry.

(ii) The surface slopes of flow boundaries being equal giving equal flow directions at the boundaries

$$x_e = \frac{\pi}{2y_0} \int_0^x f(x)\, dx$$

The eqivalent 2-*D* geometry is given by[8]

Inlet

$$(x + 11.4340)^2 + (y + 5.0754)^2 = 10.9311^2, \quad -11.54 < x < -4.553$$

Throat

$$(x + 0.0432)^2 + (y - 8.2194)^2 = 6.5867^2, \quad -4.553 < x < 1.6347$$

Exhaust

$$y = 0.2679\, x + 1.4125, \quad 1.6347 < x < 15.5789$$

The above described equivalent 2-*D* nozzle is used for simulation of the gas flow on the water table. Fig. 1 gives this nozzle shape along with the equivalent 2-*D* model. Table 1 gives the corresponding co-ordinates at eight chosen sections.

Table 1. Equivalent 2-*D* geometry of a rocket nozzle

Stn	x (cm)	y (cm)	x_e (cm)	y_e (cm)
1	−5.08	3.9370	−11.5451	5.8552
2	−3.81	3.7264	− 7.8338	5.2458
3	−2.54	3.0080	− 4.5529	3.4181
4	0.00	2.0790	0.0000	1.6329
5	1.02	2.2130	1.6347	1.8501
6	2.54	2.6207	4.4122	2.5946
7	5.08	3.3015	10.0947	4.1170
8	7.11	3.8458	15.5811	5.5872

3. Description of Water Table

The diagram of the set up used for present investigations is shown in Fig. 2. The nozzle model, 2½ times enlarged in size is prepared in wood and laminated on the channel side with plastic sheet for obtaining a smooth surface. The model is placed at the centre of a flow visualization tank, the working section of which is a smooth glass plate of dlmensions 213 × 60 cm. Water is circulated over the glass plate by an axial flow pump and the depth of water on the table is controlled by a variable height weir located at the down stream end of the working section. Further, very small changes in the flow could be obtained by a fine adjustment valve incorporated in the line. The working section is near horizontal with an extremely small gradient of a slope of 0.15° in the down stream direction. A lengthwise sliding bracket facilitated the mounting of depth measuring apparatus and a scale arranged on one side gives the axial positions at which the measurements were taken. Provision is also

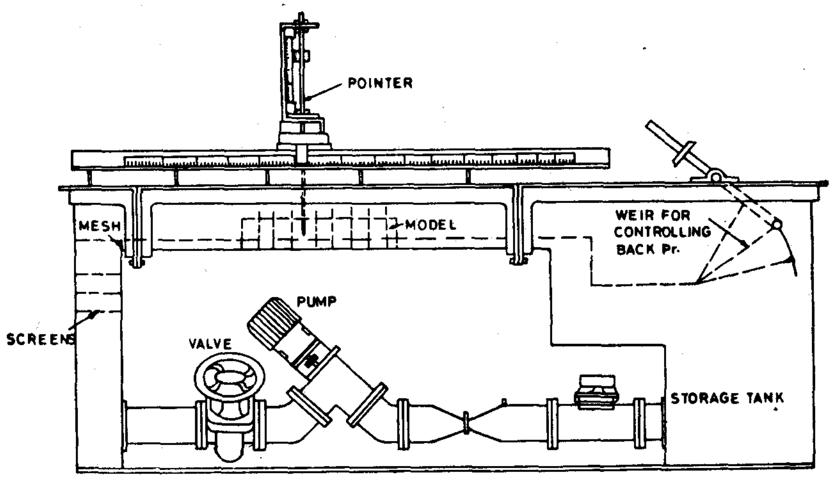

Figure 2. Experimental set-up.

made to move the depth measuring apparatus across the plate with an additional position indicating scale. A precalibrated venturimeter (with an accuracy of ± 0.5 percent) incorporated in the line gives the quantity of water passing through the model. The water depths are measured using pointer fitted with a vernier of least count 0.1 mm. Average velocities are obtained assuming uniform flow throughout the cross section. Axial positions along the water table model at which measurements are taken are shown in Fig. 1. These correspond to the stations given in Table 1.

4. Simulation of Isentropic Flows

4.1 *Isentropic Subsonic Flow*

(a) *Modelling for a hypothetical gas flow.* The nozzle described in section 2 is considered first to simulate a hypothetical gas flow with specific heat ratio 2 on the water table. Since the classical analogy is directly applicable to the flow of gases with specific heat ratio equal to 2, the values obtained from the water table can directly be compared with the gas dynamic values of the hypothetical gas flow when it is simulated. This facilitates the understanding of the inherent limitations, and the errors that are involved in the simulation of gas flows through classical analogy. The errors may be due to viscous effects, boundary layer effects, surface roughness, and other non-analogous effects mentioned earlier.

The throat is taken as reference section and a Mach number of 0.5178 is assumed in the throat of the nozzle (a value of Froude number obtained on water table in the throat) With this throat Mach number, the hypothetical gas dynamic solution is first obtained following John[10]. The flow is then simulated on the water table in the equivalent 2-*D* geometry nozzle described in section 3. Since Froude numbers on the water table correspond directly with Mach numbers in the hypothetical gas flow, for the purpose of simulation the throat Froude number in the actual model on the water table should also be 0.5178. This condition was achieved on the water table with a depth of 1.8 cm, and corresponding velocity of 21.76 cm/sec, and with a flow rate of 0.333 kg/sec through the model. The depths at the other stations for the above mentioned flow are measured, and the velocities and Froude numbers at various stations are calculated.

The Froude numbers obtained through the experiment are directly compared with the hypothetical Mach numbers obtained through the gas dynamic solution. The comparison is shown in Table 2, along with percentage errors obtained for the two solutions.

Table 2. Comparison of Froude numbers with hypothetical Mach numbers

Stn	h (cm)	V (cm/sec)	F	M ($k = 2.0$)	Error (%)
1	2.11	5.18	0.1139	0.1209	+5.79
2	2.11	5.78	0.1270	0.1353	+6.13
3	2.10	8.90	0.1961	0.2117	+7.37
4	1.80	21.76	0.5178	0.5178	0.00
5	1.80	19.21	0.4571	0.4327	−5.64
6	1.85	13.32	0.3126	0.2865	−9.10
7	1.89	8.22	0.1909	0.1738	−9.83
8	1.90	6.02	0.1394	0.1268	−9.94

In the classical hydraulic analogy for ideal gases the depth ratios on water table correspond to the temperature ratios, and depth ratio squares correspond to the pressure ratios in the corresponding gas flow. Since depths on water table for subsonic flow under consideration are available now, the pressure and temperature ratios in the corresponding hypothetical gas flow can be readily obtained from the squares of the depth ratios, and depth ratios respectively. Taking throat as reference section, these ratios are determined and compared with the pressure and temperature ratios obtained in the hypothetical gas dynamic solution. The comparison is shown in Table 3. From Table 2, it can be seen that the Froude numbers on water table are

Table 3. Comparison of pressure and temperature ratios in classical analogy (Subsonic flow)

Stn	h/h_4 (1)	$(h/h_4)^2$ (2)	P/P_4 ($k = 2.0$) (3)	T/T_4 ($k = 2.0$) (4)	Error (%) (2)−(3)	Error (%) (1)−(4)
1	1.1722	1.3741	1.2675	1.2590	− 8.41	−4.11
2	1.1722	1.3741	1.2629	1.1238	− 8.81	−4.31
3	1.1666	1.3610	1.2303	1.1092	−10.62	−5.18
4	1.0000	1.0000	1.0000	1.0000	0.00	0.00
5	1.0000	1.0000	1.0753	1.0370	+ 7.00	+3.56
6	1.0277	1.0562	1.1867	1.0894	+10.99	+5.66
7	1.0500	1.1025	1.2481	1.1172	+11.66	+6.01
8	1.0555	1.1151	1.2657	1.1251	+11.97	+6.19

lower than the gas dynamic values in the converging section, and correspondingly they are more in the divergent section. On the contrary, the pressure and temperature ratios obtained from water table are more in converging section and less in the divergent section compared to the gas dynamic solution. These errors may be attributed to the growth of the boundary layer and viscous effects of the flow especially at higher Froude numbers. Since these errors are obtained in the modelling of a hypothetical gas, they are likely to occur in whatever form the analogy may be modified and used subsequently, since practical simulating conditions on water table cannot avoid these errors.

(b) *Modelling for a real gas flow.* The Froude numbers obtained from the free surface incompressible flow on water table will not be same as the Mach numbers obtained for the compressible flow of gas with $k = 1.4$, since the classical analogy is valid only for $k = 2.0$. Corresponding Mach numbers in a gas flow can be obtained by correcting the Froude numbers measured on the water table based on the condition that A/A^* is same for hypothetical gas and gas with any specific heat ratio. The corrected Mach number from the Froude number can be obtained by[7]

$$M_c = C_M \cdot F \tag{1}$$

where

$$C_M = \frac{1.8371 \left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2} M_c^2\right)\right]^{\frac{k+1}{2(k-1)}}}{\left(1 + \frac{F^2}{2}\right)^{1.5}} \tag{2}$$

Equation (1) is a transcendental equation and M_c can be obtained by an iterative procedure through a computer program. Since the throat Froude number on the water table for the subsonic flow under consideration is 0.5178, the corresponding throat Mach number in the gas flow from Eqn. (1) above will be 0.5405. In order to model the real gas flow through the nozzle, the gas dynamic solution is found with the throat Mach number equal to 0.5405 first. Then the Froude numbers obtained at different stations on water table (Table 2) are modified with the correction factor given in Eqn. (2) and the modified Froude numbers thus obtained from free surface incompressible water flow are compared with Mach numbers obtained from the gas dynamic solution obtained earlier. The comparison is given in Table 4.

When a real gas flow with a specific heat ratio equal to 1.4 is simulated on water table, the temperature and pressure ratios in the gas flow can be obtained from the depth ratios and depth ratio squares respectively by applying suitable correction factors. These corrections are[8]

$$\frac{T}{T'} = C_T \frac{h}{h'} \tag{3}$$

Table 4. Comparison of corrected Froude numbers with Mach numbers in the gas flow (subsonic flow)

Stn	F	M_c	M ($k = 1.4$)	Error (%)
1	0.1139	0.1209	0.1283	+5.76
2	0.1270	0.1348	0.1436	+6.12
3	0.1961	0.2078	0.2243	+7.35
4	0.5178	0.5405	0.5405	0.00
5	0.4571	0.4790	0.4541	−5.48
6	0.3126	0.3300	0.3029	−8.95
7	0.1909	0.2023	0.1845	−9.65
8	0.1394	0.1480	0.1346	−9.95

where

$$C_T = \frac{\left(1 + \frac{k-1}{2} M_c'^2\right)\left(1 + \frac{F^2}{2}\right)}{\left(1 + \frac{F'^2}{2}\right)\left(1 + \frac{k-1}{2} M_c^2\right)} \tag{4}$$

($'$) represents the chosen reference section.

Similarly

$$\frac{P}{P'} = C_P \left(\frac{h}{h'}\right)^2 \tag{5}$$

where

$$C_P = \left[\frac{\left(1 + \frac{F^2}{2}\right)}{\left(1 + \frac{F'^2}{2}\right)}\right]^2 \left[\frac{\left(1 + \frac{k-1}{2} M_c'^2\right)}{\left(1 + \frac{k-1}{2} M_c^2\right)}\right]^{\frac{k}{k-1}} \tag{6}$$

The depth ratios obtained from simulated water flow model are modified by applying the correction factors given above taking throat as the reference section and the results are compared with the gas dynamic solution in Tables 5 and 6.

Table 5. Comparison of modified depth ratio squares with pressure ratios (subsonic flow)

Stn	h/h_4	$(h/h_4)^2$	C_P	$C_P\ (h/h_4)^2$	P/P_4	% Error modified analogy
1	1.1722	1.3741	0.9511	1.3069	1.2059	− 8.30
2	1.1722	1.374	0.9517	1.3077	1.2025	− 8.74
3	1.1666	1.3610	0.9561	1.3043	1.1779	−10.47
4	1.0000	1.0000	1.0000	1.0000	1.0000	0.00
5	1.0000	1.0000	0.9889	0.9889	1.0590	+ 6.62
6	1.0277	1.0562	0.9676	1.0220	1.1450	+10.73
7	1.0500	1.1025	0.9557	1.0536	1.1913	+11.56
8	1.0555	1.1141	0.9524	1.0612	1.2046	+11.90

Table 6. Comparison of modified depth ratios with temperature ratios (subsonic flow)

Stn	h/h_4	C_T	$C_T\ (h/h_4)$	T/T_4	% Error modified analogy
1	1.1722	0.9366	1.0979	1.0545	−4.11
2	1.1722	0.9374	1.0988	1.0573	−3.93
3	1.1666	0.9431	1.1002	1.0475	−5.03
4	1.0000	1.0000	1.0000	1.0000	0.00
5	1.0000	0.9856	0.9856	1.0161	+3.00
6	1.0277	0.9580	0.9845	1.0389	+5.24
7	1.0500	0.9426	0.9897	1.0508	+5.81
8	1.0555	0.9383	0.9904	1.0542	+6.05

Comparison of Tables 2 and 4, and Tables 3, 5 and 6, indicate that the order of errors obtained in modified analogy is exactly the same as that obtained in the classical analogy. This fact demonstrates the effectiveness of the correction factor proposed. Thus, the errors obtained in the modified analogy also may be attributed to the same in classical analogy, viz., viscous effects, boundary layer effects and other non-analogous effects.

4.2 *Isentropic Supersonic Flow*

(a) *Modelling for a hypothetical gas flow.* The nozzle given in section 2 is considered for simulating the isentropic supersonic flow. The Mach number at the throat of the nozzle for the flow of a gas whether it is real or hypothetical will have a value of unity under supersonic flow conditions. Hence the gas dynamic solution for a hypothetical gas flow with a throat Mach number of unity is first obtained and the flow is simulated on the water table, in the equivalent 2-*D* nozzle given in section 2.

For the purpose of simulating the flow through the equivalent 2-*D* geometry nozzle, back pressure is completely removed, and the possibility of a compression shock in the diverging section is eliminated. In view of the small thickness of the film of water obtained in the divergent section with low flows, the flow is increased to 0.5773 kg/sec, and a minimum of 4 mm thickness of water film is obtained in the divergent section. The height of water obtained in the throat section is 1.81 cm, with a velocity of 37.52 cm/sec, when the back pressure is zero. The maximum value of the throat Froude number achieved under these conditions is only 0.8904. In order to have an estimate of errors due to viscous effects, boundary layer effects, and surface roughness etc. under supersonic flow conditions, the Froude numbers obtained on the water table are again directly compared as in the subsonic case with hypothetical Mach numbers obtained through gas dynamic solution in Table 7, and the errors between the two are tabulated. The pressure ratios and temperature ratios are calculated from depth ratio squares and depth ratios respectively, and they are compared with pressure and temperature ratios obtained from the hypothetical gas dynamic solution. The comparison is given in Table 8.

Table 7. Comparison of Froude numbers with hypothetical Mach numbers (supersonic flow)

Stn	h (cm)	V (cm/sec)	F	M ($k = 2.0$)	Error (%)
1	2.57	7.36	0.1468	0.1545	+ 4.98
2	2.61	8.06	0.1600	0.1732	+ 7.62
3	2.55	12.72	0.2543	0.2748	+ 7.46
4	1.81	37.52	0.8904	1.0000	+10.96
5	1.31	45.76	1.2765	1.5199	+16.01
6	0.79	54.10	1.9433	2.2292	+12.83
7	0.52	51.80	1.2935	3.1563	+27.34
8	0.41	48.41	2.4138	3.8312	+36.98

Table 8. Comparison of pressure and temperature ratios in classical analogy (supersonic flow)

Stn	h/h_4 (1)	$(h/h_4)^2$ (2)	P/P_4 (3)	T/T_4 (4)	% Error (2)−(3)	% Error (1)−(4)
1	1.4199	2.0161	2.1972	1.4823	+ 8.24	+ 4.21
2	1.4420	2.0794	2.1840	0.4778	+ 4.79	+ 2.42
3	1.4088	1.9847	2.0893	1.4454	+ 5.00	+ 2.53
4	1.0000	1.0000	1.0000	1.0000	0.00	0.00
5	0.7237	0.5237	0.4845	0.6960	− 8.09	− 3.98
6	0.4365	0.1905	0.1853	0.4305	− 2.81	− 1.39
7	0.2873	0.0825	0.0629	0.2508	−31.16	−14.55
8	0.2265	0.0513	0.0324	0.1799	−58.33	−25.90

(b) Modelling for a real gas flow. Now a real gas with a specific heat ratio 1.4 is assumed to be flowing through the nozzle under supersonic conditions. The throat is taken as reference section, and a Mach number of unity is considered in the throat. The gas dynamic solution for the supersonic flow with throat Mach number unity is first obtained. The Froude numbers obtained from the water table are modified with the correction factor given in Eqn. (2) and the modified Froude numbers thus obtained are compared with the Mach numbers in the real gas flow in Table 9.

Table 9. Comparison of modified Froude numbers with Mach numbers (supersonic flow)

Stn	F	M_e	M ($k = 1.4$)	Error (%)
1	0.1468	0.1558	0.1640	+ 5.01
2	0.1600	0.1698	0.1837	+ 7.56
3	0.2543	0.2690	0.2907	+ 7.46
4	0.8904	0.8958	1.0000	+10.42
5	1.2765	1.2390	1.4350	+13.65
6	1.9433	1.7390	1.9271	+ 9.76
7	1.2935	1.9650	2.4522	+19·86
8	2.4138	2.0440	2.7851	+26.60

The depth ratio squares, and depth ratios are modified with the correction factors given in Eqns. (6) and (4), and the pressure and temperature ratios in the free surface incompressible water flow are obtained. These are compared with the pressure and temperature ratios obtained from the gas dynamic solution for a real gas. The comparison is shown in Table 10.

Comparison of Tables 7 and 9, and Tables 8 and 10 indicate that the order of errors in classical analogy and modified analogy are not same in the supersonic case. The errors in the classical analogy particularly in the divergent section are more. It may be mentioned here that the throat Mach number is taken as unity in the hypothetical gas flow, whereas the throat Froude number actually obtained is far less than this value. Hence comparison of these two solutions is bound to give considerable error. The larger errors obtained in the supersonic flow, particularly in the divergent section may be attributed to the following:

(i) The existence of a velocity profile due to the growth of the boundary layer. The boundary layer calculations[11] indicated that it is fully developed by the time it has reached the throat of the model.

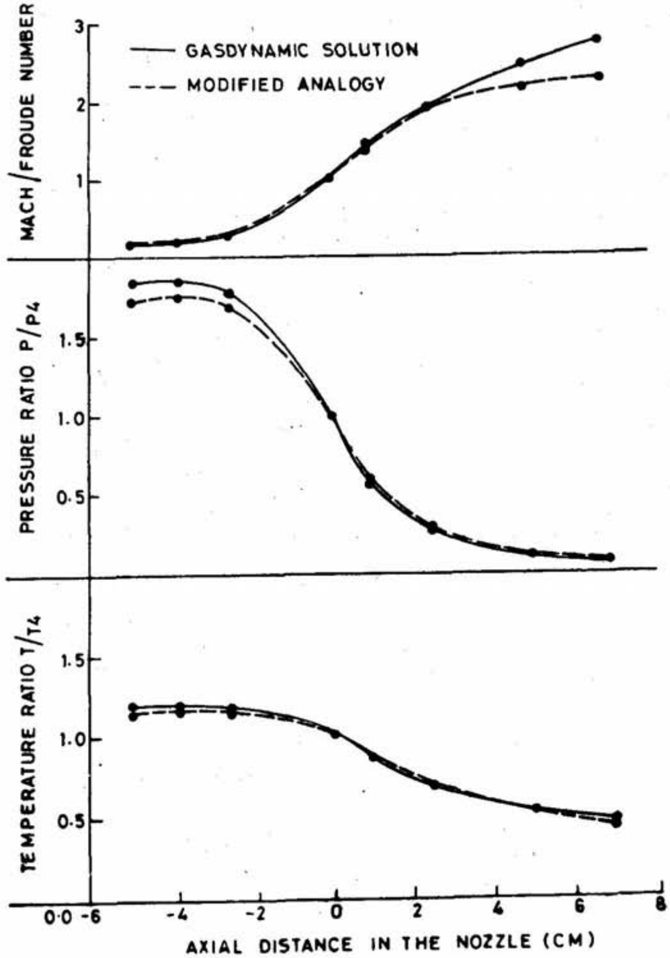

Figure 3. Comparison of mach numbers, pressure and temperature ratios, modified supersonic flow for the case in free steam velocities.

Table 10. Comparison of modified depth ratio squares and depth ratios with pressure and temperature ratios (supersonic flow)

Stn	h/h_4	$(h/h_4)^2$	C_P	$C_P\ (h/h_4)^2$	P/P_4	Error (%)	C_T	$C_T\ (h/h_4)$	T/T_4	Error (%)
1	1.4199	2.0161	0.8672	1.7484	1.8577	+ 5.88	0.8358	1.1868	1.1935	+ 0.56
2	1.4420	2.0794	0.8679	1.8048	1.8489	+ 2.38	0.8368	1.2066	1.1920	− 1.22
3	1.4088	1.9847	0.8749	1.7365	1.7851	+ 2.72	0.8456	1.1913	1.1801	− 0.95
4	1.0000	1.0000	1.0000	1.0000	1.0000	0.00	1.0000	1.0000	1.0000	0.00
5	0.7237	0.5237	1.1138	0.5833	0.5661	− 3.03	1.1537	0.8349	0.8499	+ 1.76
6	0.4365	0.1905	1.3752	0.2620	0.2709	+ 3.28	1.4955	0.6528	0.6886	+ 5.20
7	0.2873	0.0825	1.5353	0.1267	0.1194	− 6.11	1.7022	0.4890	0.5448	+10.24
8	0.2265	0.0513	1.5775	0.0809	0.0714	−13.31	1.7716	0.4013	0.4704	+14.69

(ii) The Reynolds number calculations indicate that the boundary layer is turbulent at the throat.

(iii) The throat Froude number calculated on the basis of average velocity is much below the value of unity for the choked flows.

(iv) The predominance of the viscous effects due to high velocities, particularly in the divergent section.

In view of the above, and the fact that the throat Froude number has not reached a value of unity, supersonic calculations for a real gas flow are repeated using free surface velocity as the reference velocity for the calculation of Froude numbers. In order to estimate the free surface velocity at any section beyond the throat, the velocity profile is assumed to follow 1/7th power law[11]. The ratio of average velocity to free surface velocity is taken as 0.875 (this type of correction was not applied in the subsonic case as the water depths are comparatively large in subsonic flow and calculations indicated that the boundary layer never completely covered the water film thickness). The results obtained using free stream velocities have considerably improved the accuracy of analogy. The comparison of Mach numbers, pressure ratios and temperature ratios for the case in which free stream velocities are used is given graphically in the Fig. 3.

The errors even with using free surface velocities particularly in the divergent section are considerably large. In this connection, it may be mentioned that the depths in the supersonic region with very high Froude numbers are very small (about 0.4 cm) and the accuracy of measurement of the depth plays an important role in the experimental procedure.

5. Conclusions

The hydraulic analogy can be utilised effectively to study the flow field of a two dimensional compressible isentropic gas flow. The correction factors for non-ideal gases (specific heat ratio not equal to 2) can be used effectively to model real gases on a water table. However, conventional limitations of hydraulic analogy like viscous effects, three dimensional flows, surface tension effects will be present in the modified analogy. For supersonic flows, when the film thicknesses are very small, the deviations in the analogy can be attributed to the inaccuracies in the depth measurement and predominant boundary layer effects. In cases where boundary layer effects are predominant, the results obtained indicate that the accuracy of the analogy can be considerably improved by using free surface velocities as the reference velocity for the calculation of Froude numbers. More experimental data involving direct measurements of free surface velocities by sophisticated instruments (such as Laser Doppler velocity meter) are advisable in the supersonic flow experiments.

References

1. Preiswerk, E., *NACA, TM,* No. 934 (1940).
2. Harlemann, D.R.F. & Ippen, A.T., Volume for M.D. Riabouchinsky, Pub. of Scientiques, et Tech du Ministers del' Air, Paris (1954).
3. Adams, D.M., *J. of Spacecraft,* 4 (1967), 359.
4. Bryant, R.A.A., *Australian J. of App. Sci.*, 7 (1956), 296.
5. Loh, W.H.T., 'Modern Developments in Gas Dynamics' (Plenum Press, New York), 1969.
6. Laitone, E.V., *J. of Aero. Sci.*, (1952), 265.
7. Rao, J.S., 80-ID-002-1, STI Rochester, NY, (1980).
8. Rao, J.S., 80-ID-002-2, STI Rochester, NY, (1980).
9. Back, L. H., Massier, P. F. & Gier, H.L., Jet Propulsion Lab. TR 32-654, Pasadena, California, (1964).
10. John, J.E.A., 'Gas Dynamics' (Allyn & Bacon, Boston), 1969.
11. Schlichting, H., 'Boundary Layer Theory' (McGraw-Hill Book Co), 1960.