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Diffuse Sky Radiation in a Dry Turbid Atmosphere

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Abstract. Development of a simple method for the assessment of atmospheric turbidity all over the country in all seasons has been attempted. We have been able to derive a reasonably reliable equation relating diffuse sky radiation D_r on a horizontal surface to air mass m_r and Ångström Schüepp turbidity coefficient B, in a dry atmosphere with constant albedo ($A = 0.25$) of the terrain.

1. Introduction

The diffuse sky radiation or the so called D-radiation, which arrives at the earth's surface as a result of simple or multiple scattering, has a direct bearing on atmospheric turbidity. The scattering functions involved are extremely complicated and have been evaluated numerically from Mie's theory[1] using special assumption about the composition of the aerosols. The work of Deirmendjian & Sekera[2] has shown that it is possible to estimate the total diffuse sky radiation, D_r on a horizontal plane with the aid of the following simple formula proposed earlier by Albrecht[3].

$$D_r = k_r \, (I_{w,r} - I_r) \sin r \tag{1}$$

where k_r is an empirical coefficient which, for arid and semi-arid regions (albedo $A = 0.25$) is given with sufficient accuracy by

$$k_r = .5 \sin^{1/3} r \tag{2}$$

$I_{w,r}$ is the direct solar radiation in the absence of scattering by molecules and aerosols, and I_r is the direct solar radiation when scattering effects are included. r is the altitude of the Sun.

The coefficient k_r can be deduced either from multiple scattering theory with the albedo A of the terrain taken into account, or by direct observations for very different values of turbidity and albedo.

For a pure atmosphere without secondary scattering (and therefore independent of albedo), the magnitude of k_r should be 0.5 (since in pure Rayleigh scattering, forward scattering equals backward scattering). Scattering by aerosols leads to higher values of k_r because forward scattering by dust is much stronger than backward scattering. Theoretically the magnitude of the albedo exerts a considerable effect on k_r because of its influence on multiple scattering.

Based on elaborate computations, Schüepp[4] evolved a graphical method for evaluation of D_r for given albedo ($A = 0.25$), and assumed precipitable water in the atmosphere ($W = 2$ cm). He also provided correction tables and charts for values of W and A different from the above prescribed values. However, practical use of his method has been rather limited because of the complicated nature of the various steps involved.

In meteorological practice, atmospheric turbidity is measured according to methods proposed by Ångström[5] and Schüepp[6], in which measurements of direct solar radiation with and without filters are made use of. In India, very few meteorological observatories have these facilities. However, global and diffuse sky radiations are recorded at a large number of observatories. It was, therefore, considered worthwhile to study the problem afresh, and to explore the possibilities of estimating turbidity from measurements of D-radiation.

The present paper has been limited to the study of a dry turbid atmosphere with constant albedo ($A = 0.25$) of the terrain, which is representative of most regions of India, i.e. semi-arid and arid. Some idealisation of the real situation is considered necessary as a first step, and the complicating factors can be gradually introduced at a later stage.

2. Present Approach

2.1. *Effect of Turbidity at Unit Air Mass*

It is well known that diffuse sky radiation on a horizontal surface reaches its peak value at unit air mass (Zenith sun, $m_r = 1$). It also increases with increasing turbidity of the atmosphere, but approaches a saturation level. For $W = 0$, and constant albedo of the terrain, D_r for $m_r = 1$ is a function of turbidity B alone. Assuming that D_r (1) approaches the saturation level exponentially, we can write

$$D_r\,\underset{}{(1)} = D_r\,\underset{\infty}{(1)} - \left[D_r\,\underset{\infty}{(1)} - D_r\,\underset{0}{(1)} \right] e^{-kB} \tag{3}$$

where $D_r\,\underset{\infty}{(1)}$ is the saturation value of D_r (1) as $B \rightarrow \infty$, and $D_r\,\underset{0}{(1)}$ is the value of D_r (1), when $B = 0$.

2.2. *Effect of Air Mass*

It was shown earlier[7] that log I_r is linearly related to $m_r^{0.57}$, I_r being the intensity of direct solar radiation at normal incidence. However, the slope increases with increasing B. It follows from Eqn. (1) that D_r should also behave similarly. However, since the peak value D_r (1) is a function of B, it will be reasonable to assume that it is the ratio $D_r(m_r)/D_r(1)$ which should be considered and not $D_r(m_r)$ alone. In other words, the ratio $D_r(m_r)/D_r(1)$ plotted against $m_r^{0.57}$ on a semi-logarithmic graph, should yield a straight line for each B value, all the lines starting from a common point (1, 1) and having slopes increasing with increasing B. However, this implies that the ratio approaches zero as $m_r \to \infty$, for all values of B. From a critical study of Schüepps chart[8], it is seen that the ratio approaches a steady value of about 0.06 (6%), for all values of B. This is likely to be due to the curvature of the earths surface so that m_r approaches a finite limit[8] of 39.70 as $r \to 0°$. Hence the relationship should be expressed in the following form

$$\log\left[\frac{D_r(m_r)}{D_r(1)} - 0.06\right] = \log 0.94 - m(B)\,(m_r^{0.57} - 1) \tag{4}$$

where $-m(B)$, the slope of the line, is a function of B. The equation satisfies the requirement that $D_r(m_r) = D_r(1)$ when $m_r = 1$, for all values of B.

2.3. $m(B)$ *as a Function of* B

It will be again reasonable to assume that the slope of line in Eqn. (4) cannot increase indefinitely with increasing B but should approach a saturation level $m(\infty)$ exponentially. In other words, we should have

$$m(B) = m(\infty) - [m(\infty) - m(0)]\,e^{-k'B} \tag{5}$$

so that $m(B) = m(0)$, when $B = 0$ and is equal to $m(\infty)$ as $B \to \infty$.

2.4. *Validation of the Proposed Approach*

For validation of our proposed approach and evaluation of the various constants of the equations suggested in the foregoing values of D_r for different air masses m_r and turbidity coefficient B were extracted from Schüepps chart[8]. However, these values were obtained for a fixed water vapour content of the atmosphere ($W = 2$ cm). Schüepp has provided a correction table for $W \neq 2$ cm. Our immediate object was, therefore, to reduce these values to $W = 0$. It was shown by Majumdar et al[7]. that absorption of direct solar radiation by atmospheric water vapour varies linearly with $W^{0.3}$. By plotting the corrections from Schüepps correction table against $W^{0.3}$, straight lines were fitted to the points for each B value, so that the values could be extrapolated to $W = 0$. The values so obtained are given in Table 1.

Since the correction ΔD_r $(W = 0)$ approaches its limiting value exponentially with increasing B, the relationship can be expressed in the form

$$Y = Y_\infty - (Y_\infty - Y_0)\, e^{-\lambda x} \tag{6}$$

Table 1. Corrections in (m cal. cm^{-2}, min^{-1}) for D_r values in Schüepps chart for reduction to $W = 0$.

B	0.00	0.10	0.20	0.40
Correction	+4.0	+7.0	+8.4	+9.7

For three equispaced points on an exponential curve, it can be easily shown that

$$Y_\infty = \frac{Y_1 \times Y_3 - Y_2^2}{Y_1 + Y_3 - 2Y_2} \tag{7}$$

and

$$\lambda = \frac{1}{x_2 - x_1} \ln \frac{Y_2 - Y_1}{Y_3 - Y_2} \tag{8}$$

where the three equispaced points are (x_1, y_1), (x_2, y_2) and (x_3, y_3) such that $x_2 - x_1 = x_3 - x_2$.

In the present case, we have taken the three equispaced points on the curve corresponding to $B = 0.00$, 0.20 and 0.40 respectively from Table 1. With the help of Eqns. (7) and (8), we obtain

$$\lambda = 6.10,\ Y_\infty = 10.25,$$

leading to the final form for the correction as

$$\Delta D_r\ (W = 0) = 10.25 - 6.245\, e^{-6 \cdot 1 B} \tag{9}$$

This curve is shown in Fig. 1, alongwith the points from Table 1, which indicates a very good agreement with the proposed equation.

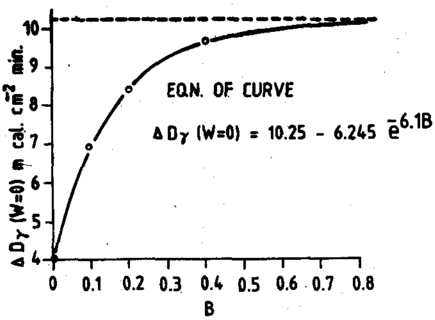


Figure 1. Exponential relationship between ΔD_r $(W = 0)$ and B.

With the help of Eqn. (9), D_r values extracted from Schüepp's chart[8] were reduced to dry atmosphere conditions ($W = 0$). The same have been presented in Table 2.

Table 2. Diffuse sky radiation, D_r in m cal. cm^{-2} min^{-1} in relation to air mass m_r and turbidity, B.

Air Mass m_r		B 0.00	0.02	0.05	0.10	0.20	0.40	0.80
1.0	Sch.	94.5	117.7	149.7	198.9	296.4	431.4	555.2
	est.	90.0	115.1	151.3	205.3	297.2	427.0	560.1
1.2	Sch.	89.5	110.2	139.7	183.9	263.4	369.7	475.2
	est.	83.4	106.2	138.8	186.7	266.3	374.9	482.1
1.5	Sch.	81.0	99.7	125.7	162.9	223.4	306.7	380.2
	est.	75.3	95.3	123.5	164.2	229.8	314.7	393.9
2.0	Sch.	68.5	84.5	105.7	134.4	178.4	234.7	281.2
	est.	64.6	81.1	104.0	135.8	184.7	243.2	293.1
3.0	Sch.	51.5	63.2	78.7	96.9	124.4	154.7	177.2
	est.	49.9	61.9	77.8	98.9	128.5	159.3	181.4
4.0	Sch.	40.5	49.7	61.7	75.4	93.4	112.7	124.2
	est.	40.2	49.3	61.2	76.0	95.5	113.4	124.6
5.0	Sch.	33.7	41.2	50.7	60.9	74.4	86.7	94.2
	est.	33.4	40.5	49.7	60.7	74.4	85.8	92.4
6.0	Sch.	29.0	34.7	42.2	50.9	60.9	67.7	74.2
	est.	28.2	34.1	41.4	49.9	60.1	68.2	73.0
7.0	Sch.	25.0	30.3	36.2	43.9	50.9	56.7	61.2
	est.	24.3	29.2	35.2	42.1	50.1	56.5	60.8
8.0	Sch.	22.0	26.9	32.2	37.9	43.9	48.2	50.7
	est.	21.3	25.4	30.5	36.2	42.8	48.4	52.8
9.0	Sch.	20.0	24.0	28.7	33.4	37.9	41.7	43.7
	est.	18.8	22.4	26.8	31.7	37.5	42.7	47.4
10.0	Sch.	18.0	22.0	25.7	29.9	34.1	37.2	38.9
	est.	16.8	20.0	23.8	28.2	33.4	38.7	43.6

Note : Sch. represents value extracted from Schüepps chart[8] and reduced to dry air conditions. est. represents values derived with the help of Eqn. (14).

2.5. *Relationship Between D_r ($m_r = 1$) and Turbidity B*

It will be seen from Table 1 that diffuse sky rediation on a horizontal surface reaches its peak value when the sun is at the Zenith ($m_r = 1$), for all values of turbidity coefficient B. The peak value increases with increasing turbidity, and appears to approach a steady value exponentially as will be apparent from Fig. 2 in which D_r ($m_r = 1$) has been plotted against B. Hence the equation of the curve should be of the form

$$D_{r1}(B) = D_{r1}(\infty) - [D_{r1}(\infty) - D_{r1}(0)]\, e^{-kB} \tag{10}$$

A smooth curve was drawn through the points in Fig. 2, with the help of french curves, and three equi-spaced points were obtained for $B = 0, 0.4$ and 0.8 respectively. The values of D_r for these points were read as 90, 426 and 560 respectively. The constants of the equation were evaluated with the help of Eqns. (7) and (9), as explained earlier. With these values, the Eqn. (10) becomes

$$D_r\,(m_r = 1) = 646.7 - 556.7\ e^{-2.324B} \tag{11}$$

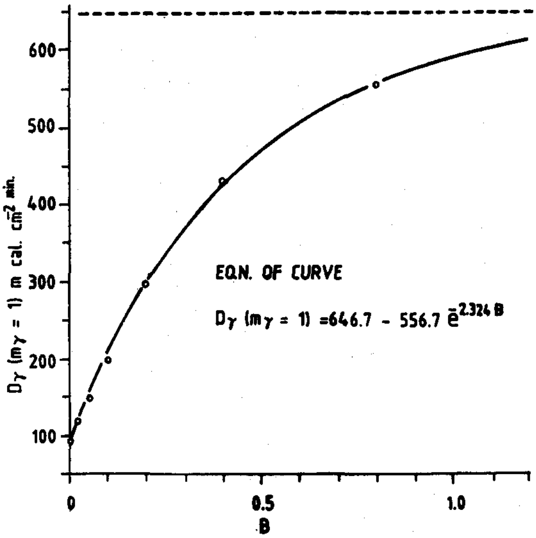


Figure 2. Exponential relationship between D_r $(m_r = 1)$ and turbidity B.

The curve represented by this equation is shown in Fig. 2 indicating thecl oseness of fit.

2.6. *D_r as a Function of B and m_r*

As shown earlier by Majumdar et al[7]., log I_r is linearly related to $m_r^{0.57}$, where I_r is the intensity of direct solar radiation at normal incidence. In view of Eqn. (1), it was felt that D_r should also have a similar relationship with m_r. But since the peak value D_r (1) is itself a function of B, it will be reasonable to plot the ratio $D_r\,(m_r)/D_r\,(1)$ against $m_r^{0.57}$ on semilogarithmic graph paper, so that the lines for different B values all have the same starting point (1, 1). Actual plotting of the values derived from the data in Table 2, however, yields curves slightly concave upwards. It is, therefore, concluded that the ratio $D_r\,(m_r)/D_r\,(1)$ does not approach Zero as $m_r \to \infty$ but tends to attain a steady value. After a few trials, this steady value worked out to be about 0.06, i.e. (6%). In Fig. 3, $D_r\,(m_r)/D_r\,(1) - 0.06$ has been plotted

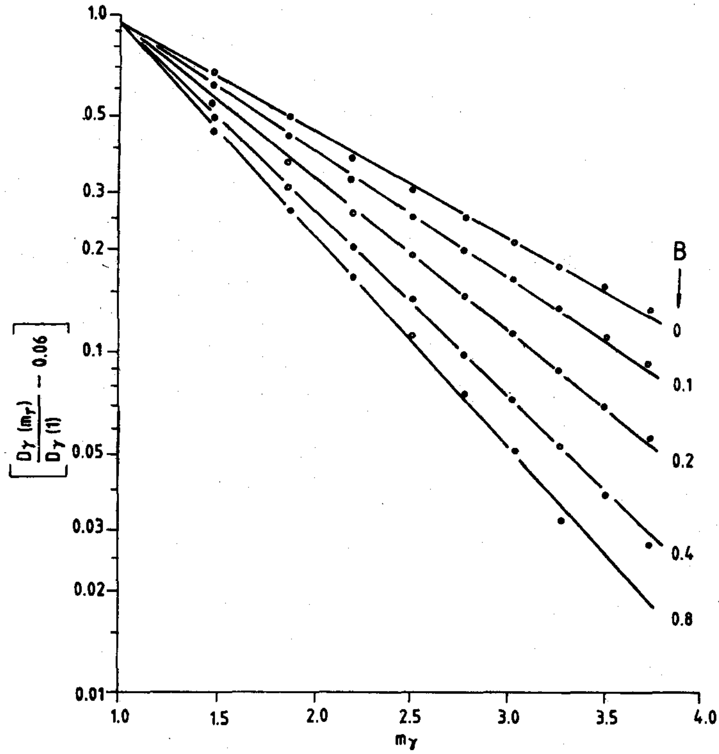


Figure 3. Relative diffuse sky radiation in relation to turbidity B and air mass (m_r).

against $m_r^{0.57}$ on semilogarithmic graph paper and straight lines fitted to the points for each B value. Equations of the lines are of the form :

$$\log\left[(D_r(m_r)/D_r(1) - 0.06)\right] = \log 0.94 - m(B)(m_r^{0.57} - 1) \tag{12}$$

values of the slope $m(B)$ obtained from Fig. 3, have been plotted against B in Fig. 4, which also indicates an exponential approach towards a steady level. By following the same method used before, and with the help of Eqns. (7) and (8) the equation connecting $m(B)$ and B is derived as foilows :

$$m(B) = (0.684 - 0.364\, e^{-2.467B}) \tag{13}$$

The curve representing the above equation is also shown in Fig. 4 indicating the closeness of fit with the plotted points.

We thus finally arrive at the equation connecting $D_r(m_r)$ with B and m_r which is

$$D_r(m_r) = D_r(1)\,[0.06 + 0.94 \times 10^{-m(B)} \times (m_r^{0.57} - 1)] \tag{14}$$

Where

$$D_r(1) = 646.7 - 556.7\, e^{-2.324B}$$

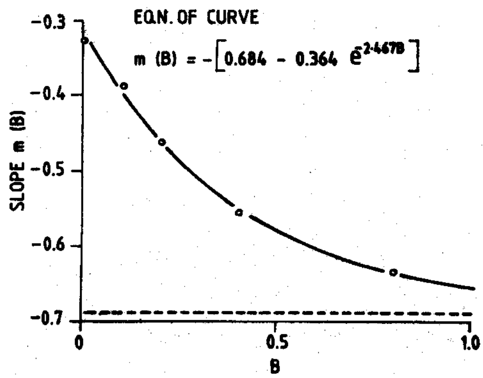


Figure 4. Exponential relationship between slope $m\,(B)$ and turbidity B.

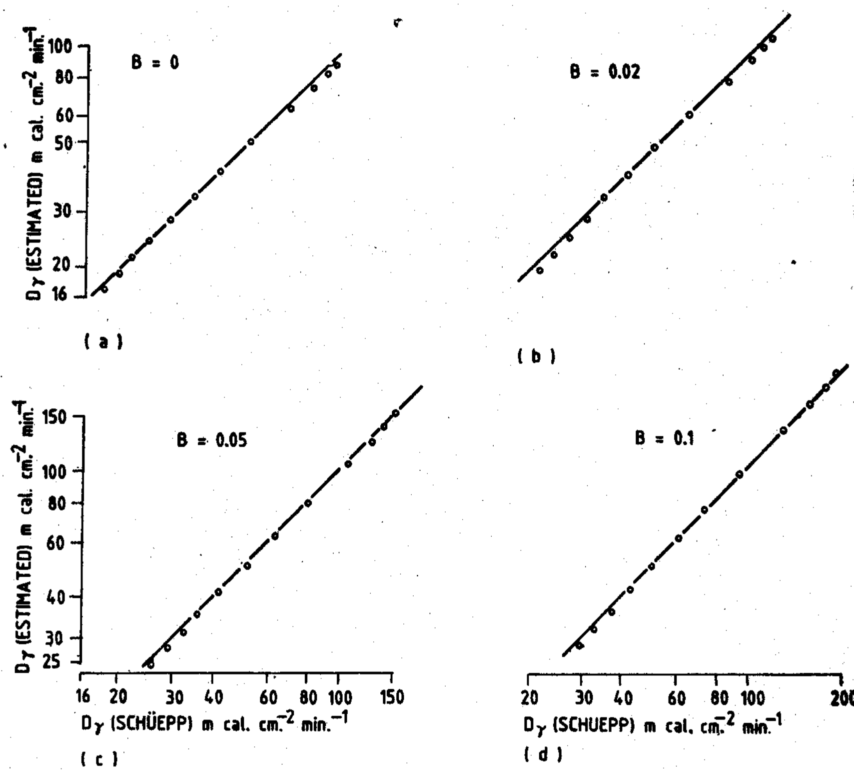
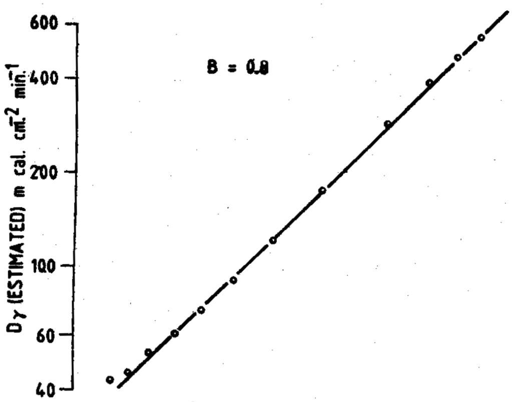
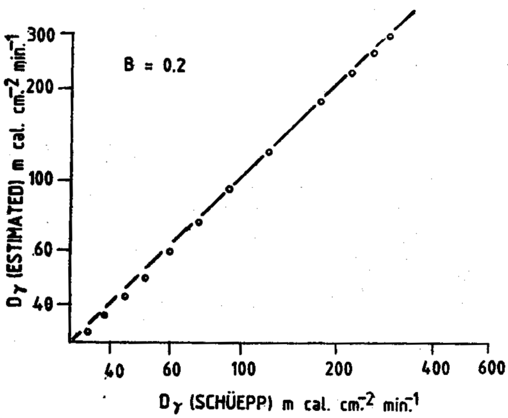


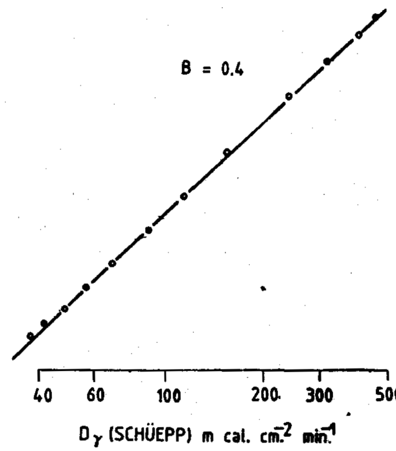
Figure 5 (a) to (g). Values of D_r estimated from Eqn. (14) compared with those obtained from Schüepps chart, reduced to dry air conditions, at different turbidity B.



(g)



(e)



(f)

and

$$m(B) = (0.684 - 0.364\, e^{-2 \cdot 467 B})$$

In order to test the validity of our present approach, D_r has been estimated with the help of Eqn. (14), and the estimated values presented in Table 2 (lower row for each m_r value). These values have also been plotted against the values derived from Schüepp's chart (Table 2 upper row) in Fig. 5, (a) to (g). The closeness of agreement between the two sets of values is evident from these figures.

3. Discussion

Development of some simple method of assessment of atmospheric turbidity all over this country in all seasons has been a long felt need. The present study may be regarded as a preliminary step in that direction. The findings appear to be quite promising, for it has been possible to derive a reasonably reliable equation, relating diffuse sky radiation, D_r on a horizontal surface, to air-mass, m_r and Ångström-Schüepp turbidity coefficient B in a dry atmosphere with constant albedo ($A = 0.25$) of the terrain. We have yet to find suitable mathematical expressions to account for the effects of variable water-vapour content of the air and albedo of the ground.

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