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Shock Waves in Dusty Gas with Radiation Effects

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Abstract : The jump conditions across a three dimensional curved shock in dusty gas with radiation have been derived. The general Rankine-Hugoniot relations for a three dimensional curved shock in dusty gas with radiation are investigated. These relations differ considerably from those without radiation effects. The most important parameter is the ratio of radiation pressure to the pressure of the mixture (gas+dust), ie the radiation pressure number R_p. The successive approximations have been used for different cases depending upon the value of radiation pressure number. In certain cases, the results may be expressed in terms of an effective ratio of specific heats and an effective dusty gas pressure. A general method of solution is given and some numerical results are obtained.

1. Introduction

The radiation phenomena can significantly modify the flow of a gas at very high temperatures. The effect of radiation on the Rankine-Hugoniot relations for a shock wave has been studied by Sachs[1] and by Guess and Sen[2,3]. We consider an optically thick dusty gas with such a high temperature that the radiation pressure number R_p is not negligible.

$$R_p = \frac{\text{radiation pressure}}{\text{pressure of the mixture (gas+dust)}} = \frac{p_R}{p}$$

In this paper our aim is to develop the results for a three dimensional curved-shock in a dusty medium, which is optically thick. To achieve the above, we have used as parameters the density shock strength and either the ordinary normal Mach number or the effective normal Mach number in front of the shock. We assume the dust particles are to be incompressible here.

2. Fundamental Equations of Motion

Proceeding in a manner similar to Praveen Dhyani[4] and considering the effects of radiations in an optically thick dusty medium, the conservation of mass, momentum and energy give the following relations

$$\rho_2 v_{2n} = \rho_1 v_{1n} = m, \tag{1}$$

$$m v_{2n} + p_{2t} = m v_{1n} + p_{1t}, \tag{2}$$

$$\tfrac{1}{2} v_{2n}^2 + \varphi C_v T_2 + (1+C)\frac{E_{2R}}{\rho_2} + \frac{p_{2t}}{\rho_2} = \frac{1}{2} v_{1n}^2 + \varphi C_v T_1 + (1+C)\frac{E_{1R}}{\rho_1} + \frac{p_{1t}}{\rho_1}, \tag{3}$$

where

$$\left.\begin{aligned} & v_n = v_i n_i,\ v_a = v_i x_{i,\alpha},\ p_t = p + p_R, \\ & p_R = \tfrac{1}{3} a_R T^4,\ E_R = a_R T^4, \\ & E_{2gR} = E_{2dR} = a_R T_2^4 = E_{2R},\ E_{1gR} = E_{1dR} = a_R T_1^4 = E_{1R}. \end{aligned}\right\} \tag{4}$$

Here a_R is the Stefan Boltzmann constant, φ is a dimensionless quantity[5], C is the concentration of dust particles. The effective normal Mach number in radiation dusty gas dynamics is given[6] by

$$M_{en}^2 = \frac{\gamma\{1+12(\gamma-1)R_p\}M_n^2}{\gamma+20(\gamma-1)R_p+16(\gamma-1)R_p^2}. \tag{5}$$

3. Derivation of the Jump Conditions

We shall introduce the following non-dimensional parameters,

$$T^* = \frac{RT}{v_{in}^2},\ P = T_1^* = \frac{1}{\gamma M_{in}^2},\ Q = \frac{a_R v_{in}^6}{R^4 \rho_1}, \tag{6}$$

and

$$R_p = \frac{p_R}{p} = \frac{1}{3}\frac{Q}{1+\delta}\left(T^*\right)^3. \tag{7}$$

From Eqns. (1), (2), (3), (4), (6) and (7), we get the following relation

$$\frac{1}{1+\delta} + T^*_2 (1+\delta)(1+R_{2p}) = 1 + P + PR_{1p}. \tag{8}$$

Using Eqns. (1), (2), (3), (4) and (6) and eliminating T^*_2 from Eqn. (8), we obtain

$$\delta\Big[l - (1+\delta)\Big(q^2\varphi + 7(1+C)R_{2p}\Big)^{-1}\Big\{(1+R_{2p})$$

$$\cdot\left(1 - \frac{1}{(1+\delta)}\frac{(1-\varphi-CR_{2p})}{(1+R_{2p})}\right) + \Big((q^2-1)\varphi + 2 + (8+6C)R_{2p}\Big)$$

$$\cdot P_{2e}\Big\}\Big] = 0, \tag{9}$$

$$P_e = (1+R_{1p})\ f(R_p)\ P, \tag{10}$$

where

$$f(R_p) = \frac{g(R_p)(1+\delta) - 1}{\delta},\quad q^2 = \frac{\gamma+1}{\gamma-1}, \tag{11}$$

and

$$g(R_p) = \frac{(1+R_p)\{(q^2-1)\varphi + 2 + (8+6C)R_{1p}\}}{(1+R_{1p})\{(q^2-1)\varphi + 2 + (8+6C)R_p\}}. \tag{12}$$

P_e is called the effective value of P in radiation dusty gas dynamics. Equation (9) gives two roots of δ, one is $\delta = 0$ which refers to no shock condition. The physically important solution is the other root, given by

$$\delta = \frac{1 - \gamma_{2e}P_{2e}}{\gamma_{2e}P_{2e} + \gamma_{2e} - A'}, \tag{13}$$

where

$$\gamma_e = (8+6C)(\gamma-1)R_p + 2(\varphi+\gamma-1)\,/\,\{(6+7C)(\gamma-1)R_p$$

$$+ (\gamma+1)\varphi - (\gamma-1)\}\left\{1 + \frac{1}{(1+\delta)}\frac{(1-\varphi-CR_p)}{\left(\frac{\gamma+1}{\gamma-1}.\varphi - 1 + (6+7C)R_p\right)}\right\}, \tag{14}$$

$$A' = (\gamma-1)\left\{(7+6C)R_{2p} + \frac{2\varphi}{\gamma-1} + 1 + \frac{1}{1+\delta}(1-\varphi-CR_{2p})\right\}\Big/$$

$$\{(\gamma+1)\varphi - (\gamma-1) + (6+7C)(\gamma-1)R_{2p}\}\left\{1 + \frac{1}{(1+\delta)}\right.$$

$$\left.\cdot\frac{(1-\varphi-CR_{2p})}{\left(\frac{\gamma+1}{\gamma-1}.\ \varphi - 1 + (6+7C)R_{2p}\right)}\right\}. \tag{15}$$

γ_e is called the effective ratio of specific heats. Equation (9) also gives the following relation which we shall use later on

$$K(1+R_{2p})\delta^2 - [(1+R_{2p})(q^2\varphi - K) + \{6(1+C) - (q^2-1)\varphi\}$$

$$(R_{2p} + PR_{2p} - PR_{1p}) + (1+C-\varphi)R_{2p}]\,\delta - (R_{2p}-R_{1p})P\{6(1+C)$$

$$-(q^2-1)\varphi\} = 0, \tag{16}$$

where

$$K = \left[\left(1 - \frac{1}{(1+\delta)}\frac{(1-\varphi-CR_{2p})}{(1+R_{2p})}\right) + \left((q^2-1)\varphi + 2 + (8+6C)R_{1p}\right)P\right].$$

The equation corresponding to Eqns. (9) in ordinary shocks is simple. Due to the inclusion of radiation effects and dust particles, this equation becomes very complicated. In fact, to determine the values of δ explicitly in terms of known quantities in front of the shock, we have to solve a higher degree equation in δ, as Eqns. (13) and (16) contain R_{2p} and f (refer to Eqn. 11) which themselves are function of δ. To get the solution for different cases offered by different values of R_{1p}, we take the approximations and for this purpose Eqn. (13) is the most convenient form.

4. Limiting cases of the Rankine-Hugoniot Relations in Radiation Dusty Gas Dynamics

(a) Weak shock in cold Dusty Gas ($R_{1p} << 1$)

In this case, the temperature in front of the shock is not very high. Therefore. the radiation effects may be neglected in front of the shock. From Eqns. (6) and (8), we obtain

$$T_2^* = \frac{1+\delta(1+\gamma M^2_{1n})}{\gamma M^2_{1n}(1+\delta)^2(1+R_{2p})}. \tag{17}$$

Using the above relation alongwith Eqns (4), (6), (7) and (16), we obtain

$$\frac{T_2}{T_1} = \frac{T^*_2}{P} = \frac{1+\delta(1+\gamma M^2_{1n})}{(1+\delta)^2(1+R_{2p})},$$

$$\frac{P_2}{P_1} = \frac{\rho_2 T_2}{\rho_1 T_1} = \frac{1+\delta(1+\gamma M^2_{1n})}{(1+\delta)(1+R_{2p})},$$

$$\frac{P_{2R}}{P_{1R}} = \left(\frac{T_2}{T_1}\right)^4 = \left\{\frac{1+\delta(1+\gamma M^2_{1n})}{(1+\delta)^2(1+R_{2p})}\right\}^4, \tag{18}$$

$$\frac{R_{2p}}{R_{1p}} = \frac{P_{2R}}{P_{1R}}\cdot\frac{P_1}{P_2} = \frac{1}{(1+\delta)}\left\{\frac{1+\delta(1+\gamma M^2_{1n})}{(1+\delta)^2(1+R_{2p})}\right\}^3, \tag{19}$$

$$K_1(1+R_{2p})\delta^2 - [(1+R_{2p})(q^2\varphi - K_1) + \{6(1+C)-(q^2-1)\varphi\}(1+P)R_{2p} + (1+C-\varphi)R_{2p}]\,\delta - P\{6(1+C) - (q^2-1)\varphi\}R_{2p} = 0, \tag{20}$$

where

$$K_1 = \left[\left\{1 - \frac{1}{(1+\delta)}\cdot\frac{(1-\varphi-CR_{2p})}{(1+R_{2p})}\right\} + \left\{(q^2-1)\varphi + 2\right\}P\right]. \tag{21}$$

(b) Very Strong Shock in a Cold Dusty Gas ($R_{1p} << 1$ And $R_{2p} >> 1$)

In this case, we have $R_{1p} << 1$ and $R_{2p} >> 1$. For a very strong shock $M^2_{1n} >> 1$. Using these facts in Eqns. (10), (11), (12), (14), (15) and using Eqns. (13), we have

$$\delta = 6\,(1 + C), \tag{22}$$

which determines the value of δ completely in terms of known quantities. This is the maximum attainable value of δ in radiation dusty gas dynamics. Pai[7] has obtained $\delta = 6$ for radiation gas dynamics irrespective of the value of γ, while in ordinary gas dynamics it is 5 for $\gamma = 1.4$.

From Eqns. (19), we have

$$R_{2p} = \left\{\frac{1 + \delta\,(1 + \gamma M^2_{1n})}{(1 + \delta)^2}\right\}^{3/4} \cdot \left\{\frac{R_{1p}}{1 + \delta}\right\}^{1/4}. \tag{23}$$

From Eqns. (18), (19) and (23), we have for this case the following relations.

$$\left.\begin{aligned}
\frac{T_2}{T_1} &\cong \left\{\frac{1 + \delta\,(1 + \gamma M^2_{in})}{(1 + \delta)\,R_{1p}}\right\}^{1/4},\\
\frac{P_2}{P_1} &\cong (1 + \delta)\left\{\frac{1 + \delta\,(1 + \gamma M^2_{1n})}{(1 + \delta)\,R_{1p}}\right\}^{1/4},\\
\frac{P_{2R}}{P_{1R}} &\cong \left\{\frac{1 + \delta\,(1 + \gamma M^2_{1n})}{(1 + \delta)\,R_{1p}}\right\},\\
\frac{R_{2p}}{R_{1p}} &\cong \frac{1}{(1 + \delta)}\left\{\frac{1 + \delta\,(1 + \gamma M^2_{1n})}{(1 + \delta)\,R_{1p}}\right\}^{3/4},\\
\frac{\rho_2}{\rho_1} &= (1 + \delta) \text{ and } v_{2i} = v_{1i} - \frac{\delta}{1 + \delta}\, v_{1n}\, n_i,
\end{aligned}\right\} \tag{24}$$

where δ is given by Eqn. (22).

The above relations determine the flow variables behind the shock in terms of the known flow quantities in front of the shock.

(c) Shock Wave in a very hot plasma ($R_{1p} >> 1$)

In this case, the temperature of the dusty gas in front of the shock is high and therefore the radiation effects in front of the shock cannot be neglected and consequently, they cannot be neglected behind as well, i.e., $R_{2p} >> 1$, irrespective of the shock strength.

$$A' = \left(\frac{7 + 6C}{6 + 7C} \right) \cdot \frac{\left\{1 - \frac{C}{(7 + 6C)(1 + \delta)}\right\}}{\left\{1 - \frac{C}{(6 + 7C)(1 + \delta)}\right\}},$$

Taking the value of A' and using Eqns. (10) to (12), (14) and (13), we obtain

$$\delta = \frac{3\left\{ 4(1 + C)\, M^2_{eln} - (4 + 3C) \right\}}{2M^2_{eln} + 3(4 + 3C)}, \tag{25}$$

where the effective normal Mach number in front of the shock for this case is

$$M^2_{eln} = \frac{3\gamma\, M^2_{ln}}{4R_{lp}}.$$

From Eqn. (25), we draw the following inferences.

(1) The value of M_{eln} tends to 0.8660 when C tends to infinity.

(2) There is very little variation in the slope of curve, when there is little variation in the concentration of dust particles.

A graph between the effective normal mach number M_{eln} and concentration of dust particles C has been drawn (Fig. 1) when density shock strength $\delta = 0$. A graph between M_{eln} and δ has been drawn for different concentration of dust particles C, viz. 0.01, 0.02, and 0.05 and 0.10 as shown in Fig. 2.

Using Eqns. (7), Eqn. (8) transforms to

$$(T^*_2) + A^{-1}\, T^*_2 - A^{-1}\, B = 0, \tag{26}$$

where

$$A^{-1} = \frac{P^3(1 + \delta)}{R_{1p}} \text{ and } B = \frac{(1 + P + PR_{1p})(1 + \delta) - 1}{(1 + \delta)^2}. \tag{27}$$

For $R_{1p} >> 1$, $(T^*_2)^4 >> A^{-1}\, T^*_2$, (Pai[7]). Therefore, from Eqns. (26) using Eqns. (6), (7), (4) and (27), we obtain the following relations

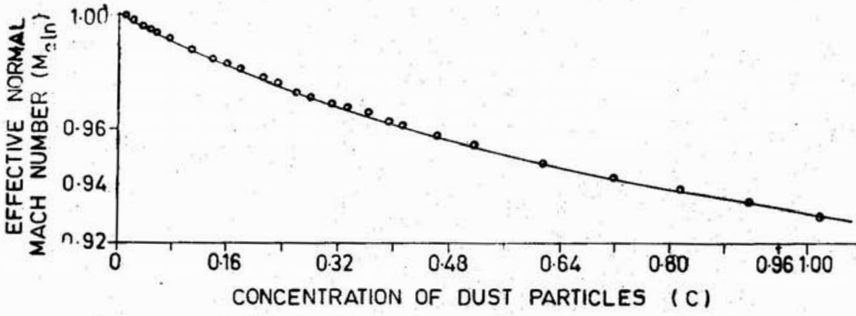

Figure 1. A graph between the effective normal mcah number M_{eln} and concentration of dust particle *(c)*.

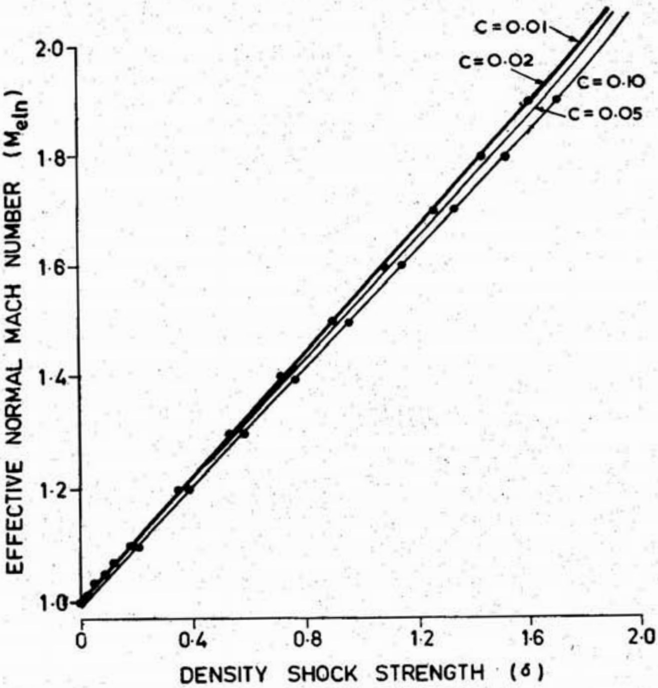

Figure 2. A graph between the effective mach number M_{eln} and density shock strength (δ) for different concentration of dust particles, $C = 0.01,\ 0.02,\ 0.05,\ 0.10$

$$\left.\begin{aligned}
\frac{T_2}{T_1} &\cong \left\{1 + \frac{4}{3}\cdot\frac{\delta}{1+\delta}\, M_{eln}^2\right\}^{1/4},\\
\frac{P_2}{P_1} &\cong (1+\delta)\left\{1 + \frac{4}{3}\cdot\frac{\delta}{1+\delta}\, M_{eln}^2\right\}^{1/4},\\
\frac{P_{2R}}{P_{1R}} &\cong \left\{1 + \frac{4}{3}\cdot\frac{\delta}{1+\delta} M_{eln}^2\right\},\\
\frac{R_{2p}}{R_{1p}} &\cong \frac{1}{(1+\delta)}\cdot\left\{1 + \frac{4}{3}\cdot\frac{\delta}{1+\delta}\cdot M_{eln}^2\right\},\\
\frac{\rho_2}{\rho_1} &= 1+\delta \text{ and } V_{2i} = V_{1i} - \frac{\delta}{1+\delta}\cdot V_{1n}\, n_i,
\end{aligned}\right\} \tag{28}$$

where δ is given by Eqns. (25).

From Eqn. (28), we draw the following inferences

(1) The temperature ratio increases with the increased value of M_{eln}. It also increases with the increased value of δ. However, for large value of δ, the slope of the curve is less as shown in Fig. 3.

(2) The radiation pressure ratio increases with the increased value of M_{eln}. It also increases with the increased valve of δ. However, for large value of δ, the slope of the curve is less as shown in Fig. 4.

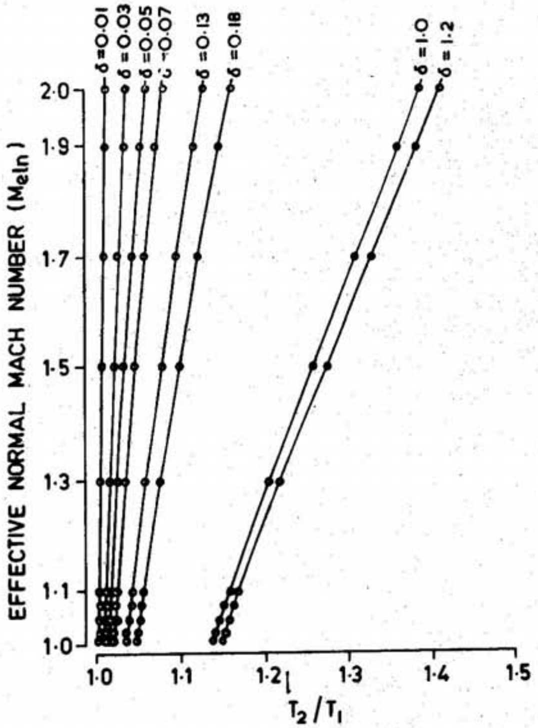

Figure 3. A graph between M_{eln} and ratio of temperature T_2/T_1 for different values of $\delta = 0.01, 0.03, 0.05, 0.07, 0.13, 0.18, 1.0$ and 1.2.

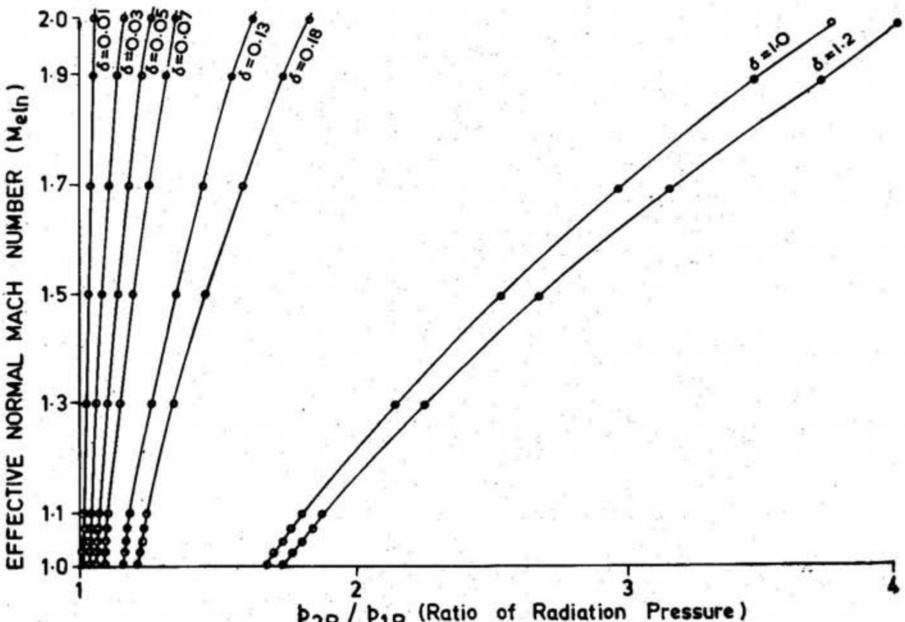

Figure 4. A graph between M_{eln} and ratio of radiation pressure P_{2R}/P_{1R} for different values of $\delta = 0.01, 0.03, 0.05, 0.07, 0.13, 0.18, 1.0$ and 1.2.

(d) Weak shock in high temperature dusty gas (When R_{1p} is not too large compared to unity but still $(T^*_2)^4 >> A^{-1} T^*_2$ holds)

If the temperature of the dusty gas is initially very high, R_{1p} is then not negligible. If in addition the shock is weak, R_{2p} will be approximately equal to R_{1p}. For this case we put $R_{1p} \cong R_{2p} \cong$ constant into Eqns. (10), (11), (12), (14) and (15) and obtain P_{2e}, γ_{2e} and A'. Putting the value of P_{2e}, γ_{2e} and A' in Eqn. (13), we get

$$\delta = \frac{2\gamma M_{1n}^2 \{ 3(\gamma-1)(1+C)R_{1p}+\varphi \} - (1+R_{1p}) \{ (8+6C)(\gamma-1)R_{1p} + 2(\varphi+\gamma-1) \}}{\gamma M_{1n}^2 \{ (\gamma-1)(1+R_{1p}) \} + (1+R_{1p}) \{ (8+6C)(\gamma-1)R_{1p} + 2(\varphi+\gamma-1) \}}, \tag{29}$$

and from Eqns. (4), (6), (7) and (27). we get the following relations

$$\left.\begin{aligned}
\frac{T_2}{T_1} &\cong \left\{ \frac{1+R_{1p}}{R_{1p}} + \frac{\delta\gamma M_{1n}^2}{(1+\delta)R_{1p}} \right\}^{1/4}, \\
\frac{P_2}{P_1} &\cong (1+\delta) \left\{ \frac{1+R_{1p}}{R_{1p}} + \frac{\delta\gamma M_{1n}^2}{(1+\delta)R_{1p}} \right\}^{1/4}, \\
\frac{P_{2R}}{P_{1R}} &\cong \left\{ \frac{1+R_{1p}}{R_{1p}} + \frac{\gamma M_{1n}^2}{(1+\delta)R_{1p}} \right\}, \\
\frac{R_{2p}}{R_{1p}} &\cong \frac{1}{(1+\delta)} \left\{ \frac{1+R_{1p}}{R_{1p}} + \frac{\delta\gamma M_{1n}^2}{(1+\delta)R_{1p}} \right\}^{3/4},
\end{aligned}\right\} \tag{30}$$

where δ is given by Eqn. (29).

These relations give the variables behind the shock in terms of known variables in front of the shock.

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