

Underwater Applications of Acoustical Holography

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Abstract. The paper describes the basic technique of acoustical holography. Requirements for recording the acoustical hologram are discussed with its ability for underwater imaging in view. Some practical systems for short-range and medium-range imaging are described. The advantages of acoustical holography over optical imaging, acoustical imaging and sonars are outlined.

1. Introduction

Since acoustical waves are able to penetrate through optically opaque media to significant distances, they can provide a valuable tool for underwater imaging where the medium may consist of turbid water, mud, silt, etc. Optical visibility in clear sea water extends to about 50 m but under practical circumstances this range may drop to as low as 1 m depending on the condition of the sea (Table 1). Under favourable conditions and with a cooperative type of target, ranging by laser beams may extend beyond 300 m if sufficient sensitive detectors are available¹.

To see with sound waves like bats and dolphins has long been an intriguing concept. Human ears and brain are not equipped for data processing the acoustical waves scattered from the object to produce a mental image. Therefore, one has to depend on external means to convert acoustical waves in a form suitable to get a visible image. However, of all the forms of radiation, acoustical waves have been

Table 1. Underwater optical visibility ranges

Place	Range (m)
Very clear water	50
Deep ocean water (undisturbed)	6-15
Near shore water	1-6
Disturbed water	~ 1

among the last to be exploited for producing images. It is only the last 50 years has man learnt to use acoustical waves for imaging.

With the discovery of holography in 1948 it became known that the hologram can be recorded at one wavelength (say acoustical) and reconstructed at another wavelength (say optical). However, the concept of acoustical holography was reported only in 1965 although a coherent acoustical source in the form of piezo-electric transducer has been on the scene since 1880. This is because of the difficulties in the recording and the visualisation of the hologram.

The acoustical holography is growing at a slow pace. During the last two decades some practical systems have been made for medical diagnosis and non-destructive testing applications^{2,3}. The use of acoustical holography for underwater imaging is a recent area, and because of presence of sonars and other acoustical lens imaging devices already available, the advantages of holographic imaging systems could not be immediately recognised.

The present paper underlines the basic technique of acoustical holography and surveys its underwater applications.

2. Hologram Construction

In principle, the concept of acoustical holography is similar to that of optical holography. A transducer insonifies the object. The scattered beam from the object is interfered with acoustical reference beam generated by the same transducer or by another transducer excited by the same oscillator as used for the object transducer. The interference pattern is recorded on a suitable detector. The hologram after suitable scaling is reconstructed by a laser beam. Unlike optical holography two separate sources can be used for object beam and reference beam. The reference beam can also be simulated electronically and added to the object signal.

Let O be the complex object amplitude and R the complex reference amplitude at the hologram recording plane, given by

$$O = A_1 \exp i [\omega t + \phi_1(x, y)]$$

and

$$R = A_2 \exp i [\omega t + \phi_2(x, y)] \quad (1)$$

where A is the amplitude of the wave, ω the angular frequency of the oscillator and $\phi(x, y)$ describes the phase of the wave.

The interference of O and R can be square-law detected as in optical holography. The recorded pattern is characterized by

$$\left. \begin{aligned} H &= (O+R)(O+R)^* \\ &= O^2 + R^2 + OR^* + O^*R \end{aligned} \right\} \quad (2)$$

The third term $OR^* = A_1 A_2 \exp i(\phi_1 - \phi_2)$ carries the object information which can be retrieved by illuminating the recorded hologram by the reference beam.

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The position of the reconstructed image is given by

$$R_i = \left[\frac{1}{R_c} \pm \frac{\mu}{R_o m^2} \right]^{-1}$$

where

R_i = distance of reconstructed image from the hologram

R_c = distance of reconstructing point source from the hologram

R_o = distance of object from the hologram

μ = $\frac{\text{reconstructing wavelength}}{\text{recording wavelength}}$

m = $\frac{\text{hologram aperture at } \Lambda}{\text{hologram aperture at } \lambda}$

$+$ = sign for virtual image

$-$ = sign for real image.

Since the image has been recorded at **A** and reconstructed at **A**, the condition for no distortion in the reconstructed image is that the longitudinal image magnification must be equal to the transverse image magnification. This imposes the conditions

$$\left. \begin{aligned} m &= \mu \\ R_c &= \mu R_r \end{aligned} \right\} \quad (4)$$

The angular resolution is given by

$$\alpha = \frac{1.22 \Lambda}{D} \quad (5)$$

The effect of third, fifth and seventh order aberrations have been studied by a number of workers⁵⁻⁸ in the case where $m = \mu = 1$.

For recording an acoustical hologram the requirements are as follows :

- (i) means for insonicating the object
- (ii) means for providing the reference beam
- (iii) visualisation of the image.

The object can be insonified in a number of ways. The important ones are

- (i) single large transducer
- (ii) a two-dimensional array of transducers
- (iii) a linear array scanned normally to the array by a mirror or by electronic means
- (iv) a single line element scanned as in (iii)
- (v) a small transducer scanned to synthesise the effect of a large transducer.

Usually a single large transducer is used but to insonify a large object scene an array of transducers are required.

The reference beam can be generated either acoustically or electronically. The available choices for providing acoustical reference beam are similar to those for insonicating the object. The major deviation in acoustical holography from its optical counterpart is the ability to simulate reference beam electronically. The amplitude and phase of the object signal is detected in a signal processor.

The main problem in the development of acoustical holography system is the detection and visualisation even though both square-law as well as linear detectors are available in the acoustical wavelength region unlike in optical holography where phase is detected indirectly by square-law detectors only. Table 2 lists the square-law and linear detectors for the detection of acoustical waves.

It was first reported by Greguss⁹ that the photographic emulsion becomes sensitive when it is stimulated by ultrasonic wavelength before or during the development. However, the sensitivity is very low. The deformation of a liquid surface is widely used as a real-time detector device in acoustical holography systems for clinical applications^{31,34&35}.

A piezo-electric transducer is a linear detector of acoustical waves. As in the case of means for insonicating the object, the receiver may be a single transducer

Table 2. Detectors for acoustical waves

Detectors	Sensitivity (W/cm ²)	Resolution
<i>Square-law Detectors</i>		
Photographic film in iodine solution ¹⁰	1	—
Starch plate in iodine solution ¹¹	1	> 1 mm
Photographic film ^{9,12}	0.5	> 1 mm
Thermistor ¹³	0.1	—
Thermoplastic film ¹⁴	0.1	—
Thermopile ^{15,16}	0.01	> 5 m
Liquid crystal ^{17,18}	< 0.01	> 0.3 mm
Liquid surface levitation ^{2,19,20}	10 ⁻³	< 0.03 mm
Pyroelectric camera tube ²¹	10 ⁻³	< 3 mm
<i>Linear Detectors</i>		
Electrostatic transducer array ^{2?}	2 × 10 ⁻⁴	—
Foil electret camera ²³	2 × 10 ⁻⁶	—
Piezoelectric quartz camera ²⁴	7 × 10 ⁻⁷	4 Δ
Piezoresistive camera ²⁵	10 ⁻⁷	—
Solid surface optical detection ^{26,27}	4 × 10 ⁻⁸	3 Δ lateral 15 Δ longitudinal
Membrane on liquid optical detection ²⁸	10 ⁻⁸	15 Δ lateral 5 mm longitudinal
Barium titanate camera ²⁹	10 ⁻⁹	—
Piezo-electric transducer scanned ³⁰⁻³²	10 ⁻¹¹	—
Foil electret array ³³	2 × 10 ⁻¹¹	—

mechanically scanned or an array of transducers electronically scanned. It is also possible to scan the source transducer (object **insonification**) keeping receiver stationary. This is a direct consequence of the fact that the source and detector are reciprocally related^{s5}. The source and receiver can be scanned simultaneously which will double the resolution.

3. Underwater Holography

3.1. *Advantages*

Acoustical holography offers following advantages for underwater imaging as compared to optical **imaging**, acoustical imaging and sonars :

- (a) Optical viewing is limited to short ranges under turbid water conditions. The ability of acoustical waves of penetration through larger distances makes acoustical lens imaging and acoustical holography suitable for underwater viewing at long ranges. However, the resolution capability of acoustical imaging is significantly lower than the optical imaging due to larger wavelength.
- (b) The acoustical **imaging** systems are limited by lens aberrations and provide a limited depth of field. Acoustical lenses are very heavy and bulky and difficult to move. Acoustical holography does not use any lenses, therefore, it should produce better images.
- (c) Lens imaging uses **information** of only amplitude from the transducer whereas holography utilises **both** amplitude and phase. Thus the quality of images produced by holography should be better than those produced by lens imaging.
- (d) Holographic concepts when applied to **Bragg** imaging can increase the signal-to-noise ratio.
- (e) **A** holographic sonar can give information about the range as well as it can produce images. The electronics become simpler in holographic case as compared to that in preformed beam sonars.

3.2. *Choice of Operating Frequency*

In order to obtain best possible resolution, it is necessary to use lowest possible operating wavelength or the highest possible acoustical frequency. But at higher frequency the attenuation of sound waves in sea water is very high^{s6}. At 10 **KHz** frequency the attenuation is only 0.5 **dB/km** whereas at **1000 KHz** it is more than 100 **db'km**. The operating frequency can be selected based on sea water attenuation, frequency sensitive noise, required range and resolution. The choice of the operating frequency depends on the total attenuation, therefore, for shorter object ranges higher frequency should be selected to obtain best resolution. For short-range imaging over

a few metres, acoustical frequency in the range of MHz may be used whereas for long-range imaging, frequency in the range of a few KHz is required.

3.3. Receiver System

As stated earlier, the angular resolution increases for larger hologram aperture and with lower operating wavelength. For a large hologram area, detectors such as liquid surface and Bragg cell are not suitable. For underwater applications, therefore, scanning techniques are employed. Scanning of a single detector works well under controlled conditions. But in actual underwater conditions an array of detectors (hydrophones) are used. Figure 1 shows the possible arrangement of the detectors in the array. Circular scanning techniques are faster than the linear scanning systems. The size, separation and spatial frequency response of the detector elements affect the quality of the reconstructed image. The image resolution and depth of field depend on the size of the detector whereas the spatial frequency response determines the angular field of view⁴⁰. The spacing A , of the elements of the array is related to the field of view of the system

$$\lambda_0 = \frac{A}{\sin \theta}$$

where θ is the desired angular field of view.

Let us examine how the object signal information is preserved in the scanning receiver case. When the receiver is in motion both the amplitude and phase terms of the received signal become the functions of time. If the receiver is moving with a velocity of v in x -direction, the signal at the receiver output is

$$S_1(vt, y) = A_r(vt, y) \cos [\omega t + \phi(vt, y)]$$

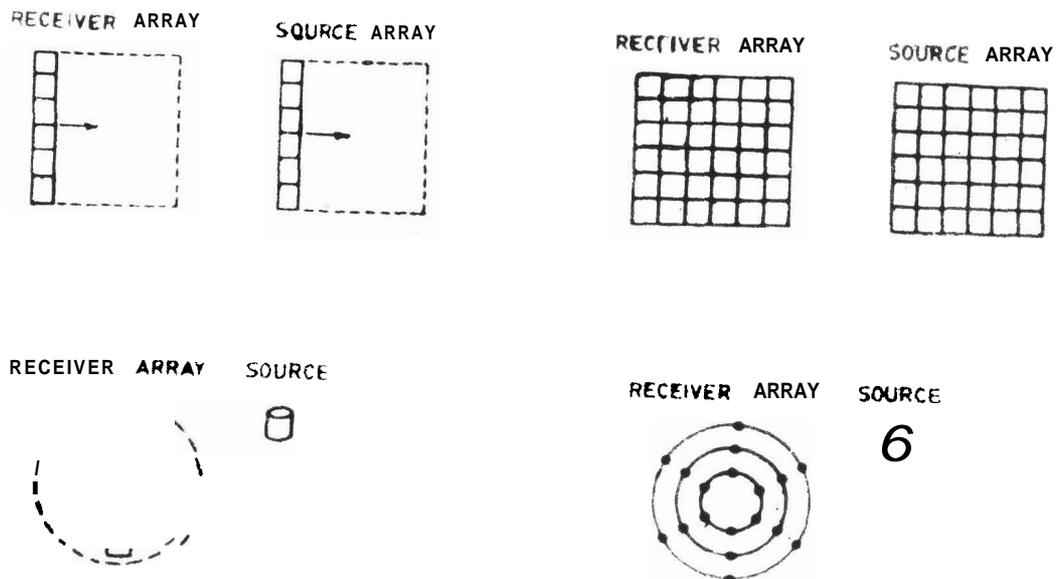


Figure 1. Source and receiver transducer array

This signal is fed to the signal processor where its amplitude and phase are detected. This can be done in following sources :

In the first method² the signal is multiplied by a part of the signal used to drive the object transducer. Thus the new signal becomes

$$S_2(t) = A_0 A_1 \cos [\omega t + \phi(vt, y)] \cos \omega t \\ = \frac{A_0 A_1}{2} \cos [2\omega t + \phi(vt, y)] + \cos \phi(vt, y)$$

Low pass filtering of this signal gives

$$S_3(t) = \frac{A_0 A_1}{2} \cos \phi(vt, y).$$

Thus we have preserved the phase of the signal. The total signal is the sum over all the values of y .

In the second method² the received signal is added to a portion of the signal used to drive the object transducer. Thus the new signal in this case becomes

$$S_2(t) = A_0 \cos \omega t + A_1 \cos [\omega t + \phi(vt, y)]$$

This signal is square-law detected. The output of the detector is

$$S_3(t) = \frac{1}{2} [A_0^2 + A_1^2 + A_0^2 \cos 2\omega t + A_1^2 \cos (2\omega t + 2\phi)] \\ + 2 A_0 A_1 \cos \phi + 2 A_0 A_1 \cos (2\omega t + \phi).$$

The low-pass filtering yields the desired signal

$$S_4(t) = \frac{1}{2} [A_0^2 + A_1^2 + 2 A_0 A_1 \cos \phi(vt, y)]$$

In this case again the signal is preserved but with additional disturbing terms.

The signal output E_0 of each receiver element depends on the sensitivity of the transducer, reflectivity of the target, range of the target and power P of the transmitter. For specularly reflecting target of area A_t and detector sensitivity of -102 dB (re $1 \text{ V}/\mu \text{ bar}$) $E_0 = -64.2 - 2 aL - 40 \log L + 10 \log (PA_t)$ where a is the attenuation in the water³². The performance of the holographic system will be dependent on the level of signal output.

3.4. Range and Depth of Field

The range of the system depends on the operating frequency and the power of the source transducer. The range ambiguity (i.e. detection of received signal from different ranges at the same time) produces overlapping images of terrain at different ranges. A low-pulse repetition rate can overcome this problem.

The depth of field depends on the recording time of the hologram. For a recording time of $150 \text{ } \mu \text{sec}$ the depth of field will be 22.5 cm at a range of 100 m . The picture depth can be increased by increasing the pulse length of the transmit

pulse. For transmit pulse larger than the recording time, the picture depth will be proportional to the transmit pulse length. The minimum target size that can be detected is a function of range. At a range of 30 m the minimum detectable target size is 1 cm on a side³². Similarly at a range of 100 m, only targets of 1 m or more on a side will be observed.

3.5. Gating

For recording the hologram, the phase change of the received signal should not be more than $\Delta/4$. This calls for stability of the target and the holographic equipment. Reducing the time of recording of the hologram will also ensure acceptable phase variations. For operating frequency of 250 KHz, the phase of the received signal from target at a range of 100 m moving with a speed of 10 knots will change by $\Delta/4$ in 150 μ sec. The receiving array is, therefore, gated for this time. This gating time can, however, be made variable for viewing stationary targets.

3.6 Visualisation of Hologram

The electrical signal from the signal processor must be converted in the form of an image. This may be done by converting the electrical signal in the form of a hologram which can be illuminated after proper scaling for reconstruction. There are basically three methods for converting the electrical signals to the required optical density variations of the film :

- (a) A scanning light source or an array of light sources can be intensity modulated by the electrical signal and photographed. The scanning light source is moved in relation to the detector's position and velocity. The commonly used light source is a focussed incandescent bulb or an array of LEDs can be used.
- (b) A CRT with a moving light spot can be used to display the information as variations in brightness.
- (c) An electron beam can be used directly to expose a photographic film.

For near real-time operation, the film is continuously moved against the CRT face. By using hot developers, the film can be developed³⁶ in 10 sec. Thus there is a delay of 10 sec between the electrical recording and optical viewing of the hologram.

A CRT type display system consisting of a thick DKDP crystal at its face has been developed for real-time display³⁷. The polarisation of the DKDP crystal depends on the electronic charge deposited on its surface. The scanning electron beam is modulated by the holographic signal which develops a positive charge pattern thereby modulating its refractive index by the electro-optic effect. A coherent beam of light from a laser when passed through or reflected from the crystal will be modulated and will reconstruct the image. The system is quite fast and a rate of more than 15 frames per second has been achieved. The resolution of the tube is inversely proportional to the thickness of the crystal.

4. Practical Systems

As discussed earlier, in acoustical holography narrow band signal is spatially sampled by the transducer array and is converted to a hologram by mixing to the signal present at each receiver with a reference signal of the same frequency. The signal at each array element is processed in parallel. The acquisition of the hologram data may be separated from the signal processing and the reconstruction part of the system. This ability provides the advantage of using existing high speed general purpose signal processors. The signal processing unit need not form an integral part of the transducer array^{ss}.

A number of research laboratories are currently working on acoustical holography systems for short-range (upto 50 m) and medium-range (upto 500 m) viewing. The choice of the receiving system is based on size, economy and practicality considerations.

4.1. Search System for Divers

The system may use a single source operating at 500 KHz and linear array of receivers scanning in a circular mode forming an aperture of about 60 cm. The transmitter receiver unit may be carried by divers or be fitted on a platform. The velocity of the revolving array of receiver must be fast enough so that platform motion during one rotation is negligible ($\lambda/4$ phase variation restriction), at the same time the rotation should not produce bubbles. A horizontal resolution of 2.5 cm and a vertical resolution of 30 cm can be achieved^z for targets at a distance of 10 m. Figure 2 shows the search system. This system can be used for location of vessels, obstacles and for searching targets of interest.

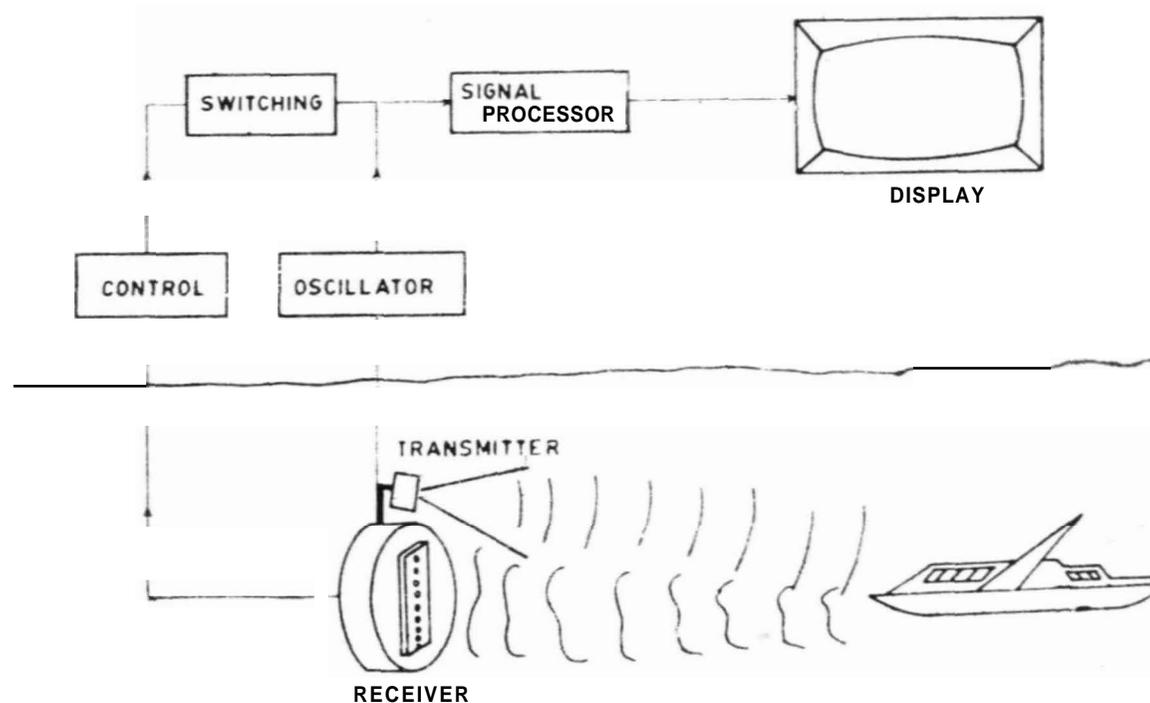


Figure 2. Holographic search system for divers.

Sutton *et al.*³⁹ have developed a system for use by U.S. Navy as a viewing aid aboard deep diving (3700 m) submersibles. The system has resolution of 0.3" and range 1-30 m to fill in the performance gap between sonar and underwater television cameras. The receiver has an array of 48 × 48 hydrophones and the transmitter output is 250 W at 642 KHz.

A 16 element transmitting array generating 200 W at 200 KHz and 32 × 32 array receiving elements form the part of a holographic system developed by OKI Industry Co. of Japan. The system is capable of providing a 40" field of view and resolution of 0.4". The processing system includes a digital computer and a FFT unit. The image is displayed on a TV monitor in 2 sec.

4.2. Harbor Surveillance System

A short-range application of the acoustical holographic system may be in harbor surveillance⁵⁰ (Fig. 3). In this system the source and the receiver remain stationary

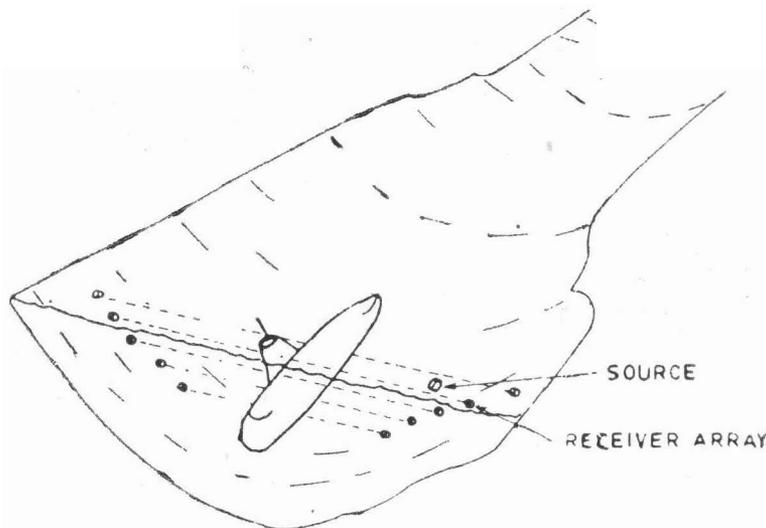


Figure 3. Holographic concept for harbor surveillance.

and the straight motion of the target provides required scanning for the hologram formation. The system can be used at the harbor or narrow channels where the submarines may try to penetrate its defences.

4.3. Medium-Range System

A underwater acoustical holographic system for range of 1.5 to 100 m has been proposed by Farrah *et al.*³² The system is designed to provide a 40° field of view and an angular resolution of 0.4". The system operates at 250 KHz and a gate time of 150 μ sec. A receiving array of 100 × 100 is aimed at. The system will be able to provide range gating similar to sonar. It is proposed to integrate each receiver transducer with a synchronous phase detector, an integrator for storing the information and a suitable gating system for interrogation. An important aspect of the system is that it takes into account the relative speed of the system and target of

10 knots. The system³² is shown in Fig. 4. The system would give both ranges as well as optical images.

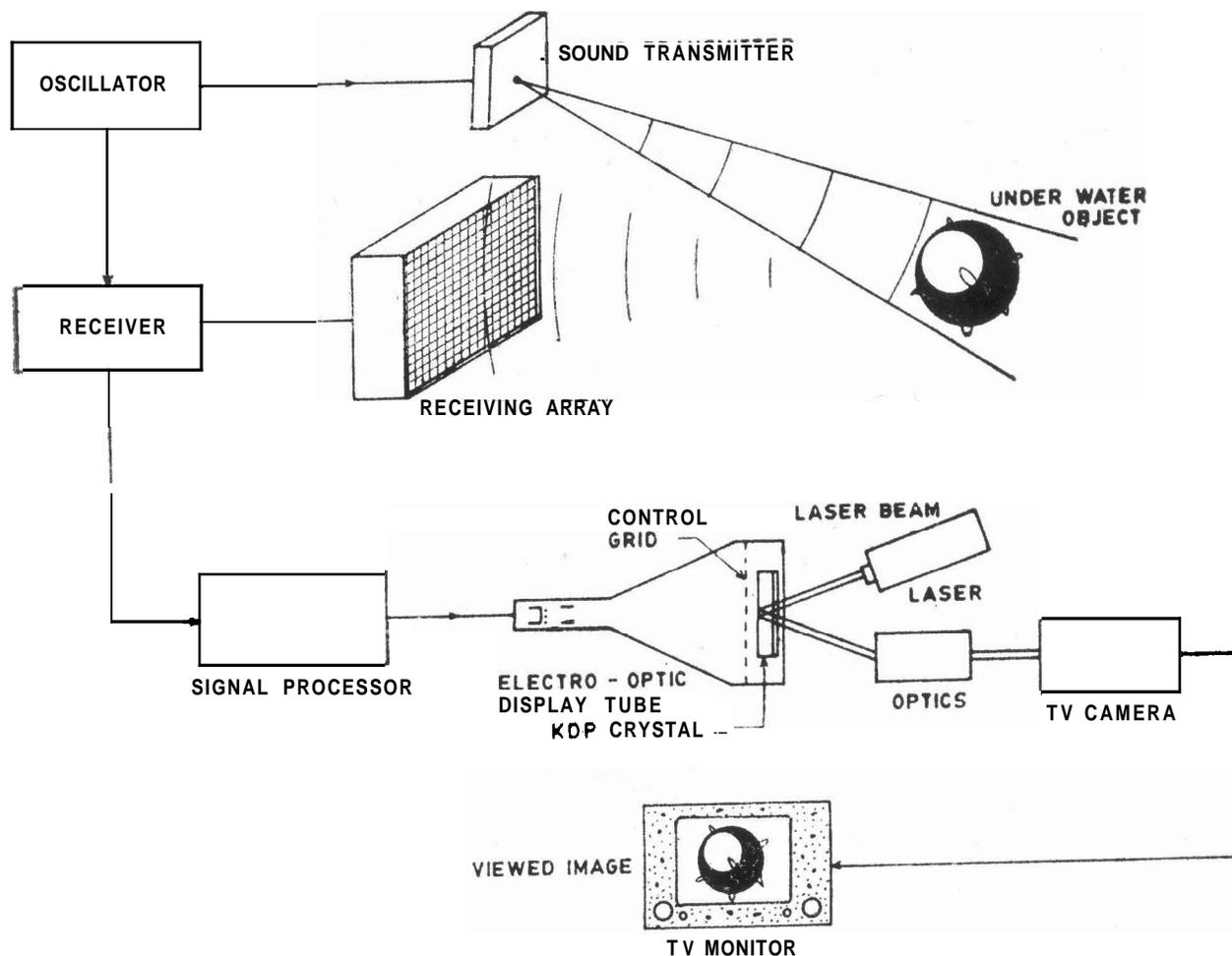


Figure 4. Underwater medium-range holographic system.

In the above system though a 100×100 receiver array is aimed at, its feasibility has been shown by using a 20×20 array and a 5×5 transmit array.

4.4. Holographic Sonar

Since its first use during World War I, sonars have developed into a highly sophisticated technology. The steered beam or preformed beam sonars are already using phase information, therefore, some may feel that holography will provide nothing new. The holographic concept achieves the performance of sonars in much simpler way.

For example, in a sonar using 100×100 elements thousands (8100 for 90° coverage) of preformed beams of 1° width are established^{d0}. The required phase shifts for 8100 beams and 100×100 elements is a major task for digital techniques. In the holographic technique this task is easier to perform. The received signal at each transducer is mixed with an electronically simulated plane reference wave. The received signal and the simulated reference wave produce holographic interference pattern.

Another advantage of holographic concept is its inherent redundant nature. The hologram recorded by each array transducer is combined coherently with other points during reconstruction to produce the image. Higher signal-to-noise ratio is expected in holographic imaging.

A holographic sonar system can initially use a very broad range gate to see all the ranges. The operator can initially select the area of interest and then can reduce the size of the range gate to see only the target of interest. The seeing capability of hologram can minimise the detection of false targets unlike conventional sonars.

The coherence requirement for preformed or steered beam sonars are the same as for recording holograms. However, holographic concept would provide imagery in a simpler way.

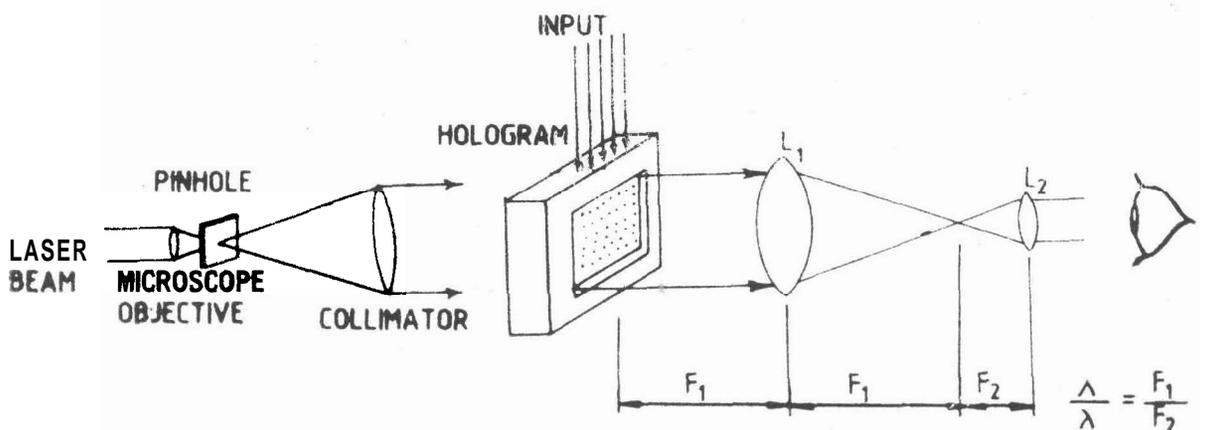


Figure 5. Holographic sonar system.

Figure 5 shows a possible reconstruction scheme of a holographic sonar proposed by Penn & Choran³⁶. The hologram is formed by combining the received signals from sonar hydrophone array with the electronic reference beam and is square-law detected. The electric signal is transferred to an optical modulator (reconstruction tube). In order to obtain distortion free images the hologram should be scaled down in such a way that the element spacing $\Delta/2$ in the sonar array should correspond to $\lambda/2$ for optical viewing. To do this a combination of two lenses is used as a correction optics. The main advantage of this arrangement is that the resolution requirement on the lenses is not very critical as these are required only to resolve the image and not the fringe structure.

5. Conclusion

Almost all the basic components required for acoustical holography are available. Real-time reconstructor tube has also been developed and is under further refinement. The development of suitable hydrophone array with associated electronics is also under development at various laboratories^{38,41}.

Till now the efforts have mainly been confined to acoustical holographic systems for medical diagnosis and non-destructive inspection work. It is only recently that holography for underwater application has received attention. The holographic sonar does provide the possibility of unambiguous range detection. The difficulties in making the system seaworthy are the same as encountered in other acoustical imaging systems.

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