TEMPERATURE PROFILE OF A FLUID BETWEEN TWO ROTATING POROUS CYLINDERS

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An exact expression for the temperature profile between two concentric rotating porous cylinders has been obtained. The results are presented graphically. For the wide gap, there is a sharp rise in temperature when the ratio between the angular velocities of the outer and the inner cylinders tends to zero.

Schlichting[1] calculated the velocity and temperature profiles for viscous flow between two coaxial non-porous rotating cylinders. Mishra[2] studied the velocity distribution of a flow between two porous rotating cylinders. Recently Bahl[3] has studied the stability of such flows for narrow gap. The present study investigates, in detail, the temperature profiles for the viscous fluid contained between two coaxial rotating porous cylinders. It is assumed that the rate of suction at the wall of one cylinder is equal to the rate of injection at the other. The study has its practical utility when it becomes necessary to inject coolant in the system to counteract heat generated due to viscous forces present in the flow.

Let R_1 and R_2 be the radii of the inner and outer cylinders, rotating with angular velocities Ω_1 and Ω_2 and having temperatures T_1 and T_2 respectively. The energy equation in cylindrical coordinates is given by

$$c\left[u\frac{dT}{dr}\right]=\frac{\lambda}{\rho}\left[\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}\right]+\nu\left[\;2\left\{\left(\frac{du}{dr}\right)^2+\left(\frac{u}{r}\right)^2\right\}+\left(\frac{dv}{dr}-\frac{v}{r}\right)\right] \qquad (1)$$

where c, λ, ρ and ν are constants and the boundary conditions are

$$\left.\begin{array}{llll} u = U_{R_1} & v = R_1\Omega_1 & T = T_1 & \text{at } r = R_1 \\ u = R_1 U_{R_1}/R_2 & v = R_2\Omega_2 & T = T_2 & \text{at } r = R_2 \end{array}\right] -\infty < z < \infty \qquad (2)$$

Following Mishra[2], velocity components for the flow are given by

$$u = R_1 U_{R_1}/r \qquad (3)$$

and

$$v = \frac{R_1\Omega_1}{K^{\sigma+2}-1}\left[R_1{}^{+\sigma}(K^2\mu-1)+\frac{K^2\;(K^{\sigma}-\mu)}{R}\right] \qquad (4)$$

where $\dfrac{r}{R_1} = R, \Omega_2/\Omega_1 = \mu \;\; R_2/R_1 = K$ and $\dfrac{R_1 U_{R_1}}{\nu} = \sigma$, Suctional Reynold Number

The solution of equation (1), after substituting the velocity components from (3) and (4) and applying the boundary conditions (2), is given by

$$\theta = (\beta-1)\frac{(R^{\sigma Pr}-1)}{(K^{\sigma Pr}-1)}+m\left[\frac{2}{(2+\sigma Pr)}\left\{\frac{(R^2-1)}{R^2}-\frac{(K^2-1)}{K^2}\cdot\frac{(R^{\sigma Pr}-1)}{(K^{\sigma Pr}-1)}\right\}\right.$$
$$+\frac{4B_1\;(TA)^2\,A_2{}^2}{\sigma(K^{\sigma Pr}-1)\;\{\sigma^2+(TA)^2\,A_2{}^2\}}\left\{\frac{(K^{\sigma Pr}-1)\;R^{\sigma}-(K^{\sigma}-1)\;R^{\sigma Pr}+K^{\sigma}-K^{\sigma Pr}}{(1-Pr)}\right\}$$
$$+\frac{\sigma^2 B_1{}^2\;(TA)^2\;A_2{}^2}{2\,(K^{\sigma Pr}-1)\;\{2-(2+Pr)\,\sigma\}\;\{\sigma^2+(TA)^2\,A_2{}^2\}}$$
$$\left.\times\left\{\frac{(1-R^{2+2\sigma})\;(K^{\sigma Pr}-1)-(1-K^{2+2\sigma})\;(R^{\sigma Pr}-1)}{(1+\sigma)}\right\}\right] \qquad (5)$$

where

$$\theta = \frac{T - T_1}{T_2 - T_1}, \; m = Pr \times E, \; Pr = \frac{\nu c \rho}{\lambda}, \; E = \frac{U^2_{R_1} + A_1^2}{c T_1}$$

$$A_1 = R_1 \, \Omega_1 A_2, \; A_2 = \frac{K^2 (K^\sigma - \mu)}{(K^{\sigma+2} - 1)}, \; B_1 = \frac{(K^2 \mu - 1)}{K^2 (K^\sigma - \mu)}$$

$$TA = \frac{R_1^2 \Omega_1}{\nu} \text{ and } \beta = \frac{T_2}{T_1}$$

For the computation of θ, it is worthwhile to consider the following particular cases:

Case I : $\sigma = 0$

In the absence of suction or injection at the inner wall $R = 1$ of the cylinder, that is, $\sigma = 0$, the solution reduces to that of Schlichting[1]*. For ready reference it is presented here.

$$\theta = \frac{\log R}{\log K} + m \left[\frac{K^2 - 1}{K^2} \left\{ \frac{(R^2 - 1) K^2}{(K^2 - 1) R^2} - \frac{\log R}{\log K} \right\} \right]$$

Case II : $Pr = 1$

In the limiting case when $Pr = 1$, equation (5) becomes

$$\theta = (\beta - 1) \frac{(R^\sigma - 1)}{(K^\sigma - 1)} + E \left[\frac{2 (K^2 - 1)}{(2 + \sigma) K^2} \left\{ \frac{(R^2 - 1) K^2}{(K^2 - 1) R^2} - \frac{(R^\sigma - 1)}{(K^\sigma - 1)} \right\} \right.$$

$$+ \frac{4 B_1 (TA)^2 A_2^2}{\{(TA)^2 A_2^2 + \sigma^2\}} \left\{ R^\sigma \log R - \frac{(R^\sigma - 1)}{(K^\sigma - 1)} K^\sigma \log K \right\}$$

$$\left. + \frac{B_1^2 \sigma^2 (TA)^2 A_2^2}{2 (1 + \sigma)(2 + \sigma) \{(TA)^2 A_2^2 + \sigma^2\}} \left\{ 1 + R^{2+2\sigma} - (1 - K^{2+2\sigma}) \frac{(R^\sigma - 1)}{(K^\sigma - 1)} \right\} \right]$$

Case III : $\sigma \to -1$

When $\sigma \to -1$, (5) reduces to

$$\theta = (\beta - 1) \frac{(R^{-Pr} - 1)}{(K^{-Pr} - 1)} + m \left[\frac{2 (K^2 - 1)}{K^2 (2 - Pr)} \left\{ \frac{(R^2 - 1) K^2}{(K^2 - 1) R^2} - \frac{(R^{-Pr} - 1)}{(K^{-Pr} - 1)} \right\} \right.$$

$$+ \frac{4 B_1 (TA)^2 A_2^2}{(K^{-Pr} - 1) \{(TA)^2 A_2^2 + \sigma^2\}} \left\{ \frac{K^{-Pr} (1 - R^{-1}) + R^{-1} - K^{-1} - R^{-Pr} (1 - K^{-1})}{(Pr - 1)} \right\}$$

$$\left. - \frac{B_1^2 (TA)^2 A_2^2}{(K^{-Pr} - 1) \{(TA)^2 A_2^2 + \sigma^2\}} \left\{ \frac{(K^{-Pr} - 1) \log R - (K^{-Pr} - 1) \log K}{Pr} \right\} \right]$$

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*There is a misprint in the expression for U_1. For r_2 one should read r_1 (see ref. 3, p. 83, line 5th).

Case IV : $\sigma \to 2/(Pr-2)$

When $\sigma = 2/(Pr-2)$,

we get

$$\theta = (\beta - 1)\frac{(R^{2+2\sigma}-1)}{(K^{2\sigma+2}-1)} + m\left[\frac{2(K^2-1)}{(2+2\sigma+2)K^2}\left\{\frac{(R^2-1)K^2}{(K^2-1)R^2} - \frac{(R^{2\sigma+2}-1)}{(K^{2\sigma+2}-1)}\right\}\right.$$
$$+\frac{4B_1(TA)^2A_2^2}{(K^{2\sigma+2}-1)\{(TA)^2A_2^2+\sigma^2\}}\left\{\frac{R^\sigma K^{2\sigma-2} - R^\sigma + K^\sigma - K^{2\sigma-2} + R^{2\sigma+2} - K^\sigma R^{2\sigma+2}}{(\sigma-2\sigma-2)}\right\}$$
$$\left. - \frac{B_1^2\sigma^2(TA)^2A_2^2}{2(K^{2\sigma+2}-1)\{(TA)^2A_2^2+\sigma^2\}}\left\{\frac{(RK)^{\sigma Pr}(\log R/\log K) + K^{\sigma Pr}\log K - R^{\sigma Pr}\log R}{(1+\sigma)}\right\}\right]$$

CONCLUSION

The non-dimensional form of the temperature profile between two coaxial rotating porous cylinders is given by the expression (5) which is a function of the ratio of their temperatures β radii K, angular velocities μ and Prandtl number, Pr Eckert number, E, Taylor number, TA, and suction Reynolds number, σ. Numerical computation shows that the temperature profile does not vary significantly with the changes in E, TA, and Pr. It is also not difficult to estimate from the expression (5) the change in the temperature profile for these three parameters and β. But on the other hand temperature profile is very significantly affected by the change in the parameters K, σ and μ. Figs. 1, 2 and 3 show the variation of θ with R for fixed values of E, Pr, β and TA, but different values of K, σ and μ.

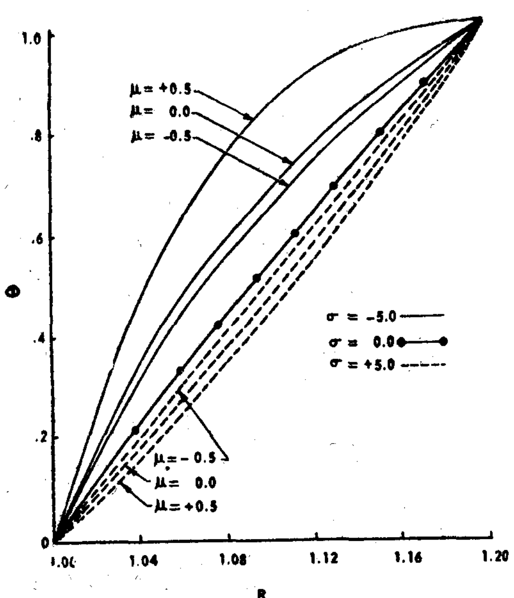

Fig. 1—Variation of θ with R for $R_2/R_1 = 1 \cdot 2$, $E = 0 \cdot 8$, $Pr = 0 \cdot 9$, $T_2/T_1 = 2$, $R_1^2\,\Omega_1/\nu = 600$ and three different values of μ and σ

In Fig. 1 the variation of θ with R is given for $K = 1 \cdot 2$, and for three different values of μ and σ. We find that when the cylinders are rotating in the same direction, by blowing in the flow region a hot fluid there is increase in the temperature profile as compared to the case when the outer cylinder is either stationary or moving in the opposite direction. As expected reverse is the case when the coolant is injected into the flow region.

Fig. 2 gives the variation of θ with R for $K = 1 \cdot 5$. We find that with the increase in the gap ratio, the blowing in of the hot fluid in the flow region, shows a marked increase in the temperature profile when the inner cylinder is rotating but the outer cylinder is kept fixed. In other words blowing in of the hot fluid between the cylinders varies in a disorderly way with μ. But the injection of a coolant causes the decrease in the temperature as μ varies from $-0 \cdot 5$ to $+0 \cdot 5$. This effect is pronounced when the cylinders are rotating in

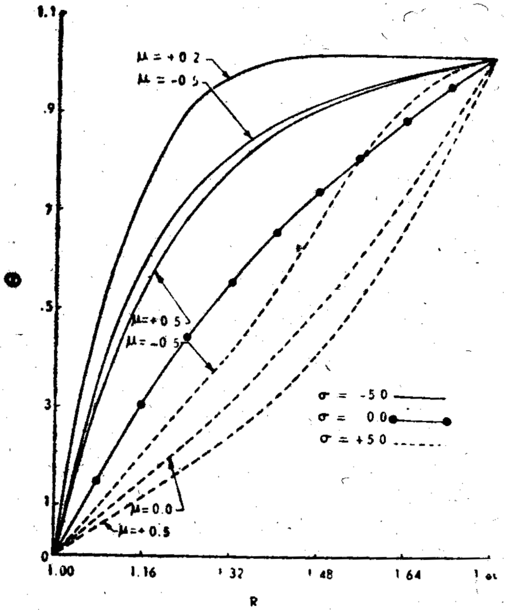

Fig. 2—Variation of θ with R for $R_2/R_1 = 1\cdot5$, $E = 0\cdot8$, $Pr = 0\cdot8$, $T_2/T_1 = 2$, $R_1^2\, \Omega_1/\nu = 600$ and three different values of μ and σ.

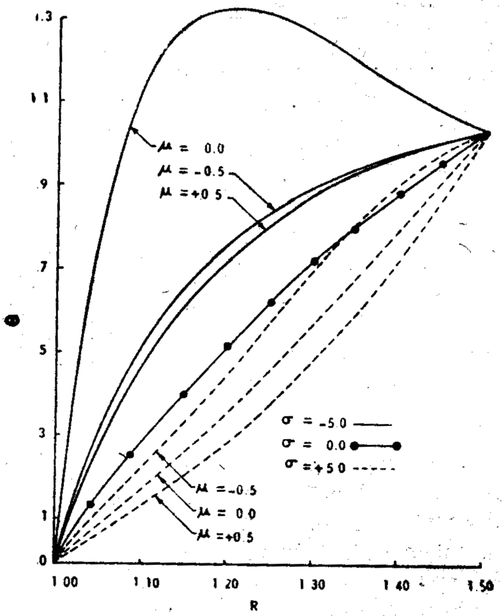

Fig. 3—Variation of θ with R for $R_2/R_1 = 1\cdot8$, $E = 0\cdot8$, $Pr = 0\cdot9$, $T_2/T_1 = 2$, $R_1^2\, \Omega_1/\nu = 600$ and three different values of μ and σ.

the same direction. Whereas for $\mu < 0$, the temperature profile has a point of inflection and the fluid temperature in the neighbourhood of the outer cylinder becomes higher than the case when $\sigma = 0$.

Fig. 3 shows the variation of θ with R for $K = 1\cdot8$. Again we find that for small values of $|\mu|$ there is a rise in the temperature profile when a hot fluid is blown in the flow region. For $\mu = 0$ the maximum value of $\theta = 3\cdot499742$; it cannot be drawn with this scale. But this does indicate that for very small values of $|\mu|$ there is a sharp rise in the value of θ. Whereas for moderate values of $|\mu|$ there is an increase in the temperature as compared to the case when $\sigma = 0$. The injection of the coolant in the flow region shows that the temperature decreases as μ varies from $-0\cdot5$ to $+0\cdot5$. This behaviour is similar to that observed for $K = 1\cdot5$.

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REFERENCES

1. SCHLICHTING, H., *ZAMM*, **31** (1961), 78.
2. MISHRA, S. P., *Proc. National Inst.*, India, **29**, Nov. (1963).
3. BAHL, S. K., *Def. Sci. J.*, **20**, April (1970).