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Biomathematical Approach Towards A Linear Climatic Index

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Abstract. The approach is aimed towards integration of major climatic elements for evolving a linear climatic index based on annual averages of three climatic parameters, viz; diurnal range of temperatures, daily mean temperature and precipitation respectively. The index is a measure of aridity of a place, being positive for all arid regions and negative for all humid regions and is effectively applicable to high altitudes. The climatic index, so derived, is termed as 'Linear Aridity Index' and has been compared against Thornthwaite's Moisture Index for 32 meteorological stations in India and neighbourhood (including coastal, non-coastal and high altitude) which reveals a close correlation between the two indices. More stations in India and neighbourhood have been analysed in terms of the index value for studying the climatic pattern in India. A nomogram has also been developed for quick evaluation of the index from the given values of the three parameters used.

1. Introduction

Climate plays an important role in our lives and determines the living conditions, clothing needs, production of food, conditions of health etc. The qualitative picture of climates and terrains is fairly known but their quantitative description is indeed difficult due to paucity of meteorological data. In the plains, one is reasonably satisfied if the meteorological stations are placed 100 km apart because interpolation and extrapolation of data over such distances is more or less permissible. The situation is far more complicated in the case of high altitudes on account of wide variations in the regional microclimates over small areas, depending on altitude and topography.

Climatologists all over the world have been engaged in the problem of classification of climates into various climatic types for over hundred years, but little

*Presently attached to Defence Institute of Physiology and Allied Sciences, Delhi Cantt-110010 success seems to have been achieved for formulating a general classification scheme which would meet the requirements of all concerned and at the same time enable us to classify the climates in a quantitative manner. The most important elements which determine the climate are temperature, wind, humidity, precipitation and evapotranspiration. The purpose for which climatic classification is needed may decide about the elements to be incorporated and emphasis be laid down to make it as quantitative as possible.

The early efforts towards classification of climates in a semi-quantitative manner were carried out by Richard Brinsley Hinds in the year 1842 as reported by Court[1]. Since then this study is drawing the attention of various workers and the first modern climatic scheme showing the effects of climate on vegetation was described by the German climatologist Koppen[2] in the year 1900. Dzerdzeevskii[3] has characterised the climatic features of regions all over the world in a quantitative manner but these approaches have been esseentially in relation to plant and forest ecology.

2. Thornthwaite's Moisture Index

Thornthwaite[4] introduced a new concept of 'Potential Evapotranspiration' (PE) which is defined as the total amount of water that would evaporate and transpire if it is always made available for full use. His system of classification of climates, based on a critical study of water balance of the soil in relation to certain meteorological parameters, is generally regarded as the most rational approach and, perhaps, enjoys the widest popularity.

For the purpose of classification of climatic types, Thornthwaite evolved an empirical formula defining a moisture index (I_m), as the dimensionless ratio which is expressed as

$$I_m = \frac{s - 0.6\, d}{n} \times 100 \qquad \text{...(1)}$$

where s = surplus moisture in the soil,

d = deficient moisture in the soil, and

n = necessary quantity of moisture for plants (or PE)

His simplified expression for the computation of PE or n requires a knowledge of mean monthly temperature and precipitation together with the latitude of the place. According to notations used, positive values of the index represent moist climates and negative values the dry climates. Thornthwaite's scheme for classification of climates in terms of moisture index has been presented in Table 1.

The concept of potential evapotranspiration and water balance proved useful for the classification of climates but the computational procedure involved in the

Table 1. Thornthwaite's scheme for classification of climates in terms of moisture index, I_m.

Climatic type	Moisture Index
A Perhumid	+ 100 and above
B_4 Humid	+ 80 to + 100
B_3 Humid	+ 60 to + 80
B_2 Humid	+ 40 to + 60
B_1 Humid	+ 20 to + 40
C_2 Moist subhumid	0 to + 20
C_1 Dry subhumid	— 20 to 0
D Semiarid	— 40 to — 20
E Arid	— 60 to — 40

evaluation of I_m is rather elaborate and somewhat restricts its practical usefulness. The other limitation is the nonlinearity of the climatic scale, being highly compressed for the arid type (I_m : —60 to — 40) and spreads out towards the humid type (I_m : + 20 to + 100). Moreover, no upper limit has been prescribed for the prehumid type where the index value exceeds + 100.

Evidently, the climatic scale with I_m, having a low sensitivity in the arid zone, is not suited for the purpose of grading it in terms of relative degrees of aridity. Based on these considerations, it was considered worthwhile to attempt a fresh approach to the problem from a general stand point for evolving a climatic index on a linear scale which should be equally sensitive to all types of climates and easily evaluable from routine meteorological data.

3. Climatic Index on a Linear Scale

Efforts were initiated in this direction by Majumdar and Sharma[5] to evolve a climatic index on a linear scale which should meet the following basic requirements :

1. It should reasonably account for the recognised climatic types.
2. It should, in a fair measure, reflect the combined effect of major climatic elements.
3. The climatic scale should be equally sensitive throughout the range.

Majumdar and Sharma[5] considered the concept of diurnal range of temperatures, which is simply the difference between daily maximum and minimum temperatures, useful towards the development of a climatic index. Accordingly, a climatic index (I_{clm}), based on the annual means of diurnal range of temperatures (D) and rainfall (R) was developed which is expressed as

$$I_{clm} = a \log\left(\frac{D/R}{D_0/R_0}\right) \tag{2}$$

where a is a dimensionless constant and its value depends only on the arbitrary choice of the size of the unit of I_{clm} and D_0/R_0 is the value of the ratio D/R when $I_{clm} = 0$, the index being positive for all dry climates and negative for all moist climates.

The above climatic index having linearity on the climatic scale was succesfully employed for the sub-classification of arid climates but could not be used with accuracy for high altitudes. Its failure in this respect could be ascribed to non-inclusion of a third important parameter namely 'potential evapotranspiration' or n which is closely related to mean temperature which falls with increasing altitude. Furthermore, the assumption of equal weightage for D and R could not be justified on the basis of available data.

4. Present Approach in the Evalution of a New Climatic Index

In order to overcome the defects in the above scheme, the inclusion of the third parameter n was considered and the index so arrived at termed as Linear Aridity Index (I_a). Since a change in I_a should be proportional to the relative changes in D, R and n, and also the index increases with increasing D and n but decreases with increasing R, we may write

$$\delta I_a = a\frac{\delta D}{D} + b\,\frac{\delta n}{n} - c\,\frac{\delta R}{R} \tag{3}$$

where a, b and c are positive constants.

Since both n and R are expressed in cm., the ratio n/R becomes dimensionless but the same cannot be said about the exponent of D which depends on a number of climatic variables. Eqn. (3) may be put in the form

$$I_a = A \log\frac{D^p.n}{R} \tag{4}$$

where p being a positive constant. Let the value of the expresson $D^p.n/R$ when $I_a = 0$, be denoted by $(D^p.n/R)_0$, so that Eqn. (4) takes the form

$$I_a = A\ \log\frac{D^p.n/R}{(D^p.n/R)_0} \tag{5}$$

The linearity of the climatic scale is ensured by the logarithmic scale and the choice of the value of A will determine the size of the unit of I_a.

5. Development of Relation Between I_a and I_m

I_a being an index of aridity, $(-I_a)$ may be regarded as a linearised version of Thornthwaite's moisture index, I_m. Eqn. (5) may be expressed as

$$-I_a/A = \log\frac{R/D^p.n}{(R/D^p.\,n)_0} \tag{6}$$

The nonlinear moisture index, I_m, can be obtained by removing the logarithm in the above equation and Eqn. (6) takes the form

$$10^{-I_a/A} = \frac{R/D^p.n}{(R/D^p.n)_0} = q\,I_m + r \tag{7}$$

By making the zero of the I_a scale to coincide with the zero of the I_m scale, we have $r = 1$. Also the limiting value of aridity as per Thornthwaite's scheme is reached with $I_m = -60$ as $R \rightarrow 0$, it follows from Eqn. (7) that $1 - 60\,q = 0$ or $q = 1/60$, thus Eqn. (7) reduces to

$$10^{-I_a/A} = \frac{R/D^p.n}{(R/D^p.n)_0} = \frac{I_m + 60}{60} \tag{8}$$

or

$$I_a = A \log\frac{60}{I_m + 60} \tag{9}$$

In general, we may write

$$I_a = A \log\frac{C}{I_m + C} \tag{10}$$

where C is a constant having a value close to 60 and can be determined more accurately from an analysis of the data on R, D and n, together with the computed values of I_m.

6. Evaluation of the Exponent of D

The computed values of potential evapotranspiration (n), mean annual temperature (t_m), mean annual precipitation (R) and moisture index (I_m) in respect of climates of 32 stations (including coastal, non-coastal and high altitude) in India and neighbourhood were available from a study made by Subrahmanyam[6]. These values together with the computed values of diurnal range of temperatures (D) have been presented in Table 2 and the data utilised for the evaluation of constants of Eqns. (5) and (10).

From Eqn. (8), we find that $(I_m + 60)\,.n/R$ is proportional to D^{-p} or say $\bar{D}^m$ (replacing $-p$ by m). In Fig. 1, $\log\,(I_m + 60)\,.n/R$ has been plotted against $\log \bar{D}$ for all the 32 stations. A strong negative correlation is apparent and only four stations

Table 2. Climatic data of 32 stations in India and neighbourhood.

S/No.	Station	t^*_m (°C)	D (°C)	R^* (cm)	n^* (cm)	I^*_m	I_a	I^{**}_{am}
1	2	3	4	5	6	7	8	9
1.	Agra	26.1	15.2	64.4	149.9	− 34.2	+28.2	+25.0
2.	Allahabad	25.6	13.2	95.0	147.3	− 18.4	+13.3	+10.9
3.	Calcutta	26.1	10.2	162.0	152.0	+ 21.9	− 5.9	− 9.3
4.	Chittagong	25.0	8.8	261.0	142.7	+ 84.4	−29.6	−26.4
5.	Darjeeling	11.7	6.0	303.9	65.5	+364.0	−59.8	−59.0
6.	Delhi	25.6	13.5	68.7	146.9	− 31.9	+23.7	+22.5
7.	Gauhati	23.9	10.1	170.0	133.6	+ 29.7	−11.8	−12.1
8.	Jodhpur	26.7	14.0	34.7	151.4	− 46.3	+46.8	+43.2
9,	Karachi	25.0	6.9	19.3	144.0	− 51.5	+50.8	+56.4
10.	Lahore	23.9	15.7	51.3	136.2	− 37.4	+37.1	+28.8
11.	Lucknow	25.6	13.2	100.0	144.0	− 14.7	+11.8	+ 8.4
12.	Patna	22.8	10.4	118.7	148.6	− 6.0	+ 5.1	+ 3.2
13.	Peshawar	22.5	13.6	32.1	126.5	− 44.7	+41.0	+40.1
14.	Simla	13.1	7.2	157.5	70.7	+123.0	−34.4	−33.6
15.	Srinagar	12.8	13.3	65.7	71.2	+ 6.3	+ 1.0	− 3.0
16.	Veraval	25.6	7.6	57.7	148.9	− 36.8	+20·0	+28.1
17.	Bangalore	25.0	11.1	87.0	156.4	− 26.6	+12.2	+17.4
18.	Bombay	27.1	7.2	192.5	169.4	+ 35.6	−14.6	−14.0
19.	Cuttack	27.5	10.4	153.9	169.0	+ 3.5	− 5·3	− 1.7
20.	Cuddapah	29.4	11.5	75.5	187.3	− 35.8	+26.6	+26.8
21.	Madras	28.3	9.6	125.4	172.3	− 8.4	+ 5.0	+ 4.5
22.	Mangalore	27.3	7.1	329.4	173.0	+105.2	−30.8	−30.5
23.	Nagpur	24.2	12.2	119.2	153.8	− 6.3	+ 2.6	+ 3.3
24.	Pune	25.1	13.9	67.9	140.1	− 30.9	+23.4	+21.5
25·	Trivandrum	26.4	5.3	163.2	163.6	+ 7.8	−15.6	− 3.7
26.	Ootacumund	14.2	9.5	139.6	69.6	+100.5	−24.5	−29.6
27.	Port Blair	26.3	6.3	313.1	159.9	+103.5	−32.9	−30.1
28.	Rangoon	27.4	8.5	261.8	173.5	+ 81.2	−20.9	−25.7
29.	Akyab	25.9	7.8	515.3	152.9	+244.6	−45.6	−49.0
30.	Mandalay	26.8	11.5	87.1	159.9	− 27.3	+16.0	+18.0
31.	Sukkur	26.8	14.4	6.3	152.3	− 57.5	+87.1	+86.5
32.	Colombo	26.9	6.0	236.5	168.7	+ 40.5	−24.0	−15.5

*Subrahmanyam

**I_{am} = Linearised version of I_m according to equation (17)

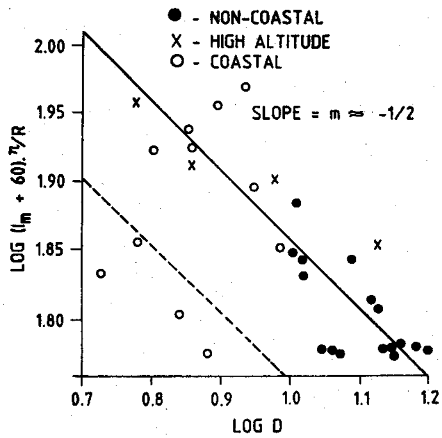

Figure 1. Evaluation of m in the exponent of D.

(all coastal) shown below the broken line, fall out of the general pattern. Excluding these four points, the mean slope does not differ significantly from $-$ 0.5 and we may accept

$$m = -\tfrac{1}{2} \text{ or } p = \tfrac{1}{2}$$

Eqn. (5), thus takes the form

$$I_a = \text{A} \log \frac{n\sqrt{D}}{R} \bigg/ \left(\frac{n\sqrt{D}}{R}\right)_0 \tag{11}$$

7. Relationship Between Potential Evapotranspiration (n) and Mean Annual Temperature (t_m)

The computation of potential evapotranspiration (n) by Thornathwaite's method is rather elaborate and troublesome since it involves the latitude of the place from which a theoretical value of incoming solar radiation is derived and, therefore, ignores the local factors like cloudiness, turbidity etc. Since the vapour pressure, which determines the rate of evaporation, increases almost exponentially with temperature, it was thought that log n should be linearly related to t_m. In Fig 2, n has been plotted against t_m on a semi-logarithmic scale for all the 32 stations and the expected relationship is quite evident. The regression equation statistically fitted to the points by the method of least squares is

$$n = 31.85 \times 10^{0.0264 t_m} \tag{12}$$

8. Relationship Between I_a And I_m in terms of R, D and t_m

With the help of Eqn. (12), we may put Eqn. (11) in the form

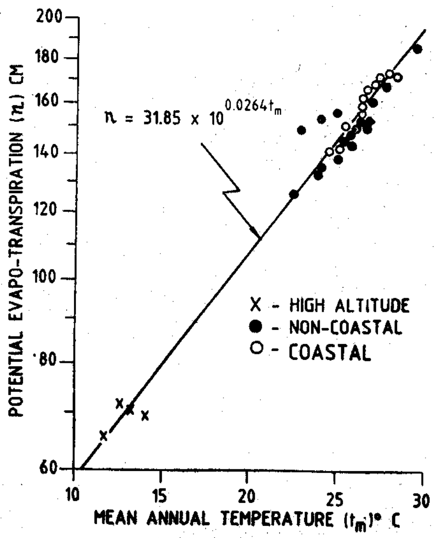

Figure 2. Relationship between potential evapo-transpiration (n) and mean annual temperature (t_m)

$$I_a = A \log \frac{\sqrt{D} \times 10^{0.0264\, t_m}}{k.R}$$

or

$$-\ I_a/A = \log \frac{k.R}{\sqrt{D} \times 10^{0.0264 t_m}} \tag{13}$$

Also from Eqn. (10), we have

$$-\ I_a/A = \log \frac{I_m + C}{C} \tag{14}$$

From Eqns. (13) and (14), it follows that

$$I_m = Ck. \frac{R}{\sqrt{D} \times 10^{0.0264 t_m}} - C \tag{15}$$

In Fig. 3, I_m has been plotted against $R/(\sqrt{D} \times 10^{0.0264}\, t_m)$ which yields a straight line and the regression equation obtained is

$$I_m = 7.152 \;\; R\,(\sqrt{D} \times 10^{0.0264}\; t_m) - 61.046 \tag{16}$$

which gives the value of $C = 61.046$ and Eqn. (10) takes the form

$$I_a = A \log \frac{61.046}{I_m + 61.046} \tag{17}$$

Since the value of A may be chosen arbitrarily, say $A = 70$ for convenience, so that I_a may lie roughly between $-$ 100 to + 100 for the entire range of practical

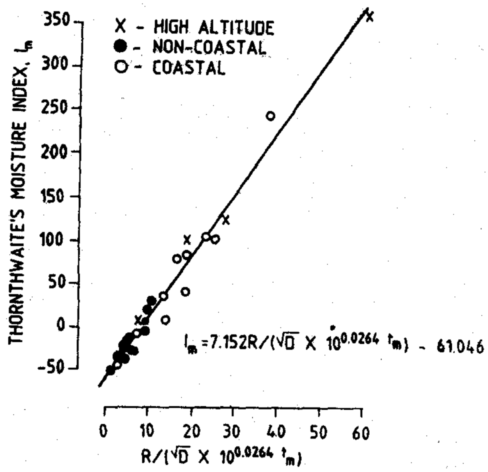

Figure 3. Linear relationship between I_m and $R/(\sqrt{D} \times 10^{0.0264t_m})$

conditions all over the world. Substituting the value for I_m from Eqn. (16) and after some simplifications, Eqn. (17) finally reduces to

$$I_a = 65.19 + 70 \log \frac{\sqrt{D} \times 10^{0.0264\, t_m}}{R} \tag{18}$$

The index value is the measure of aridity of a place, being positive for arid regions and negative for humid regions. A nomogram, as shown in Fig. 4, has also been developed from which I_a can be quickly evaluated from given values of D, t_m and R.

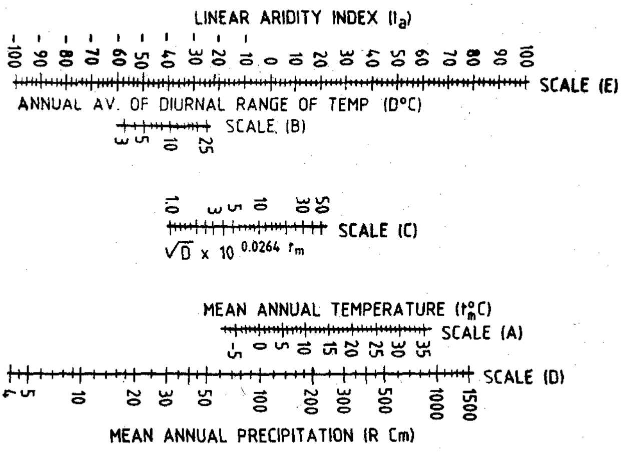

Figure 4. Nomogram for evaluation of linear aridity index (I_a). Straight line through scales (A) and (B) gives ($\sqrt{D} \times 10^{0.0264t_m}$) on scale ($C$). Straight line through this point and scale (D) gives I_a on scale (E).

9. Comparison Between I_a and I_m

With the help of the nomogram in Fig. 4, I_a was computed for all the 32 stations listed in Table 2 (column 8) and compared against reported I_m, values as shown in Fig. 5. The closeness of agreement between the two indices is quite apparent and the curve in the figure represents Eqn. (17) with $A = 70$.

However, for a quantitative estimation of degree of correlation between I_a and I_m, the linearised version of I_m designated as I_{am}, has been used. The computed values of I_{am} have been presented in column 9 of Table 2. In Fig. 6, I_a has been

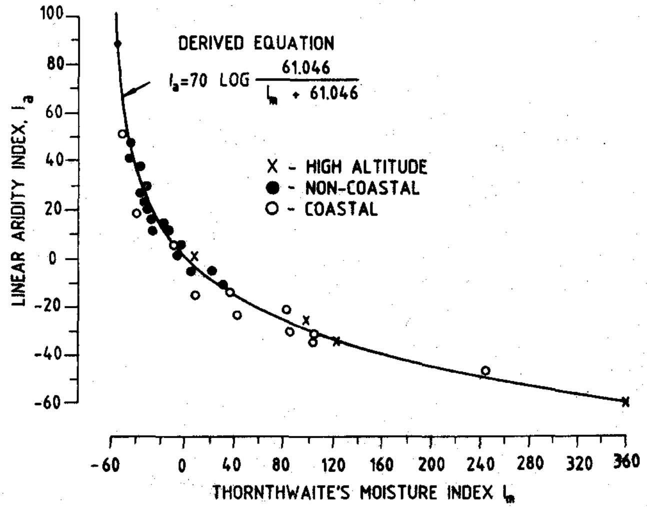

Figure 5. I_a compared against I_m for 32 stations in India and neighbourhood.

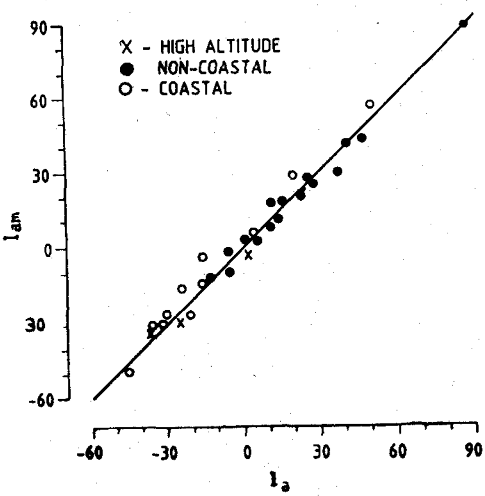

Figure 6. Comparison between I_{am} and I_a for 32 stations.

compared against I_{am} which reveals a high degree of positive correlation between the two sets of values.

10. Classification of Climates and Sub-classification of Arid and Perhumid Zones

The new climatic index, so derived, has reasonable linearity on the climatic scale and is equally sensitive throughout the range. It has, therefore, been possible to sub-classify arid and perhumid zones and to study the climatic features all over the world in terms of this index. A tentative scheme for classification of climates in terms of linear aridity index has been put forward as presented in Table 3.

Table 3. Tentative scheme for Classification of climates in terms of linear Aridity Index I_a,

Climatic type		Range of I_a
	Extreme	$I_a > +70$
Arid	Moderate	$+50 < I_a \leqslant +70$
	Mild	$+30 < I_a \leqslant +50$
Semiarid		$+10 < I_a < +30$
	Dry	$0 < I_a \leqslant +10$
Subhumid		
	Moist	$-10 < I_a \leqslant 0$
Humid		$-30 < I_a < -10$
	Mild	$-50 < Ia \leqslant -30$
Perhumid	Moderate	$-70 < I_a \leqslant -50$
	Extreme	$Ia < -70$

The curve in Fig. 7, represents classification of climates both along the I_m and I_a scale. It will be observed that the former is highly compressed in the arid zone, and gradually spreads out towards the humid and perhumid zones. The I_a scale, on the other hand, is more or less uniformly distributed throughout the zones, each zone covering a range of 20 units. This supports that the I_a scale is reasonably linear and is equally sensitive for the various climatic zones.

For the purpose of studying the climatic pattern in India, more stations in India and neighbourhood for which meteorological data were available have been studied in terms of linear aridity index values. The study reveals that most of western Rajasthan is mildly arid with the exception of Jaisalmer ($I_a = 61.8$), Phalodi ($I_a=57.0$), Bikaner ($I_a=50.7$), Ganganagar ($I_a = 50.4$) and Barmer ($I_a = 50.2$) which are moderately arid. Aridity increases further west and ranges from moderate to extreme

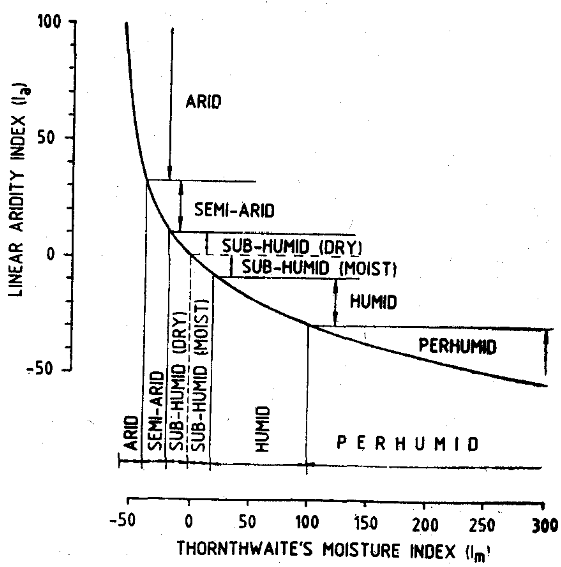

Figure 7. Broad classification of climates in terms of I_m and I_a.

in Pakistan, Iran, Arabia and so on. Among the perhumid stations, Cherrapunji and Gnatong top the list with I_a values of $-$ 88.5 and $-$ 78.2 respectively. Mahabaleshwar ($I_a = -61.4$), Darjeeling ($I_a = -53.3$) and Gangtok ($I_a = -52.0$) are moderately perhumid. Most of Kerala is mildly perhumid.

11. Conclusion

The new climatic index, based on routine meteorological data, represents reasonable linearity on the climatic scale and is equally sensitive for all types of climates. It is easily evaluable and is capable of reflectting the combined effect of major climatic elements like humidiy, turbidity, wind, solar radiation, cloudiness, nature of terrain etc. in the form of diurnal range of temperatures. Incorporation of the concept of daily mean temperature in the index has made the climatic scale applicable to high altitudes as well. In general, the index serves the purpose of characterising the climatic features all over the world in terms of degree of aridity.

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