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# Note on Free Convection Past a Vertical Porous Plate in a Rotating Fluid

S. NARASIMHA MURTHY* & H. M. THIMMARAYAPPA
Department of Mathematics, College of Basic Sciences & Humanities
UAS, Bangalore - 560065

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**Abstract**. An exact solution for the free convection flowp ast a vertical infinite porous plate in a rotating conducting fluid in the presence of a transverse applied magnetic field is attempted. The plate and the fluid are assumed to rotate in a solid body rotation. The effects of the Ekman number and the magnetic parameter, on the flow characteristics are discussed.

## 1. Introduction

The rotating fluid is receiving the attention of many workers[1,2,3,4 & 5] because of their applications in geophysical and cosmical sciences. It is very well known that in a rotating fluid near a plate Ekman layer exists wherein the viscous and coriolis forces are of the same order of magnitude. Recently the free convection flow past, a vertical infinite porous plate in a rotating fluid is discussed by Soundalgekar and Pop[6]. The expressions for axial velocity and transverse velocity for the case of air and water are derived. The local skin friction shows a decrease with increase in the Ekman number for both air and water. They[6] concluded with the results, that the rate of heat transfer is exactly equal to the Prandtl number of the fluid. The object of the present study is to examine the influence of an applied magnetic field which is transverse to the direction of the flow to the problem investigated by Soundalgekar and Pop[6]. The study concentrates on the precise effect of applied magnetic field on the flow velocity and on temperature distribution. We feel that such a model is of importance in aero-dynamic cooling.

## 2. Formulation of the Problem

We consider a cartesian coordinate system rotating uniformly with a conducting fluid in a rigid state of rotation with angular velocity $\Omega$ about $z$ axis taken positive in the

*Present Address : Department of Mathematics, Dr. Ambedkar Institute of Technology, Rajajinagar, Bangalore - 560010.

direction normal to the vertical plate which is assumed to be electrically non-conducting. The plate is further assumed to coincide with $z = 0$ plane. The applied magnetic field is in the $z$ direction which is transverse to the flow with a strength $B_0$. The fluid is incompressible viscous and conducting fluid.

Since the plate is infinite extent and the flow is steady, the physical variables are functions of $z$ only. We further assume that the magnetic Reynolds number is small as is the case, hence the induced magnetic and electric fields can be neglected.

The governing equations of the problem are

$$- w_0 \frac{du}{dz} - 2\Omega\, v = \nu \frac{d^2u}{dz^2} + g\beta\,(T - T_\infty) - \frac{\sigma B_0^2 u}{\rho}, \tag{1}$$

$$- w_0 \frac{dv}{dz} + 2\Omega\, u = \nu \frac{d^2v}{dz^2} - \frac{\sigma B_0^2 v}{\rho}, \tag{2}$$

$$- w_0 \frac{dT}{dz} = \frac{K}{\rho c_p} \frac{d^2T}{dz^2}, \tag{3}$$

with the boundary conditions

$$\left.\begin{array}{l} u = 0 \;\; v = 0 \;\; T = T_w \text{ at } z = 0 \\ u \to 0 \;\; v \to 0 \;\; T \to T_\infty \text{ as } z \to \infty \end{array}\right\} \tag{4}$$

The equation of continuity under the above approximations yields

$$\frac{\partial w}{\partial z} = 0 \tag{5}$$

which yield for constant suction

$$\text{as } w = - w_0 \tag{6}$$

In the above equations $(u, v, w)$ are the components of velocity in $(x, y, z)$ directions respectively. $\nu$ is the kinematic viscosity, $g$ is the acceleration due to gravity, $\beta$ is the coefficient of volume expansion, $T$ is the temperature of the fluid, $T_\infty$ is the ambient temperature. $\sigma$ is the electrical conductivity and $\rho$ is the density of the fluid, $w_0$ is the constant suction velocity. If the flow of the fluid is slow, viscous dissipation effects can be neglected to make the problem linear. If one neglects viscous dissipation, the ohmic dissipation which is always less than the viscous dissipation can also be neglected.

Let us introduce the following dimensionless quantities :

$$\left.\begin{array}{l} z = \dfrac{w_0 z}{\nu},\; U = \dfrac{u}{w_0} + \dfrac{iv}{w_0},\; \Theta = \dfrac{T - T_\infty}{T_w - T_\infty} \\ P = \dfrac{\rho c_p}{K},\; G = \dfrac{\nu g\beta\,(T_w - T_\infty)}{w_0^3} \\ E = \dfrac{\Omega \nu}{w_0^2},\; M = \dfrac{\sigma B_0^2 \nu}{\rho w_0^2} \end{array}\right\} \tag{7}$$

in Eqns. (1)—(3) and in boundary conditions (4) and obtain the following equations

$$\frac{d^2U}{dz^2} + \frac{dU}{dz} - 2iEU - MU = -G\,, \tag{8}$$

$$\frac{d^2\Theta}{dz^2} + P\frac{d\Theta}{dz} = 0 \tag{9}$$

The boundary conditions are reduced to

$$\left.\begin{array}{l} U = 0 \quad \Theta = 1 \text{ at } z = 0 \\ U = 0 \quad \Theta \to 0 \text{ as } z \to \infty \end{array}\right\} \tag{10}$$

It is to be noted that Eqns. (1) and (2) are combined into one equation as Eqn. (8) through the definition of complex velocity $U = u + iv$. The numbers $P$, $G$, $E$ and $M$ are the dimensionless quantities which are the Prandtl number, Grashof number the Ekman number and the magnetic parameter respectively of the problem.
The solutions of Eqns. (8) and (9) under the boundary conditions (10) are

$$U = \frac{u}{w_0} + \frac{iv}{w_0} = \frac{G}{P(P-1) - (2iE + M)}\left[e^{-m_2 z} - e^{-Pz}\right], \tag{11}$$

$$\Theta = e^{-Pz}\,, \tag{12}$$

where $m_2 = \frac{1}{2}\left(1 + \sqrt{1 + 4(2iE + M)}\right)\cdot$

we observe that in the limiting case as $M \to 0$, the solutions of Soundalgekar and Pop[6] will follow.
Separating the real and imaginary part in Eqn. (11), we obtain

$$\frac{u}{w_0} = \frac{G}{\{P(P-1) - M\}^2 + 4E^2}\Big[\{P(P-1) - M\}\, e^{-\lambda_r z}\cos\lambda_i z$$
$$+ 2E\, e^{-\lambda_r z}\sin\lambda_i z - \{P(P-1) - M\}\, e^{-PZ}\Big] \tag{13}$$

$$\frac{v}{w_0} = \frac{G}{\{P(P-1) - M\}^2 + 4E^2}\Big[2Ee^{-\lambda_r z}\cos\lambda_i z$$
$$- \{P(P-1) - M\}\, e^{-\lambda_r z}\sin\lambda_i z - 2E\, e^{-PZ}\Big]. \tag{14}$$

Here

$$\lambda_r = \frac{1}{2} + \left\{\left(\frac{1}{4} + M\right)^2 + 4E^2\right\}^{\frac{1}{4}}\cos\theta/2$$

$$\lambda_i = \left\{\left(\frac{1}{4} + M\right)^2 + 4E^2\right\}^{\frac{1}{4}}\sin\theta/2$$

Knowing the velocity field, we can now calculate the skin friction at the plate which is given by

$$\tau = \mu \left[ \frac{\partial u}{\partial z} + i \frac{\partial v}{\partial z} \right]_{z=0} \tag{15}$$

in dimensionless form

$$\tau = \left. \frac{\partial U}{\partial z} \right|_{z=0}$$

The skin friction $x$ and $y$ directions respectively are of the type

$$\frac{\tau_{xz}}{G} = \frac{\{P(P-1) - M\}(P - \lambda_r) + 2E\lambda_i}{\{P(P-1) - M\}^2 + 4E^2}, \tag{16}$$

$$\frac{\tau_{yz}}{G} = \frac{\{2E(P - \lambda_r) - \{P(P-1) - M\}\lambda_i\}}{\{P(P-1) - M\}^2 + 4E^2}. \tag{17}$$

The rate of heat transfer in dimensionless form is

$$q = - \left. \frac{\partial \Theta}{\partial z} \right|_{z=0}$$

using Eqn. (12), follows that $q = P$

This means that the rate of heat transfer is equal to the Prandtl number of the fluid. It is noticed that the magnetic field has no influence on the local rate of heat transfer. This is due to the assumption that the Joule dissipation is neglected since viscous dissipation effects are neglected.

## 3. Discussion and Results

In Figs. 1-4, we computed the axial velocity and transverse velocities for the case of air and Mercury separately, when $E = 0.2, 0.4$ and for various values of $M = 0, 5, 10$. The Prandtl number for air $P = 0.71$ and for mercury is 0.0249. In the case of air, we observe from Fig. 1, that the axial velocity decreases with increase in the rotation parameter in the absence of magnetic parameter. However even in the presence of magnetic parameter, it is observed that there is a further decrease in the axial velocity. Thus rotation and magnetic parameter acting separately or in a combined way has the same effect.

In the case of transverse velocity from Fig 2, we observe that it is negative for small values of $E$ and by suitable chosen values of $M$ and $E$, it is possible that the transverse velocity becomes positive.

In the case of mercury as the conducting fluid, it is noticed from Fig. 3 that with increase in rotation alone or magnetic parameter alone or both together we find that the axial velocity increases which is entirely an opposite behaviour in compansion with air. Again the transverse velocity from Fig. 4 in case of mercury shows a negative tendency and with increase in $M$ and $E$ it shows a positive nature.

The numerical values of skin friction $\tau_{xz}/G$ and $\tau_{yz}/G$ are given in Table 1 and Table 2 separately for air and mercury. It is observed from Table 1 that $\tau_{xz}$

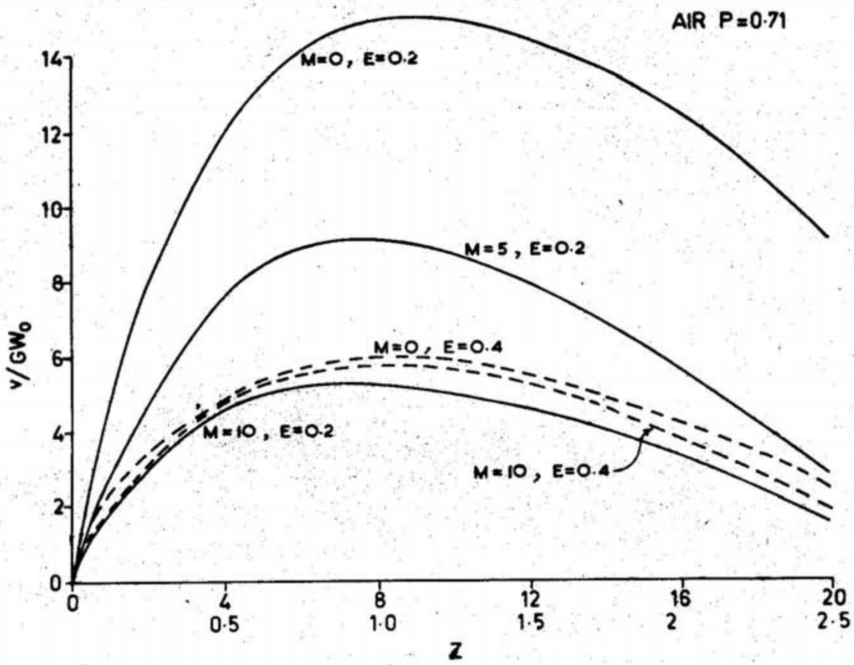


**Figure 1.** Axial velocity (air).

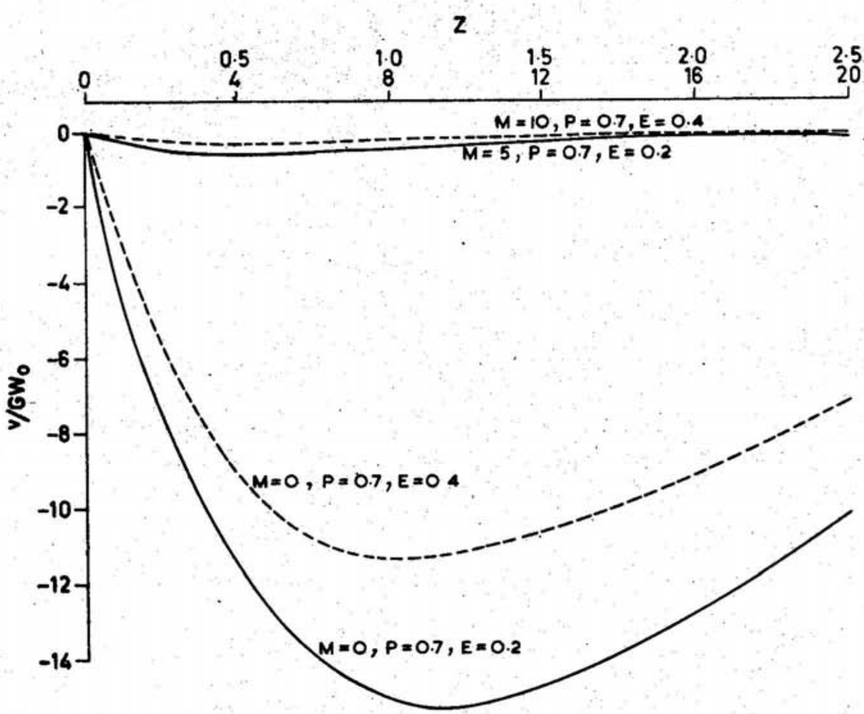


**Figure 2.** Transverse velocity (air).

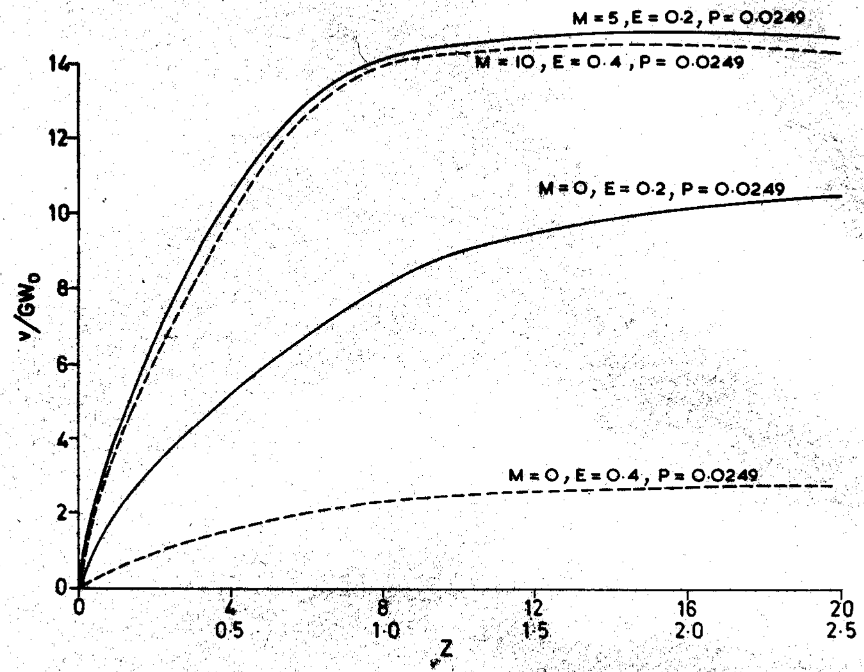


**Figure 3.** Axial velocity (mercury).

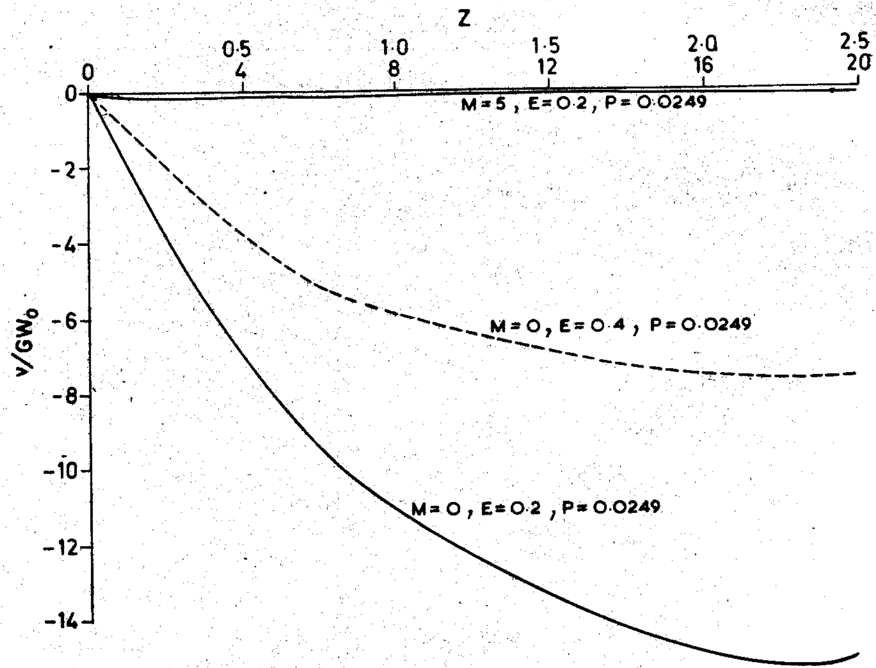


**Figure 4.** Transverse velocity (mercury)

**Table 1.** The numerical values of skin friction ($\tau_{xz}/G$) for air and mercury

| | AIR ($P = 0.7$) | | Mercury ($P = 0.0249$) | |
|---|---|---|---|---|
| $E/M$ | 0.2 | 0.4 | 0.2 | 0.4 |
| 0 | 1.06479 | 0.79919 | 0.9919507 | 0.72320 |
| 5 | 0.40064 | 0.39840 | 0.548837 | 0.543651 |
| 10 | 0.293849 | 0.29340 | 0.366501 | 0.365599 |

**Table 2.** The numerical values of skin friction ($T_{yz}/G$) for air and mercury.

| | AIR ($P = 0.7$) | | Mercury ($P = 0.0249$) | |
|---|---|---|---|---|
| $E/M$ | 0.2 | 0.4 | 0.2 | 0.4 |
| 0 | −0.442750 | −0.46223 | −2.62930 | −1.49394 |
| 5 | −0.0140177 | −0.02776 | −0.02633 | −0.0519175 |
| 10 | −0.005394 | −0.0112663 | −0.00839 | −0.016729 |

decreases with increase in the magnetic parameter and rotation parameter. However the decrease is rapid with increase in magnetic parameter than with the rotation parameter. But in the case of component of skin friction as observed from Table 2. It is seen that increase in magnetic parameter decreases the skin friction while increase in the rotation parameter increases also the skin friction. The skin friction in $y$ direction namely $\tau_{yz}$ is negative since it is in the opposite direction to that of gravitational force.

The above results are in confirmity with the results obtained by Soundalgekar and Pop[6] in the non-magnetic case.

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