Def Sci J, Vol 33, No. 1, January 1983, pp 53–58

Folding Detonation Waves

V. P. SINGH
Terminal Ballistics Research Laboratory, Chandigarh-160020

Received 27 February 1982

Abstract. Propagation of converging detonation waves in solid explosive is discussed. Whitham's method modified for solid explosives is used. Using folding coordinates, it is found that the strength of detonation waves increases as it moves towards the centre of implosion.

1. Introduction

Study of overdriven detonation waves has gained momentum in recent years due to their applications in the production of high pressure and temperature. Conventional explosives give pressure of the order of few hundreds of kilobars but when in the same explosive the detonation wave is made to converge at a point, pressure of the order of mega bars is generated[1-5]. Singh[1], found that as detonation wave moves towards its centre or axis of convergence, its velocity and pressure increase and become infinite at the point or axis of convergence. Conger[6] has shown that as imploding detonation wave moves towards its centre or axis of symmetry, it becomes folded and its surface does not remain same in shape. A conformal transformation was suggested, which changes a circular front to a star shaped front as it reaches near its centre. Using characteristics method, he also found detonation velocity as the wave converges. It was assumed by Conger that Chapman-Jouguet condition is obeyed during the convergence and detonation front moves along the negative characteristic axis. These assumptions however seem unrealistic as it is known that converging detonations are overdriven and move along the positive characteristic axis[1,4&7].

In the present paper, the conformal transformations suggested by Conger[6] have been used and the convergence of spherical detonation waves found by using Whitham's method of characteristics[1,7]. Results are compared with those obtained earlier by the author.

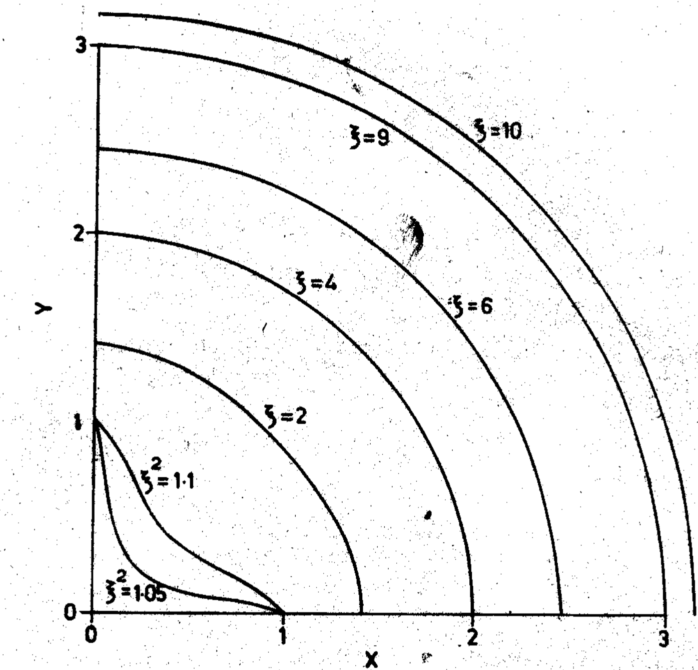

Figure 1. Shape of the detonation front as it moves towards its centre of convergence.

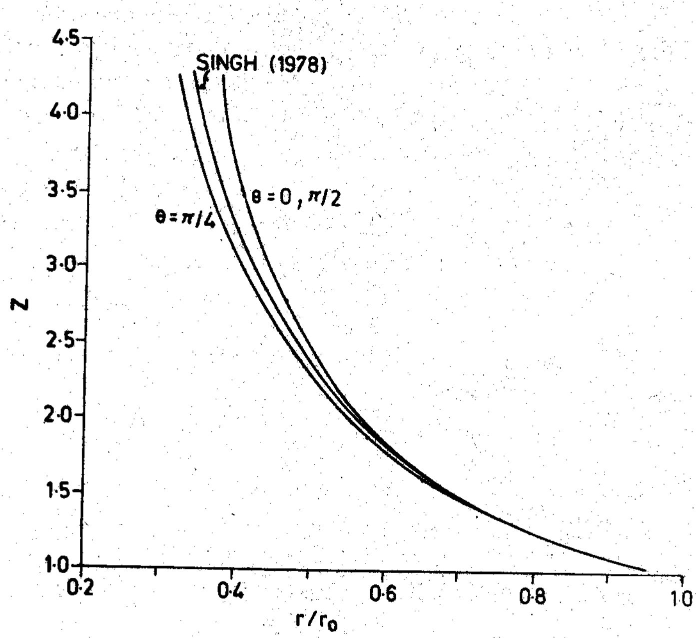

Figure 2. Variation of detonation pressure *vs.* radius.

It is found that variation of pressure does not deviate too much in this case, but it also does not become infinity at the centre In Fig. 1, the shape of the detonation front is shown and in Fig. 2, variation of dimensionless pressure z is shown versus dimensionless radius of the detonation front.

2. Formulation of the Problem

We assume that detonation wave starts from a surface of a spherical charge of radius r_0 and moves towards its centre, where the distance r of the detonation front is measured from its centre of convergence. Using the notations of the earlier paper[1], detonation velocity, particle velocity, densitv and the pressure across the detonation front are given by

$$U = Dz/(2z - 1)^{1/2}, \tag{1}$$

$$u = \frac{D}{(r+1)} (2z - 1)^{1/2}, \tag{2}$$

$$\rho = \frac{\rho_0 (r+1) z}{1 + (r-1) z}, \tag{3}$$

$$z = (r+1) p/\rho_0 D^2. \tag{4}$$

Equations (1) - (3) give the variation of detonation velocity U, particle velocity u, density ρ as a function of pressure ratio z. Here D is the C-J detonation velocity. It can be shown from (1) - (4) that

$$u + c = \frac{D}{(r+1)} \left\{ \sqrt{2z - 1} + \sqrt{r\,(1 + (r-1) z)} \right\}. \tag{5}$$

This relation shows that $u + c = D$ only when $z = 1$, i.e. detonation front is non-overdriven (Gruschka & Wecken[8]).

As detonation front approaches the centre, variation of z versus distance r is given as[1]

$$A/A_0 = g(M_2), \tag{5a}$$

$$g(M_2) = \left\{ \frac{2rM_2^2 - r + 1}{r+1} \right\}^{\frac{r+2}{2r}} \left\{ \frac{M_2 - \left(\frac{r-1}{2r}\right)^{1/2}}{M_2 + \left(\frac{r-1}{2r}\right)^{1/2}} \right.$$

$$\times \left. \frac{1 + \left(\frac{r-1}{2r}\right)^{1/2}}{1 - \left(\frac{r-1}{2r}\right)^{1/2}} \right\}^{1/2 \left(\frac{r-1}{2r}\right)^{1/2}} \times \left\{ \frac{1 + rM_2^2}{r-1} \right\}^{-\frac{1}{r}}$$

$$\times \exp\left\{ \frac{2}{\sqrt{r}} \left(\tan^{-1}\sqrt{r}M_2 - \tan^{-1}\sqrt{r} \right) \right\}, \tag{6}$$

where

$$M_2 = \left\{ \frac{1 + (r - 1) z}{r (2z - 1)} \right\}^{1/2} . \tag{7}$$

$A/A_0 = (r/r0)^j$, $j = 1, 2$ for cylindrical and spherical case respectively.

Equation (5) is an analytical relation between A/A_0 and z found by Whitham's method of characteristics. Eqns. (5) and (6) show that as $A \to 0, z \to \infty$ and $M_2 \to \left(\frac{r - 1}{2r} \right)^{1/2}$. But in actual practice, pressure does not become infinite at the centre and it has some finite value, though large.

3. Discussion of the Problem

For a spherical detonation wave, reaching near the centre, Conger[6] suggested a conformal transformation which transforms a circle to a star shape, as the radius of the circle decreases. We take into consideration only the cross section of the detonation front which is a circle. Our aim is to find such a curve, which is a circle for large radius but star shaped for small radius, i.e. a circular shock becomes folded as it approaches near the centre. We consider a conformal transformation,

$$q = a\xi\eta \, , \; s = a[(\xi^2 - 1)(1 - \eta^2)]^{1/2} ,$$

where

$$\zeta = \xi + i\eta \text{ and } \tilde{w} = q + i s \tag{8}$$

$$\xi \geqslant 1{\cdot}0 \quad , \quad | \eta | \leqslant 1{\cdot}0.$$

This transformation converts the curves ξ = constant and η = constant in ζ-plane into ellipses and hyperbolas in $\tilde{w}$-plane respectively. We consider only the curves ξ = constant. Thus a plane

$$| \eta | \leqslant 1{\cdot}0 \quad , \quad \xi_0 \geqslant \xi > 1{\cdot}0, \tag{9}$$

is converted into an area of an ellipse given by

$$\frac{q^2}{\xi^2} + \frac{s^2}{(\xi^2 - 1)} = 1 , \tag{10}$$

in the $\tilde{w}$ — plane.

Now we consider the transformation

$$\tilde{w} = \tilde{z}^n, \tag{11}$$

where

$$\tilde{z} = x + iy.$$

This conformal transformation converts an ellipse in $\tilde{w}$-plane into a curve

$$r = \left[\frac{\xi^2 (\xi^2 - 1)}{\xi^2 - \cos 2\theta} \right]^{\frac{1}{4}}, \tag{12}$$

in the $\tilde{z}$-plane. Here $r = \sqrt{x^2 + y^2}$ and θ is the angle made with the x-axis. When ξ is large, relation (12) is an equation of a circle of radius $\sqrt{\xi}$. But for small values of ξ this become a star shaped contour. The front of detonation converges as ξ decreases from its initial value ξ_0 to $\xi = 1$. The shape of detonation front for different values of ξ is shown in Fig. 1.

Now surface area of the curve (12) is given by

$$A = \frac{\xi}{2} \int_0^{2\pi} \frac{d\theta}{(1 + k^2 \sin^2 2\theta)^{1/2}}, \tag{13}$$

where

$$k^2 = \frac{1}{(\xi^2 - 1)}.$$

We express this area in the form of standard elliptic integral as

$$A = 2\xi \int_0^{2\pi} \frac{d\varphi}{\sqrt{1 + k^2 \sin^2 \varphi}}. \tag{13a}$$

This equation is integrated as

$$A = \pi\xi \left[1 - \left(\frac{k}{2}\right)^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} k^2 \right)^2 - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} k^3 \right)^2 + \left(\frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} k^4 \right)^2 - \dots \right], \tag{14}$$

where

$$k^2 \leqslant 1 \text{ and } \varphi = 2\theta.$$

We assume that the detonation front starts at point when $\xi = \xi_0$. Relation (14) gives value of A_0, i.e. the initial area of the surface. Thus knowing A/A_0 as a function of ξ, with the help of relations (4)-(6), (12) and (14), we can find the variation of detonation pressure p as a function of distance ratio r/r_0.

4. Conclusion

From Fig. 2 (variation of pressure ratio $p/p_{CJ} = z$ versus the dimensionless radius r/r_0 of the detonation front), it is found that, in the case of folding detonation wave, for the same value of r/r_0, pressure is slightly less for $\theta = \pi/4$ as compared to $\theta = 0$ and $\theta = \pi/2$. The curves have been plotted upto the value of $r/r_0 = 0.3$. These have not been extended to the centre as the elliptic integral (13) is integrable only when $\xi \geqslant \sqrt{2} = 1{\cdot}41$. Initial value of ξ is taken to be 10 in the Figs. 1 & 2. In the folding coordinate system r never approaches zero, because the shock front becomes star shaped near the centre of convergence. Results when compared with those obtained by Singh[1] for the case of spherical detonation waves (Fig. 2) show that pressure curves obtained in this paper are initially same as those obtained earlier, but deviate slightly when the detonation front reaches the centre.

We have not compared our results with those obtained by Conger[6] because we have taken different assumptions in our problem. Conger assumed that during the convergence

$$u + c = D,$$

but we have not taken this assumption into account. We have taken the detonation moving along the positive characteristic axis whereas Conger has taken the negative characteristic axis, which is not correct. His results show that front velocity does not increase during convergence, which is again against the experimental facts.

Acknowledgement

The author is thankful to Shri D. S. Murty, Assistant Director and Shri H. S. Yadav, Scientist 'C', for helpful discussions and to Shri J. P. Sirpal, Director, for providing facilities for the completion of the present work and also for giving permission to publish it.

References

1. Singh, V. P., *J. Ind. Maths. Soc.*, **42** (1978), 387–88.
2. Abarbanel, S., *Israel. J. Tech.*, **5** (1967), 238–42.
3. Lee, B.H.K., *A.I.A.A.J.*, **5** (1967), 1997–2003
4. Lee, B.H.K. & Lee, J. H., *Phys. Fluids*, **8** (1965), 2148–52.
5. Wu, J. H. T., Neemeh, R. A. & Ostrowski, P. P., *A.I.A.A.J.*, **18** (1980), 47–48.
6. Conger, R. L., *J. Math. Phys.*, **9** (1968), 1036–40.
7. Whitham, G. B., *J. Fluid Mech.*, **4** (1958), 357.
8. Gruschka, H. D. & Wecken, F., 'Gasdynamic Theory of Detonation' (Gorden & Breach Science Pubs, New Delhi), 1971.