RAYLEIGH'S PROBLEM IN MAGNETOHYDRODYNAMICS WITH SUCTION

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The extension of Rayleigh's problem to magnetohydrodynamics with suction is considered when the p'ate is non-magnetic and non-conducting. The suction velocity is assumed to be varying as (time)$^{-\frac{1}{2}}$. Series solutions for velocity and skin-friction are obtained under the assumption that the hydromagnetic parameter K is small. It is seen that the skin-friction increases with the magnetic field.

Considerable[1—4] work has been done in regard to Rayleigh's problem in magnetohydrodynamics for conducting and non-conducting plates but in the absence of suction velocity. Rossow[1] initiated Rayleigh's problem for non-conducting plate while Ludford[2] and Chang & Yen[3] have taken the plate to be perfectly conducting in a viscous incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field. Drake[4] has exended it for a non-perfect conductor and as a special case for an insulator.

In recent years the problem of boundary layer control has become very important in the field of aerodynamics. Application of suction is widely used in these days to prevent separation and to delay transition to turbulence which gives much larger maximum lift and reduced drag. Therefore we have tried to study the effect of time-dependent suction velocity on Rayleigh's problem in magnetohydrodynamics of an infinite flat plate, assumed to be non-magnetic and non-conducting. The suction velocity is assumed to be of the form $c(\nu/t)^{1/2}$, where c is a positive constant mean suction velocity, ν the kinematic viscosity and t the time. The suction velocity is taken to be normal to the plate and directed towards it.

The distribution of velocity is determined, in terms of known functions, by expansion in series of the hydromagnetic parameter and it is shown how the velocity and the skin-friction vary with and without the hydromagnetic interactions and with the variations in the suction velocity.

BASIC EQUATIONS AND THEIR SOLUTION

We consider a two dimensional incompressible and electrically conducting viscous fluid flow along an infinite porous flat plate, started impulsively into motion in its own plane with a constant velocity in the presence of a uniform transverse magnetic field of strength H_0. The flow is assumed to be independent of the distance parallel to the plate and suction velocity normal to the plate is directed towards it and varies as (time)$^{1/2}$. The x—axis is taken along the plate and y—axis normal to it. The unsteady hydromagnetic boundary layer equations relevant to the problem are :

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\mu_0 H_0}{\rho} \cdot \frac{\partial H_x}{\partial y} \tag{2}$$

$$\frac{\partial H_x}{\partial t} + v\,\frac{\partial H_x}{\partial y} = H_0\,\frac{\partial u}{\partial y} + \frac{\nu}{P_m}\cdot\frac{\partial^2 H_x}{\partial y^2} \qquad (3)$$

where H_x, t, ν, ρ, μ_0 and P_m ($= \sigma\,\mu_0\,\nu$) are the induced magnetic field in the direction of x—axis, the time, the kinematic viscosity, the density, the magnetic permeability and the magnetic Prandtl number respectively. σ being the electrical conductivity of the fluid. u, v are the velocity components parallel and perpendicular to the plate. From eqn. (1) it is clear that v is a function of time only. Hence we consider v in the form of $v = -\,v_0\,(t) = -\,c\,(\nu/t)^{1/2}$, where c is a real positive constant mean suction velocity.

Substituting $v = -\,c\,(\nu/t)^{1/2}$ and $W = H_x\;(\mu_0/\rho)^{1/2}$ in eqns. (2) and (3), we get

$$\left.\begin{aligned} \frac{\partial u}{\partial t} - c\,(\nu/t)^{1/2}\,\frac{\partial u}{\partial y} &= \nu\,\frac{\partial^2 u}{\partial y^2} + H_0\,(\mu_0/\rho)^{1/2}\,\frac{\partial W}{\partial y}\,, \\ \frac{\partial W}{\partial t} - c\,(\nu/t)^{1/2}\,\frac{\partial W}{\partial y} &= \frac{\nu}{P_m}\,\frac{\partial^2 W}{\partial y^2} + H_0\,(\mu_0/\rho)^{1/2}\,\frac{\partial u}{\partial y}\,, \end{aligned}\right\} \qquad (4)$$

where W stands for the Alfve'n wave velocity. Equations (4) are to be solved subject to the following conditions :

$$\left.\begin{aligned} &u = 0,\; W = 0 && && \text{for } t \leqslant 0\,, \\ &u = U_0,\; W = 0 && \text{at } y = 0 && \left.\right\} \text{for } t > 0\,. \\ &u \to 0,\; W \to 0 && \text{as } y \to \infty && \end{aligned}\right\} \qquad (5)$$

For small values of K, where $K^2 = t\,\mu_0\,H^2{}_0/\rho\,\nu$, we expand u and W in ascending powers of K as follows

$$\left.\begin{aligned} u &= U_0 \sum_{n=0}^{\infty} K^n\,u_n\,(\eta)\,, \\ W &= U_0 \sum_{n=0}^{\infty} K^{n+1}\,W_n\,(\eta)\,, \end{aligned}\right\} \qquad (6)$$

where $\eta = y/2(\nu t)^{1/2}$

Now substituting (6) in eqns. (4) and comparing harmonic terms, neglecting coefficients of K^4 and higher, we get

$$\left.\begin{aligned} &u_0'' + 2\,(\eta + c)\,u_0' = 0 \\ &u_1'' + 2\,(\eta + c)\,u_1' - 2\,u_1 = 0 \\ &u_2'' + 2\,(\eta + c)\,u_2' - 4\,u_2 = -\,2\,W_0' \\ &u_3'' + 2\,(\eta + c)\,u_3' - 6\,u_3 = -\,2\,W_1' \end{aligned}\right\} \qquad (7)$$

.... so on for other values.

and

$$\left.\begin{aligned} W_0'' + 2\,P_m\,(\eta + c)\,W_0' - 2\,P_m\,W_0 &= -2\,P_m\,v_0' \\ W_1'' + 2\,P_m\,(\eta + c)\,W_1' - 4\,P_m\,W_1 &= -2\,P_m\,u_1' \\ W_2'' + 2\,P_m\,(\eta + c)\,W_2' - 6\,P_m\,W_2 &= -2\,P_m\,u_2' \end{aligned}\right\} \quad (8)$$

....so on for other values.

where the dashes denote the differentiation with respect to η.

The boundary conditions (5) reduce to

$$\left.\begin{aligned} &u_0\,(0) = 1, \quad u_{r+1}\,(0) = 0, \qquad W_r\,(0) = 0 \\ &u_r\,(\infty) \to 0, \; W_r\,(\infty) \to 0\,, \end{aligned}\right\} (r = 0, 1, 2..) \quad (9)$$

The solutions of eqns. (7) with the help of eqns. (8), satisfying the boundary conditions (9), are

$$\left.\begin{aligned} u_0\,(\eta) &= \frac{Hh_0\,(\sqrt{2}\,\xi)}{Hh_0\,(\sqrt{2}\,c)} \\ u_1\,(\eta) &= 0 \\ u_2\,(\eta) &= \frac{2\,P_m^{3/2}}{(P_m-1)^2} \cdot \frac{Hh_1\,(\sqrt{2}\,c)}{Hh_0\,(\sqrt{2}\,c)} \cdot \frac{Hh_2\,(\sqrt{2\,P_m}\,c)}{Hh_1\,(\sqrt{2\,P_m}\,c)} \\ &\quad \times \left[\frac{Hh_2\,(\sqrt{2\,P_m}\,\xi)}{Hh_2\,(\sqrt{2\,P_m}\,c)} - \frac{Hh_2'\,\sqrt{2}\,\xi)}{Hh_2\,(\sqrt{2}\,c)} \right] \\ &\quad + \frac{P_m}{(P_m-1)} \left[\frac{Hh_0\,(\sqrt{2}\,\xi)}{Hh_0\,\sqrt{2}\,c)} - \frac{Hh_2\,(\sqrt{2}\,\xi)}{Hh_2\,(\sqrt{2}\,c)} \right], \; (P_m \neq 1) \\ u_2\,(\eta) &= \frac{Hh_{-1}\,(\sqrt{2}\,c)}{2\,Hh_1\,(\sqrt{2}\,c)} \left[\frac{Hh_2\,(\sqrt{2}\,\xi)}{Hh_2\,(\sqrt{2}\,c)} - \frac{Hh_0\,(\sqrt{2}\,\xi)}{Hh_0\,(\sqrt{2}\,c)} \right] \\ &\quad - \frac{Hh_{-2}\,(\sqrt{2}\,c)}{4\,Hh_0\,(\sqrt{2}\,c)} \left[\frac{Hh_2\,(\sqrt{2}\,\xi)}{Hh_2\,(\sqrt{2}\,c)} - \frac{Hh_{-2}\,(\sqrt{2}\,\xi)}{Hh_{-2}\,(\sqrt{2}\,c)} \right] \\ &\qquad (P_m = 1) \end{aligned}\right\} \quad (10)$$

$$u_3\,(\eta) = 0\,,$$

where $\xi = \eta + c$ and $Hh_n\,(x)$ is defined by[5],

$$Hh_n\,(x) = \int_x^\infty \frac{(u - x)^n}{\Gamma\,(n + 1)} \cdot e^{-\frac{1}{2}u^2}\,du.$$

Substituting (10) in the first expression of (6), we get the expression for the velocity.

The skin-friction at the plate is given by

$$\tau_\omega = -\frac{\rho U_0}{2} (\nu/t)^{\frac{1}{2}} \left[u'_0 (0) + K u'_1 (0) + K^2 u'_2 (0) + K^3 u'_3 (0) + \ldots \right] \quad (11)$$

In the absence of magnetic field the local skin-friction at the plate is given by

$$\tau_0 = -\frac{\rho U_0}{2} (\nu/t)^{\frac{1}{2}} . \; u'_0 (0) \quad (12)$$

Combining (11) and (12), we get

$$\frac{\tau_\omega}{\tau_0} = 1 + K^2 \frac{u'_2 (0)}{u'_0 (0)} + \ldots ,$$

since u_1 and u_3 are zero.

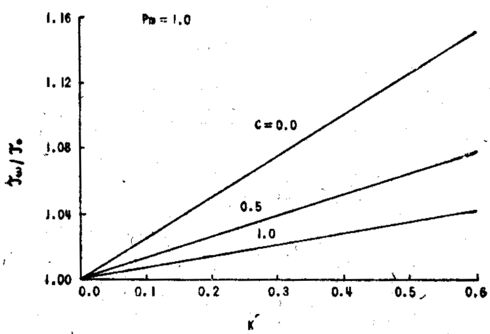

Fig. 1—Variation of skin-friction ratio against (hydromagnetic parameter) [2] for $P_m = 1$.

Fig. 1 and 2 show the variation of the skin-friction ratio (τ_ω / τ_0) with the non-dimensional magnetic parameter K^2 for different values of P_m and c. In Fig. 3 the dimensionless velocity (u/U_0) is plotted against the similarity variable η for fixed P_m and different values of c and K^2.

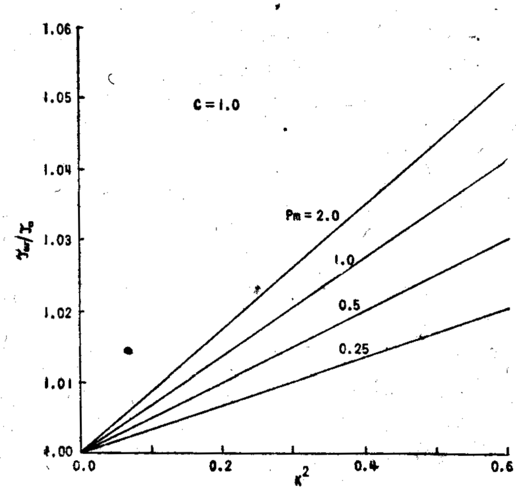

Fig. 2—Variation of skin-friction ratio against (hydromagnetic parameter)[2] for $c = 1$.

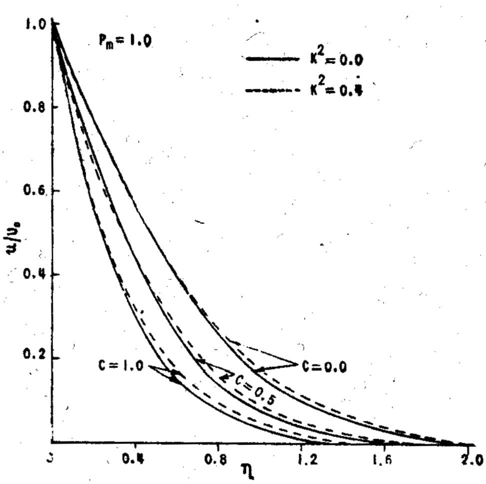

Fig. 3—Variation of dimensionless velocity against similarity variable for $P_m = 1$.

CONCLUSION

We thus conclude that

(i) The skin-friction increases with the magnetic field. Further, as the magnetic Prandtl number increases, so also does the effectiveness of the magnetic field in increasing the skin-friction for fixed c. Also for fixed magnetic Prandtl number the skin-friction increases slowly with K^2 when the suction velocity increases.

(ii) For fixed P_m, the velocity decreases as η increases. Further this decrease in velocity with η is substantially less and less as the suction velocity increases. Also in the neighbourhood of the plate, the hydrodynamic velocity is greater than the hydromagnetic velocity but after a fixed point the reverse order takes place. This point of intersection of these two velocities approaches the plate as the suction velocity increases.

REMARKS

The induced magnetic field in x-direction (H_x) can very easily be calculated with the help of (6), (7) and (8) and finally substituting the value of W in

$$H_x = W \, (\rho/\mu_0)^{\frac{1}{2}}$$

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