A NOTE ON HYDROMAGNETIC FLOW NEAR AN ACCELERATED PLATE

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The flow of a viscous incompressible and electrically conducting fluid due to accelerated motion of an infinite flat plate in the presence of a uniform transverse magnetic field is considered. The magnetic field is assumed to be fixed relative to the plate. Series solutions for velocity and skin-friction are obtained under the assumption that the magnetic Reynolds number is small. The effect of magnetic field on the velocity and the skin-friction in case of impulsive motion, uniformally accelerated motion and accelerated motion is discussed.

Recently Soundalgekar[1] has studied the flow of an electrically conducting incompressible viscous fluid due to the uniformally accelerated motion of an infinite flat plate in the presence of a uniform transverse magnetic field fixed relative to the plate.

The purpose of this note is to extend the above problem in a general case, i.e., accelerated motion. We have solved this problem by a series expansion, in which the first term represents the (similarity) flow in a non-magnetic case. Expressions for velocity and skin-friction are obtained for small values of magnetic Reynolds number. The effect of magnetic field on the velocity and the skin-friction has been discussed for impulsive motion, uniformally accelerated motion and accelerated motion respectively.

ANALYSIS

Two dimensional motion has been considered in which we take x-axis along the plate and in the direction of motion, while the y-axis is chosen perpendicular to it. It is assumed that a uniform magnetic field of strength H_0 is acting parallel to the y-axis. The momentum equations relevant to the problem are

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u , \tag{1}$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} , \tag{2}$$

Here u is the velocity along the plate, $-\frac{\sigma B_0^2}{\rho} u$ is the component of Lorentz force in the x-direction, σ the electrical conductivity, B_0 $(= \mu_0 H_0)$ the magnetic induction, μ_0 the magnetic permeability, t the time, p the pressure, ρ the density and ν the kinematic viscosity.

All the physical properties of the fluid such as σ, μ_0, ρ, ν are assumed to be constant.

In deriving the above equations, it is assumed that the magnetic Reynolds number is small so that the induced magnetic field is negligible in comparison with the imposed magnetic field. This assumption is justified as in most of the aerodynamic applications the magnetic Reynolds number is small. Further, since no external electric field is applied, the effect of polarization of ionized fluid is negligible, hence it can be assumed that the electric field is zero.

At time $t < 0$, the fluid, the plate and the magnetic field are assumed to be everywhere stationary. For $t = 0$ and for all later times the plate and the magnetic field are accelerated at velocity $u = At^{\alpha}$ (where $\alpha \geqslant 0$ and A is a real positive constant). Because the magnetic field is moving and the fluid is initially at rest, the relative motion must be accounted for (since the origin of coordinates is fixed in the space). Hence (1) becomes

$$\frac{\partial u}{\partial t} + \frac{\sigma B_0^2}{\rho} \left(u - At^{\alpha} \right) = \nu \frac{\partial^2 u}{\partial y^2} , \tag{3}$$

which is to be solved under the following initial and boundary conditions

$$\left.\begin{array}{lll} u = 0 & \text{for } t < 0 & \\ u = At^{\alpha} & \left.\begin{array}{l}\text{at } y = 0 \\ \text{as } y \to \infty\end{array}\right\} & \text{for } t \geqslant 0 \\ u \to 0 & & \end{array}\right\} \tag{4}$$

SOLUTION OF THE PROBLEM

For small values of mt, where $m = \dfrac{\sigma B_0^2}{\rho}$, we expand u in ascending powers of mt as follows

$$u = At^{\alpha} \left[u_0 (\eta) + (mt)\, u_1 (\eta) + (mt)^2\, u_2 (\eta) + \ldots\ldots\ldots\ldots\ldots \right], \tag{5}$$

where

$$\eta = \frac{y}{2\sqrt{\nu t}} ,$$

Now substituting (5) in (3) and comparing the coefficients of like powers of mt, neglecting the coefficients of $(mt)^3$ and higher, we get

$$\left.\begin{array}{l} u_0'' + 2\,\eta\, u_0' - 4\,\alpha\, u_0 = 0 , \\ u_1'' + 2\,\eta\, u_1' - 4\,(\alpha + 1)\, u_1 = 4\,(u_0 - 1) , \\ u_2'' + 2\,\eta\, u_2' - 4\,(\alpha + 2)\, u_2 = 4\, u_1 , \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right\} \tag{6}$$

where the dashes denote the differentiation with respect to η.

The relevant boundary conditions reduce to

$$\left.\begin{array}{ll} u_0 (0) = 1, & u_k (0) = 0 \\ u_0 (\infty) \to 0 , & u_k (\infty) \to 0 \end{array}\right\} (k = 1, 2, 3, \ldots\ldots) \tag{7}$$

The solutions of (6), satisfying the boundary conditions (7), are

$$
\left.
\begin{aligned}
u_0(\eta) &= D\, Hh_{2\alpha}(\sqrt{2}\,\eta), \\
u_1(\eta) &= D\left[\,2\alpha\, Hh_{2\alpha+2}(\sqrt{2}\,\eta) - Hh_{2\alpha}(\sqrt{2}\,\eta)\right] + \frac{1}{(\alpha+1)}, \\
u_2(\eta) &= D\left[\,2\,\alpha\left\{(\alpha+1)\, Hh_{2\alpha+4}(\sqrt{2}\,\eta) - Hh_{2\alpha+2}(\sqrt{2}\,\eta)\right\}\right. \\
&\qquad \left. + \tfrac{1}{2}\, Hh_{2\alpha}(\sqrt{2}\,\eta)\right] - \frac{1}{(\alpha+1)(\alpha+2)},
\end{aligned}
\right\} \tag{8}
$$

where D stands for $\dfrac{2^{\alpha+\frac{1}{2}}}{\sqrt{\pi}}\ \Gamma(\alpha+1)$ and $Hh_{2\alpha}(\sqrt{2}\ \eta)$ is defined[2]

$$
Hh_{2\alpha}(\sqrt{2}\,\eta) = \int_{\sqrt{2}\,\eta}^{\infty} \frac{(u-\sqrt{2}\ \eta)^{2\alpha}}{\Gamma(2\,\alpha+1)}\ e^{-\frac{1}{2}u^2}\, du
$$

Substituting (8) in (5), we get the expression for the velocity.

The skin-friction at the plate is given by

$$
\tau_\omega = -\tfrac{1}{2}\, A\, \rho\, \nu^{\frac{1}{2}}\, t^{\alpha-\frac{1}{2}}\ \left[u'_0(0) + (mt)\ u'_1(0) + (mt)^2\ u'_2(0) + \ \ldots\ldots\right] \tag{9}
$$

In the absence of magnetic field, the local skin-friction at the plate is given by

$$
\tau_0 = -\tfrac{1}{2}\, A\, \rho\, \nu^{\frac{1}{2}}\, t^{\alpha-\frac{1}{2}}\, u'_0(0). \tag{10}
$$

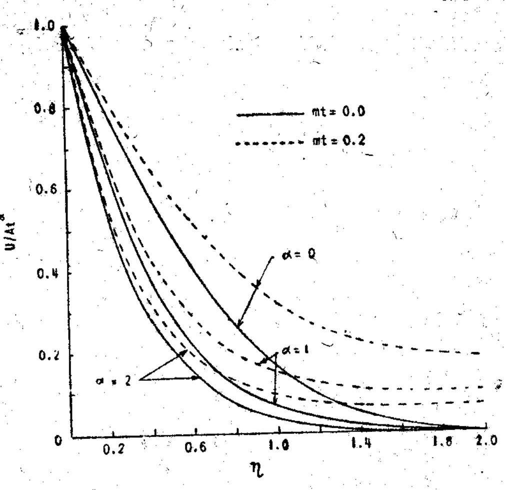

Fig. 1—Variation of velocity profile with η.

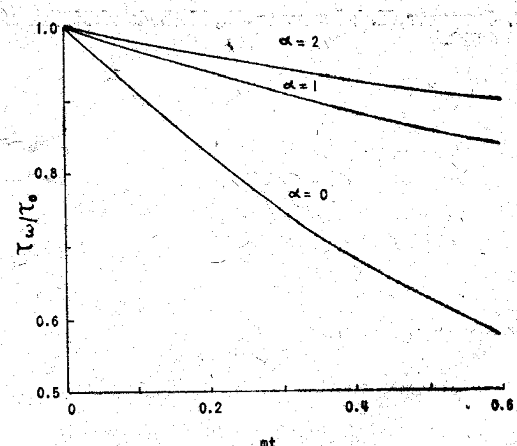

Fig. 2—Variation of skin-friction with mt.

Combining (9) and (10), we get

$$\frac{\tau_\omega}{\tau_0} = 1 + \frac{u'_1(0)}{u'_0(0)}(mt) + \frac{u'_2(0)}{u'_0(0)}(mt)^2 + \ldots\ldots\ldots$$

By utilizing (8), the ratio of the skin-friction becomes

$$\frac{\tau_\omega}{\tau_0} = 1 - \frac{1}{(2\alpha + 1)}(mt) + \frac{3}{2(2\alpha+1)(2\alpha+3)}(mt)^2 + \ldots\ldots$$

RESULTS

In Fig. 1 the dimensionless velocity (u/At^α) is plotted against the similarity variable η for $\alpha = 0, 1, 2$ and $mt = 0, 0\cdot2$. The skin-friction ratio (τ_ω/τ_0) is plotted against mt in Fig. 2 for $\alpha = 0, 1, 2$.

We thus draw the following conclusions :

(*i*) For fixed value of α, the skin-friction decreases and the velocity increases with the increase in magnetic field strength.

(*ii*) The effectiveness of magnetic field, in reducing the skin-friction and increasing the velocity, decreases respectively in the case of accelerated motion $(\alpha = 2)$ than for uniformally accelerated motion $(\alpha = 1)$ and finally for impulsive motion $(\alpha = 0)$.

(*iii*) The skin-friction is greater in the case of accelerated motion than in uniformally accelerated motion and respectively in impulsive motion.

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