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Digital Filters Using Identical Blocks

S. C. Dutta Roy, E. Gopinathan & R. K. Patney

Indian Institute of Technology, New Delhi-110016

Abstract. Improved response of non-recursive digital filters is achieved using Amplitude Change Functions (ACFs) on a prototype filter. A generalized ACF with interesting properties is suggested. Methods for achieving variable cut-off frequency and frequency transformation are explained. A modular hardware implementation is also presented.

1. Introduction

Several methods viz. windowing, frequency sampling and optimization, for the design of Finite Impulse Response (FIR) digital filters are available in the literature[1-3]. In these methods, in general, as the order of the filter increases, the computational complexity also increases. Moreover, if the specifications are to be changed, the filter is to be completely redesigned. An alternative approach to design a higher order filter could be to design a filter of lower order by using single-precision FIR filter design programmes, and then, to improve the response by repeated use of the same filter. The hardware implementation of the overall filter also becomes simpler than that of a single filter designed by conventional methods. This is particularly true when the prototype filter can be fabricated on an integrated circuit chip. The simplest example of the multiple use of the same filter is the cascading of two identical filter sections; the out-of-band rejection (stopband loss) is increased thereby, but the passband error is doubled by this interconnection. The 'twicing' method suggested by Tukey[4] squares the ripple in the passband, but doubles the stopband ripple. A procedure, called 'filter sharpening', for improving the response of a symmetrical nonrecursive filter was introduced by Kaiser and Hamming[5]. The method is based on an 'Amplitude Change Function' (ACF) of the form $H_0 = f(H)$, where f is a polynomial relationship between the amplitudes H and H_0 of the prototype and transformed filters respectively. The ACF curve passes through the points (0, 0) and (1, 1) in the (H_0, H) plane. The improvement in the passband (near $H = 1$) or in the stopband (near $H = 0$) response depends on the order of tangency of the ACF at $H = 1$ or at $H = 0$ respectively. An expression[5] for an ACF giving nth and mth order tangencies at $H = 0$ and 1, respectively, is given by

$$H_{00} = H^{n+1} \sum_{k=0}^{m} C(n + k, k) (1 - H)^k \tag{1}$$

where $C(n + k, k) = (n + k)/(n.k.)$ is the binomial coefficient and an additional subscript has been used for distinguishing between various transformed filters to be used.

In this paper, some new applications of the Amplitude Change Function are proposed. In section 2, a method for improving the response of the transformed filter by using shifting and scaling of the ACF is given. An expression for an ACF which has a similar characteristic as that of the Kaiser-Hamming Chebyshev ACF is derived in Section 3. This new ACF does not require additional correction filters and is a generalized expression for any order of tangencies at $H = 0$ and 1. In section 4, a method is suggested for continuously varying the cut-off frequency of the transformed filter. An efficient method for hardware implementation for different order of tangencies at $H = 0$ and 1, keeping the order of transformation constant, is also suggested in this Section. A method for frequency transformation of FIR digital filters using the concept of ACF is discussed in Section 5, where methods for obtaining variable centre frequency and variable cut-off frequencies in the case of lowpass to bandpass transformed filters are also suggested.

2. Selective Improvement in the Passband, Stopband or Both

In has been shown[6] that selective improvement in the passband, stopband or both can be achieved by shifting and/or scaling the ACF. By shifting the point (0, 0) to $(\alpha, 0)$ and scaling the ACF by a factor β, where $0 < \alpha, \beta < 1$, Eqn. (1) changes to

$$H_{01} = \left(\frac{H - \alpha}{\beta - \alpha}\right)^{n+1} \sum_{k=0}^{m} C(n + k, k) \left[1 - \frac{H - \alpha}{\beta - \alpha}\right]^k \tag{2}$$

Equation (2) is similar to Eqn. (1) with H replaced by $(H - \alpha)/(\beta - \alpha)$ and its derivatives vanish at the points $(\alpha, 0)$ and $(\beta, 1)$. For the simple case of third order transformation with first order tangencies at $H = \alpha$ and $\beta = (1 - \alpha)$ i.e. equal deviation from the points (0, 0) and (1, 1), Eqn. (2) reduces to

$$H_{01} = [(H - \alpha)^2/(1 - 2\alpha)^3] [(3 - 4\alpha) - 2H] \tag{3}$$

The ACF of Eqn. (3) is shown in Curve A in Fig. 1(a), for $\alpha = 0.05$. It may be noted that shifting and/or scaling reduces the effective transition width of the transformed filter; however the error in the passband/stopband is increased depending on the extent of deviations from the points (0, 0) and (1, 1). The limit on the deviation α is set by the desire to keep inband errors in H_{01} equal to those in H in the worst case. To explain this, it may be first pointed out that shifting and scaling effectively shrinks (expands) the H axis around the point $H = 0.5$ for $\alpha > 0$ (< 0). Now refer to Fig. 1(b) drawn for H_{00} and H_{01} with $\alpha > 0$ around $H = 0$. At the

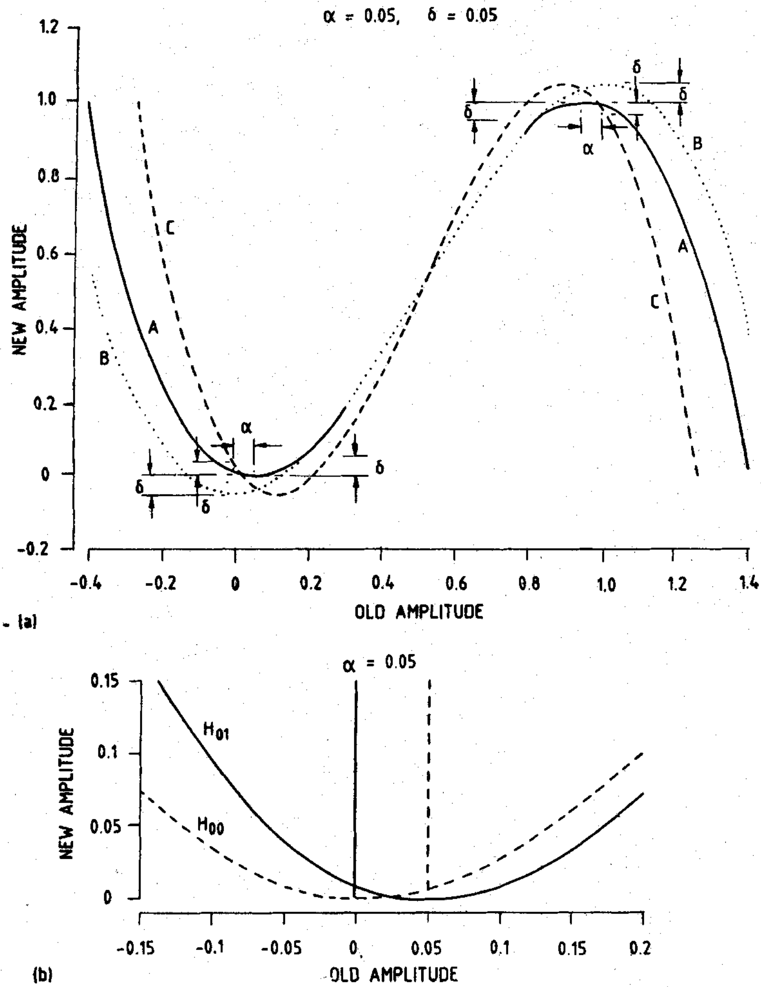

Figure 1. (a) Comparison of third order amplitude change functions.
Curve A : H_{01} for $\alpha = 0.05$
Curve B : H_{03} for $\delta = 0.05$
Curve C : H_{04} for $\delta = 0.05$

1. (b) Comparison of third order ACFs H_{00} and H_{01} around $H = 0$ (scales expanded).

two points intersected by the same horizontal line, the slope of H_{01} is larger (smaller for $\alpha < 0$) than that of H_{00}. Also, the error in H_{01} is larger than that in H_{00} around $H = 0$. For the case when H is restricted between 0 and 1, the maximum deviation occurs when α is equal to the intercept on the H_{01} axis. For $\alpha < 0$, in Fig. 1(b), the roles of H_{00} and H_{01} are switched and the dotted line becomes the vertical axis.

Both shifting and scaling need not be employed if the improvement is required either in stopband or passband alone. The ACF's for shifting (or scaling) for improvement in stopband (or passband) may be obtained from Eqn. (2) by putting $\beta = 1$ (or $\alpha = 0$).

2.1 *Example*

In this example, it has been shown that the stopband performance of the 'sharpened' filter can be improved without affecting the passband characteristic significantly, by shifting the ACF. It is also shown that this method can be applied to a rather crude filter which is otherwise not of much use. Consider the simple smoothing by 3's filter with the weighting sequence given by $h = (1/3)\{1, 1, 1\}$. Since the stopband performance of the prototype filter is very poor, the values of n and m have been chosen as 3 and 1 respectively. For increasing the out of band rejection further, shifting with $\alpha = -0.1$ has also been applied. The fifth order ACF in this case can be written as

$$H_{01} = [(H + \alpha)^4/(1 + \alpha)^5]\,[(5 + \alpha) - 4H] \tag{4}$$

When this transformation is applied to the above prototype filter, the frequency response obtained is shown in Fig. 2. Also shown are the responses of the prototype and

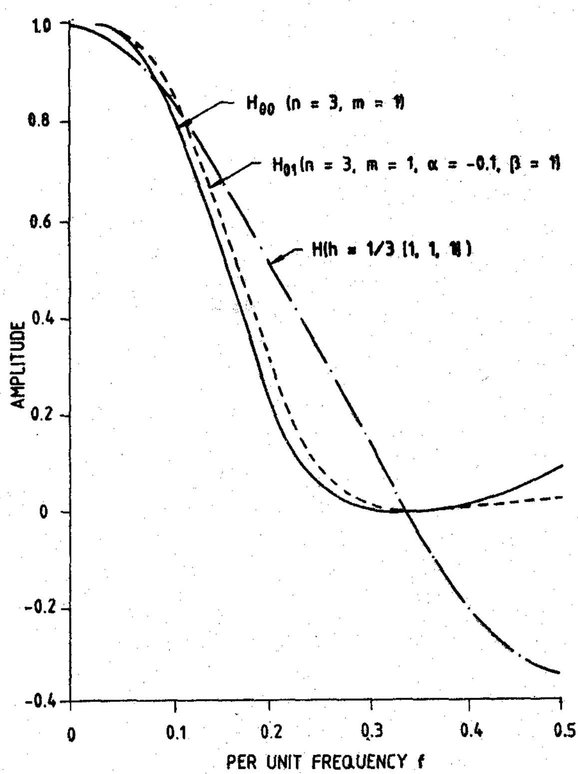

Figure 2. Comparison of frequency response of a crude filter, H, with its transformed versions.

that of H_{00} for $n = 3$ and $m = 1$. From this figure, it may be noted that H_{00} and H_{02} attain a stopband attenuation equal to 0.0781897 ($\approx -$ 22.137 dB) and 0.011844 ($\approx -$ 38.53 dB) respectively. This improvement, however, is at the cost of increased transition width of the transformed filter H_{01} as in evident from Fig. 2.

3. New ACF for Equiripple Behaviour near H = 0 and H = 1

Kaiser and Hamming[5] have obtained another ACF in which the maximum deviation in H_0 is within $\pm\,\delta$ for a given deviation $\pm\gamma$ in H. This Chebyshev type ACF (Fig. 4 in ref. 5) may be written as

$$H_{02} = H_{00} + E(H) \tag{5}$$

where $E(H)$ is a correction term. For the cubic case, for example, $H_{00} = H^2 (3 - 2H)$ and $E(H) = a_0(H - \frac{1}{2})\,[(H - \frac{1}{2})^2 - a_1]$, a_0 and a_1 being constants.

An alternate expression for obtaining similar characteristics[6] is

$$H_{03} = (1 + 2\delta)H_{00} - \delta \tag{6}$$

A relation between γ and δ can be obtained from Eqn. (6) by substituting $H = \gamma$ and $(1 - \gamma)$, and is given by

$$\delta = \frac{\gamma^{n+1} \sum\limits_{k=0}^{m} C(n+k, k)\,(1-\gamma)^k}{2(1-\gamma)^{n+1} \sum\limits_{k=0}^{m} C(n+k, k)\gamma^k} \tag{7}$$

For the third order case with $n = m = 1$,

$$H_{03} = (1 + 2\delta)\, H^2(3 - 2H) - \delta \tag{8}$$

where $$\delta = \gamma^2(3 - 2\gamma) \,/\, [2(1 - \gamma)^2\,(1 + 2\gamma)] \tag{9}$$

This is plotted as curve B in Fig. 1(a) for $\delta = 0.05$. For $\gamma < 0.05$, $\delta \simeq 3\gamma^2/2$ which is the same[5] as that obtained in ref. 5 for $\gamma < 0.1$.

Some points for comparison between Eqns. (5) and (6) are in order. The correction term in Eqn. (5) is a polynomial of the order of H_{00}, and is used to flatten the ACF around $H = 0$ and 1. As a result, the derivatives of H_{02} do not vanish at $H = 0$ and 1. H_{03} is also much simpler than H_{02} and it does not require any additional filter blocks for its implementation, as in the case for H_{02}. Further, generalization of H_{02} may be difficult while H_{03} can be used for any order of transformation. Lastly, H_{02} has equal maximum deviation on either side of $H = 0$ and 1, while H_{03} does not have this property.

Shifting and scaling may be applied to Eqn. (6) to further improve the performance of the transformed filter, resulting in the ACF

$$H_{04} = (1 + 2\delta)\, H_{01} - \delta \tag{10}$$

The number of extrema of H_{04} is equal to three in both pass and stopbands. However, the amplitude of one of the extrema is less than or equal to that of the other two, the equality being valid for the case when α takes its maximum permissible value (refer to Section 2) For the maximum value of α, the error in H_{04} is also minimum. This can be explained with the help of an example where the prototype filter amplitude lies between 0 and 1. Curve C in Fig. 1(a), which shows H_{04} for the third order transformation with $\delta = 0.05$, can be made to pass through the origin or to intersect the H_{04} axis at a point $\delta' > 0$. Let δ_1 be the maximum in band error for the $\delta' = 0$ case, and let δ_2 be the corresponding quantity for the case $\delta' > 0$. Then, assuming the same value of α in the two cases, we get

$$\delta = \alpha^2(3 - 4\alpha)/[1 - 6\alpha(1 - \alpha)] \tag{11}$$

and for $\delta = \delta_2$

$$\delta_2 = (\alpha^2/2)\,(3 - 4\alpha)/[(1 - \alpha)^2\,(1 - 4\alpha)] \tag{12}$$

It is seen that $\delta_1 \approx 2\delta_2$ for small values of α. The larger error in the first case is to be expected since, as can be derived from Curve C in Fig. 1(a), the frequency responses in the two cases will have 2 and 3 extrema, respectively.

3.1 *Example*

Consider Blackman and Tukey's simple smoothing by 3's and 5's filter giving the weightng sequence

$$h = (1/15)\,\{1, 2, 3, 3, 3, 2, 1\}$$

Since the passband response of the prototype filter is fairly good, we apply transformation H_{04} with $\alpha = 0$ to further improve the passband response. The stopband response will be almost the same as that in the H_{00}. The ACF for the cubic case, scaled by β, can be written as

$$H_{04} = [H^2/(3\beta - 2)]\,(3\beta - 2H) \tag{13}$$

The above transformation with $\beta = 0.95$ is applied to the smoothing by 3's and 5's filter. The transformed filter has two extrema in the passband, as shown in Fig. 3, which compares the frequency responses of the original filter H and the sharpened filters H_{00} and H_{04}. The improvement in the passband response is obvious.

4. Variable Cutoff Filtering

The cutoff frequency of a linear phase FIR filter can be varied using the cosine transformation[7,8]. However, in this method, the coefficients of the transformed filter are to be recalculated for varying the cut-off frequency, since, the coefficients of the filter determine the cut-off frequency of the filter.

The variations in cut-off frequency can also be achieved by constructing an ACF which is restricted to pass through a predetermined point (μ, ϵ) in the (H_0, H) plane. For $0 < \epsilon, \mu < 1$, the new ACF can be written[9] as

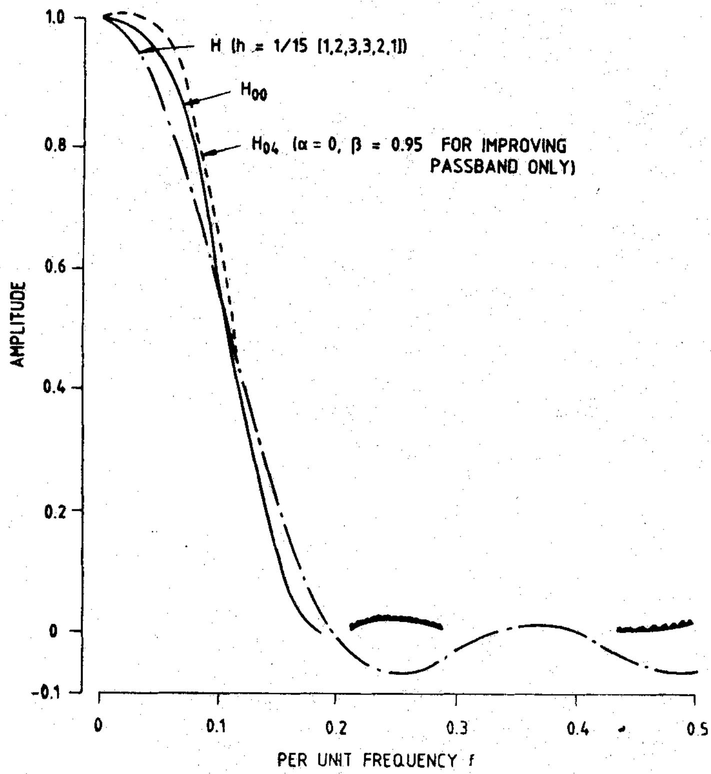

Figure 3. Comparison of the response of a prototype filter H with its sharpened version H_{00} and H_{04} having two extrema in the passband.

$$H_{05} = H_{00} + K_{n,m} H^{n+1} (1 - H)^{m+1} \tag{14}$$

where

$$K_{n,m} = \epsilon^{-(n+1)} (1 - \epsilon)^{-(m+1)} \left\{\mu - \epsilon^{n+1} \sum_{l=0}^{m} C(n + l, l) (1 - \epsilon)^{l}\right\} \tag{15}$$

The second term in Eqn. (14), which represents a modification of the ACF used in filter sharpening[5], increases the order of the transformed filter by one. For the case when ϵ is kept constant, μ can be changed to vary the cut-off frequency within certain limits, beyond which H_{05} will not remain between 0 and 1. The order of tangencies of H_{05} is the same as that of H_{00} except for the limiting values of μ for which the order increases by one at $H = 1$ ($\mu = \mu_{\max}$) or at $H = 0$ ($\mu = \mu_{\min}$). Based on this, it can be shown that

$$\mu_{\max} = \frac{\epsilon^{n+1}(1 - \epsilon)^{m+1} (n + m + 1)!}{n! \, (m + 1)!} + \epsilon^{n+1} \sum_{l=0}^{m} C(n + l, l) (1 - \epsilon)^{l} \tag{16}$$

and

$$\mu_{\min} = \frac{-\epsilon^{n+1}(1-\epsilon)^{m+1}(n+m+1)!}{(n+1)!\,m!}$$

$$+\ \epsilon^{n+1} \sum_{l=0}^{m} C(n+l,\, l)\,(1-\epsilon)^l \tag{17}$$

From Eqns. (16) and (17), it is easily seen that the extent of variation in μ, i.e., $\mu_{\max} - \mu_{\min}$, is maximum and that μ varies symmetrically on either side of μ_0, the value of μ for $K_{n,m} = 0$ and $n = m$. Also, as $|n-m|$ becomes large, $\mu_{\max} - \mu_{\min}$ decreases and the absolute value of the range shifts away from μ_0. Thus for a given order of transformation $(n + m + 1)$, the largest variation in cut-off frequency is obtained by varying μ as well as n and m such that $(n + m)$ remains a constant.

4.1 *Example*

With the help of an example, it can be shown how the largest range of variation in cut-off frequency can be achieved. Consider the simple, raised-cosine bandpass filter with the following weighting sequences,

$$h = \tfrac{1}{4}\{-1, 0, 2, 0, -1\}$$

To illustrate fairly large variation in cut-off frequency, we choose $n + m = 3$. The ACF for different values of n and m are given by

$$H_{05(0,3)} = H\,[4 - 6H + 4H^2 - H^3 + K_{0,3}(1-H)^4] \tag{18a}$$

$$H_{05(1,2)} = H^2\,[6 - 8H + 3H^2 + K_{1,2}(1-H)^3] \tag{18b}$$

$$H_{05(2,1)} = H^3\,[4 - 3H + K_{2,3}(1-H)^2] \tag{18c}$$

$$H_{05(3,0)} = H^4\,[1 + K_{3,0}(1-H)] \tag{18d}$$

How Eqn. (18) can be implemented in hardware as well as software will now be discussed.

4.2 *Hardware Implementation*

Hardware implementation of $H_{05(n,m)}$ necessitates different interconnection for different values of n and m. This problem, however, can be avoided by expressing $H_{05(n,m)}$ in terms of powers of $(1 - H)$ giving

$$H_{05(n,m)} = H\,[1 + G_1(1-H) + G_2(1-H)^2 + G_3(1-H)^3$$
$$+\ G_4(1-H)^4] \tag{19}$$

where G_1, G_2, G_3 and G_4 are functions of μ, n and m while $n + m$ is kept equal to 3.

Table 1 gives the range of values of the gain factors G for different combinations of n and m. Curves A and B in Fig. 4 show the ACF for the maximum and minimum values of μ corresponding to $n = 0$, $m = 3$, and $n = 3$ and $m = 0$ respectively while

Table 1. The range of values of the gain factors G for different combinations of n and m.

n	m	Range of μ for $\epsilon = 0.5$		G_1		G_2		G_3		G_4	
		μ_{max}	μ_{min}	max	min	max	min	max	min	max	min
0	3	0.96875	0.8125	1	1	1	1	1	1	−4	1
1	2	0.8125	0.5	1	1	1	1	1	−9	6	−4
2	1	0.5	0.1875	1	1	1	−9	−9	11	−4	6
3	0	0.1875	0.03125	1	−4	−9	6	11	−4	1	−4

curves C and D show the ACF for $n = 1$, $m = 2$, and $n = 2$, $m = 1$ for $\mu = \mu_0$. The hardware implementation of Eqn. (19) is shown in Fig. 5. The frequency responses of the transformed filters for $n = 0$, $m = 3$, $\mu = \mu_{max}$ and for $n = 3$, $m = 0$, $\mu = \mu_{min}$ are compared with that of the prototype filter in Fig. 6.

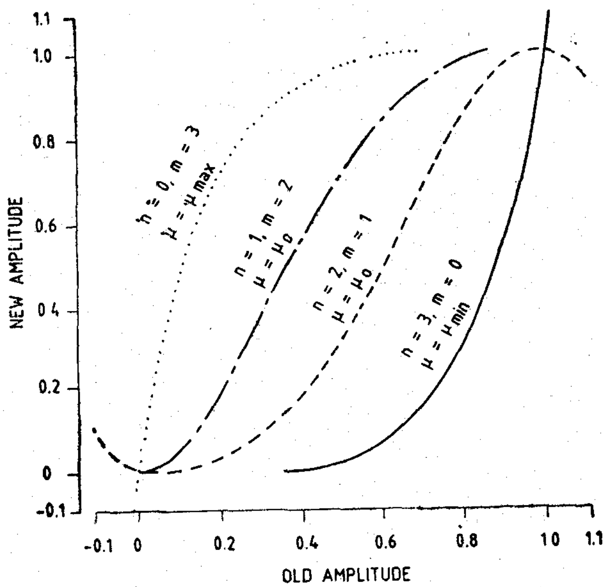

Figure 4. ACF curves.

4.3 *Software Implementation*

The filtering algorithm for H is stored as a subroutine and the overall filtering is carried by the repeated use of the subroutine. For example, the filtering operation of Eqn. (18d) starts first by passing the input signal through the filter H once. The resulting signal is subtracted from the input, delayed by N sampling intervals where N is one half of the total delay of the prototype filter. The residue thus obtained is multiplied by the constant $K_{3,0}$. This is then added with the delayed input signal once again. The overall filtering is done by repeatedly passing the resulting signal four times through the prototype filter.

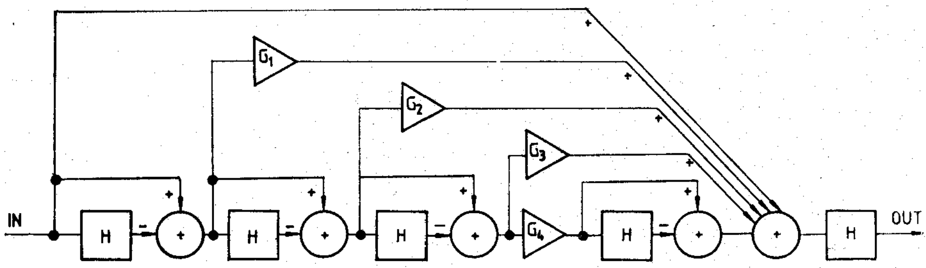

Figure 5. Hardware realization of the structure.

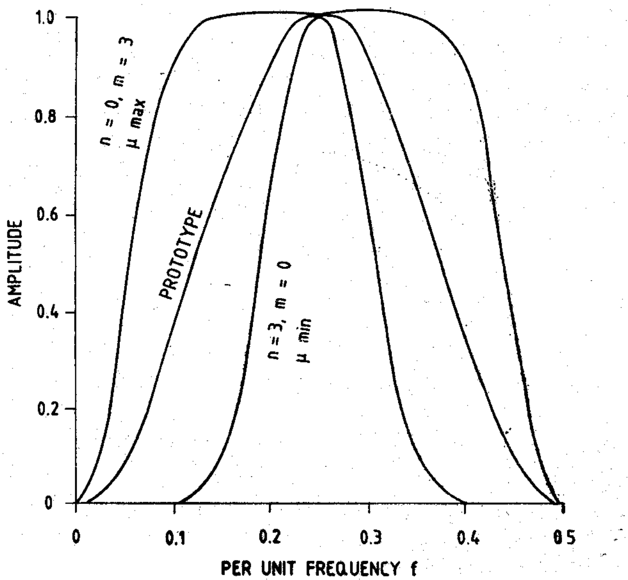

Figure 6. Comparison of the frequency responses of a raised-cosine bandpass filter with the transformed versions.

5. Frequency Transformation Through ACF

The concept of ACF can be used for the frequency transformation of linear phase FIR filters, viz. transforming a lowpass (LP) filter into highpass (HP), bandpass (BP) or bandstop (BS) filters[10].

For transforming a LP filter into a HP filter (or a HP filter into LP filter), the passband of the LP filter is to be mapped on to the stopband of the HP filter, the stopband to passband and the transition to transition. An ACF satisfying the above condition is obtained by allowing the corresponding curve to pass through the points (0, 1) and (1, 0). The expression for such a curve is obtained as

$$H_{06} = 1 - H_{00} = (1 - H)^{m+1} \sum_{k=0}^{n} C(m + k, k)\, H^k \tag{20}$$

where n and m are the order of tangencies at the points (0, 1) and (1, 0) respectively. The simplest case of first order tangencies at the above points can be written as

$$H_{06} = 1 - H^2(3 - 2H) \tag{21}$$

and is shown in Fig. 7(b), while Fig. 7(a) is the ACF for simple symmetric sharpening (Refer to Curve C, Fig. 2 in ref. 5) drawn for easy reference.

If the ACF is required to transform a LP filter into a BP filter, then the passband of the LP filter is to be mapped on to the stopband of the BP filter, stopband to stopband and the transition to passband and transition. Clearly, such a curve should pass through the points (0, 0), (η, 1) and (1, 0) where η represents the centre of the passband of the transformed filter and its value lies between 0 and 1. An expression for an ACF curve for $\eta = 0.5$ can be written as

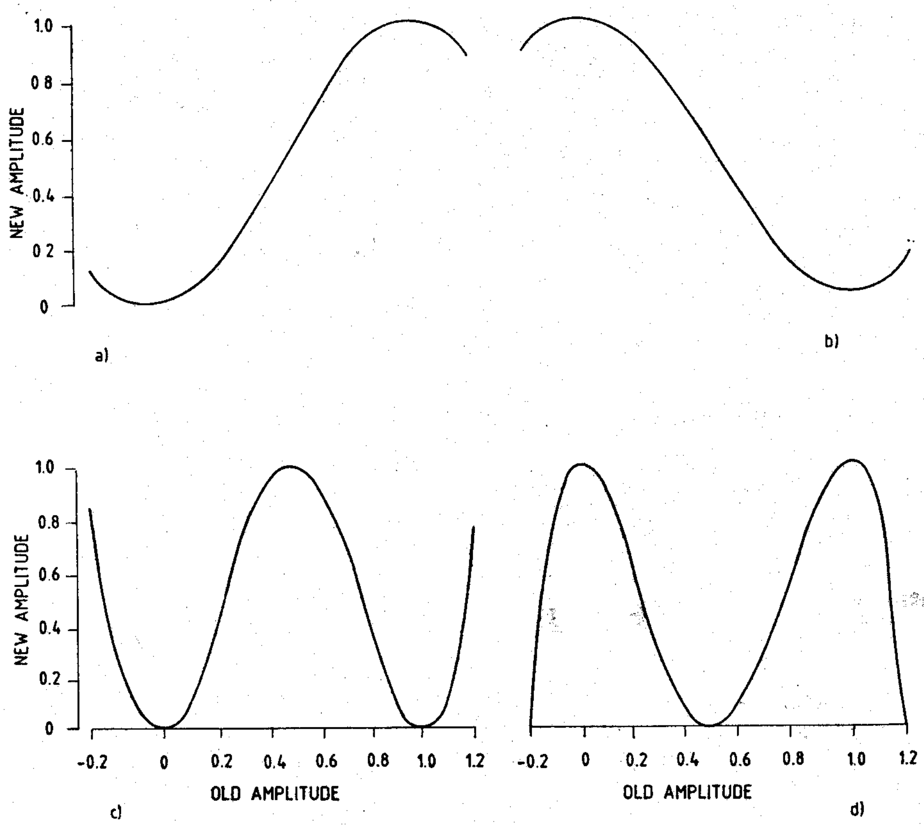

Figure 7. ACF for (a) lowpass to lowpass transformation, (b) lowpass to highpass transformation, (c) lowpass to bandpass transformation, (d) lowpass to bandstop transformation.

$$H_{07} = [4H(1 - H)]^{n+1} \sum_{\substack{k=1 \\ k \text{ odd}}}^{m} C\left(n + \frac{k-1}{2}, \frac{k-1}{2}\right)(1 - 2H)^{k-1} \quad (22)$$

where n is the order of tangency at $H = 0$ and 1, and m is that at $H = 0.5$. Equation (22) is also valid for HP to BP transformation. The fourth order transformation with first order tangencies at $H = 0$, 1 and 0.5 is given by

$$H_{07} = 16H^2(1 - H)^2 \quad (23)$$

which shows that H_{07} is a product of two cascaded, squared sections of lowpass and highpass filters. This is also evident from Fig. 7(c), a plot of Eqn. (23), which depicts a combination of LP to LP transformation for $H = 0$ to 0.5, scaled by a factor 0.5 and LP to HP transformation for $H = 0.5$ to 1, shifted by 0.5. While Eqn. (22) will also transform a HP into BP filter, $H_{08} = 1 - H_{07}$ can be used for LP to BS transformation. The ACF H_{08}, for the first order tangencies at the points (0, 1), (0.5, 0) and (1, 1) is shown in Fig. 7(d). It may be noted that the transformations H_{06}, H_{07} and H_{08} are not only used for frequency transformation of filters, but the frequency responses of the transformed filters can also be improved by properly choosing the order of tangencies at the points corresponding to the passband and stopband.

The cut-off frequency of the transformed filter can also be varied continuously within certain limits. In the case of LP to HP transformation, the method is similar to the one discussed in Section 4. For varying the cut-off frequency of the BP transformed filter with $\eta = 0.5$, it is required to simultaneously vary the following two points on the ACF : (ϵ, μ) and $(1 - \epsilon, \mu)$. The resulting ACF is given by

$$\begin{aligned} H_{09} = {} & [4H(1 - H)]^{n+1} \Bigg[\sum_{\substack{k=1 \\ (k \text{ odd})}}^{m} C\left(n + \frac{k-1}{2}, \frac{k-1}{2}\right) \\ & \times \left\{ (1 - 2H)^{k-1} - \left(\frac{1 - 2H}{1 - 2\epsilon}\right)^{m+1} (1 - 2\epsilon)^{k-1} \right\} \Bigg] \\ & + \frac{\mu}{(1 - 2\epsilon)^{m+1}} \left[\frac{H(1 - H)}{\epsilon(1 - \epsilon)}\right]^{n+1} \end{aligned} \quad (24)$$

where n and m have the same significance as in Eqn. (22). Larger variation in the cut-off frequency is achieved by varying $\epsilon(\mu = \text{const.})$ as well as n and m.

The centre frequency of the BP/BS transformed filters discussed in the previous sections is a constant for a given LP/HP prototype filter. It is equal to the frequency at which the amplitude of the prototype filter becomes equal to 0.5. An ACF can also be constructed for continuously varying the centre frequency of the transformed filter while keeping the bandwidth constant. The curve for LP to BP transformation passes through the points (0, 0), $(\eta, 1)$ and (1, 0), where η represents the centre of the passband and lies between 0 and 1. The corresponding ACF can be expressed as

$$H_{010} = \left[\frac{H}{\eta}\right]^{n+1} \left[\frac{1-H}{1-\eta}\right]^{m+1} \sum_{k=0}^{l} \left[\sum_{j=0}^{k} C(n+k-j, k-j) \right.$$

$$\left. \times\, C(m+j, j) \left(\frac{-\eta}{1-\eta}\right)^{j} \right] \left(1 - \frac{H}{\eta}\right)^{k} \tag{25}$$

where n is the order of tangency at $H = 0$, m is that at $H = 1$ and l at $H = \eta$.

The centre frequency can be varied by varying η such that H_{010} remaints within 0 and 1 for all values of H between 0 and 1. For a given order of transformation, the largest variation in centre frequency is achieved by varying η as well as by changing n and m, keeping l a constant.

5.1 *Example*

The prototype lowpass filter chosen is a maximally flat, linear phase[11] FIR filter with the following weighting sequences :

$$h = \frac{1}{256}\{-1, -5, -5, 20, 70, 98, 70, 20, -5, -5, -1\}$$

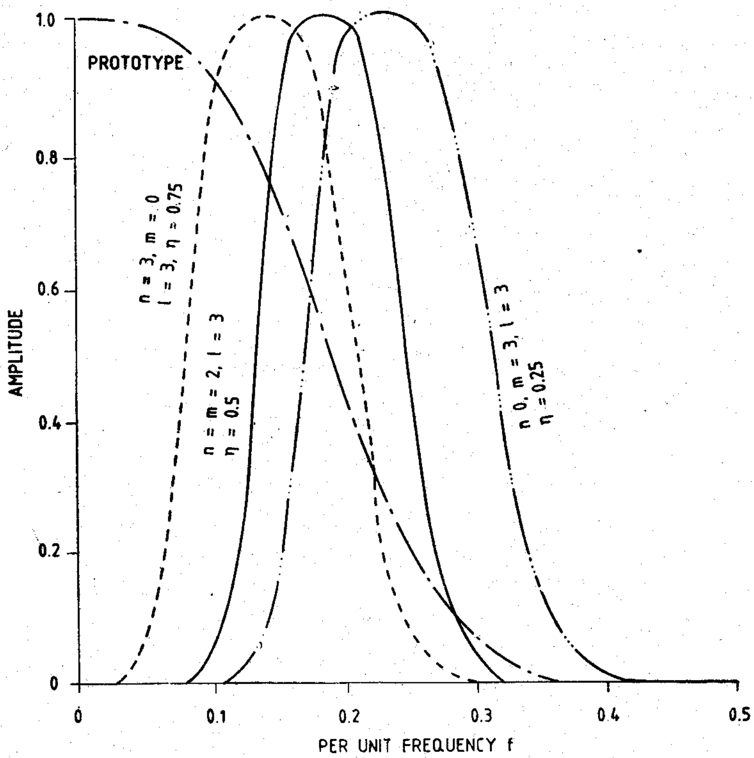

Figure 8. Comparison of frequency responses of a lowpass to variable centre frequency bandpass transformed filters.

The transformation given by Eqn. (25) is applied to this filter for several values of η, n and m while keeping l and $n + m$ a constant, equal to 3. The results of the transformation are expressed in terms of powers of $(1 - H)$ for the uniformity of hardware implementation. The gain factors associated with each of these $(1 - H)$ modules can be calculated in a similar way as was discussed in Section 4. Fig. 8 compares the frequency responses of the transformed filter for $\eta = 0.25$, 0.5 and 0.75, respectively, for the above prototype filter.

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