Def Sci J, Vol 36, No 4, October 1986, pp 395-408

Buoyancy Effect on MHD Flow Past a Permeable Bed

S. VENKATARAMANA & D. BATHAIAH
Sri Venkateswara University, Tirupati-517502

Received 29 November 1984; revised 14 January 1986

Abstract. In this paper, the effect of buoyancy force on the parallel flows bounded above by a rigid permeable plate which may be moving or stationary and below, by a permeable bed has been investigated. To discuss the solution, the flow region is divided into two zones. In Zone 1, the flow is laminar and is governed by the Navier-Stokes equations from the impermeable upper rigid plate to the permeable bed. In Zone 2, the flow is governed by the Darcy law in the permeable bed below the nominal surface. The expressions for velocity and temparature distributions, Slip velocity, slip temperature, mass flow rate and the rates of heat transfer coefficients are obtained. The effects of magnetic, porous, slip and buoyancy parameters and Biot number on the above physical quantities are investigated. The thickness of the boundary layer in Zone 2 has been evaluated.

1. Introduction

Fluid flow through porous media is of fundamental importance to a wide range of disciplines in various branches of natural science and technology. Petroleum engineers, civil engineers, mining engineers and hydrogeologists are all interested in 'seepage' problems in rockmass, sand beds and subterranean acquifers. Civil and agricultural engineers are also interested in the same phenomenon for efficient layout of drainage systems for irrigation and recovery of swampy area. Geotechnical engineers and soil physicists are all interested in the water movement in clays and other surface active soils. The chemical engineers and ceramic engineers have filteration and seepage problems in their respective fields. The nuclear engineers are interested in fluid flow through reactors to maintain a uniform temperature throughout the bed. The textile technologist is interested in fluid flow through fibres, whereas biologists are interested in water movement through plant roots and out of the cells of living systems.

In view of all these applications of flow through or past porous media, it is important and also interesting to study such flow models. In the case of flow over a permeable surface, Beavers & Joseph[1], Beavers[2], et al. and Rajasekhara[3] have shown experimentally the existence of a slip at the permeable surface. A rigorous theoretical justification for the existence of slip velocity at the permeable surface was given by Saffman[4]. Rajasekhara[3] has investigated plane Couette flow in the presence of a pressure gradient and found slight deviation between his theoretical and experimental results. Rudraiah, Rajasekhara & Ramaiah[5] have studied about the flow of an incompressible viscous fluid past a porous bed. Venugopol & Bathaiah[6] have studied the flow of an incompressible viscous and slightly conducting fluid past a porous bed neglecting the buoyancy force under the influence of uniform transverse magnetic field. In the case of parallel flows at low Prandtle numbers bounded by rigid porous walls, Sparrow[7], et al. and Gill & Casol[8] found that the buoyancy force cannot be neglected since it significantly affects the flow field.

The objective is to study the effect of buoyancy force on the parallel flows bounded above by rigid impermeable plate which may be moving or stationary and below by a permeable bed. To discuss the solution, the flow region is divided into two zones. In Zone 1, the flow is laminar and is governed by the Navier-stokes equations from the impermeable upper rigid plate to the permeable bed. In Zone 2, the flow is governed by the Darcy law in the permeable bed below the nominal surface. The expressions for velocity and temperature distributions, slip velocity, slip temperature, mass flow rates and the rates of heat transfer coefficients are obtained in Section 2.

The effects of magnetic, porous, slip and buoyancy parameters and Biot number on the above physical quantities are presented in Section 3.

2. Formulation and Solution of the Problem (Zone 1)

2.1 *Part A : Couette Flow*

We consider the flow of a viscous incompressible slightly conducting fluid between a plate moving with the velocity u_0 and a permeable bed at a depth h under the influence of a uniform transverse magnetic field. The interface is taken as the X-axis and a line perpendicular to that as the Y-axis. The magnetic field of intensity H_0 is introduced in the Y-direction. The fluid being slightly conducting, the magnetic Reynolds number is much less than unity so that the induced magnetic field can be neglected in comparison with the applied magnetic field (Sparrow & Cess[9]).

We introduce the non-dimensional quantities as follows :

$$u^* = \frac{u}{U},\ y^* = \frac{y}{h},\ p^* = \frac{p}{\rho U^2},\ x^* = \frac{x}{h},\ T^* = \frac{T - T_0}{T_1 - T_0},$$

$$u_0^* = \frac{u_0}{U},\ u_B^* = \frac{u_B}{U},\ T_B^* = \frac{T_B' - T_0}{T_1 \quad T_0} \tag{1}$$

where u is the velocity component in the Y-direction, ρ the fluid density, p the fluid pressure, u_B, T'_B are the slip velocity and. slip temperature respectively, U is the characteristic velocity, T_0 is the ambient temperature, T_1 is the temperature at y $= 1$ and T is the temperature of the fluid.

The non-dimensional momentum and energy equations are (after dropping the superscripts '*')

$$\frac{d^3 u}{dy^3} - M^2 \frac{du}{dy} = G \tag{2}$$

$$\frac{d^2 T}{dy^2} - d^2 T = P_r Rbu - P_r E \left(\left(\frac{du}{dy}\right)^2 + M^2 u^2\right) \tag{3}$$

where

$$M^2 = \frac{\sigma \mu_e^2 H_0^2 h^2}{\mu} \text{ (magnetic parameter)}$$

$$G = \frac{A\beta' g h^3}{\nu U}, \quad b = \frac{Ah}{T_1 - T_0}$$

$$d^2 = \frac{Q' h^2}{K_T} \text{ (heat source parameter)}$$

$$P_r = \frac{\mu C_p}{K_T} \text{ (Prandtle number)}$$

$$R = \frac{Uh}{\nu} \text{ (Reynolds number)}$$

$$E = \frac{U^2}{C_p (T_1 - T_0)} \text{ (Eckert number)}$$

The non-dimensional boundary conditions are

$$u = u_0, \; T = 1 \text{ at } y = 1 \tag{4a}$$

$$\frac{du}{dy} = sa\left(u_B - \frac{P}{a^2}\right), \; \frac{d^2 u}{dy^2} = -P, \; \frac{dT}{dy} = \beta a T_B \text{ at } y = 0 \tag{4b}$$

where

$$a = \frac{h}{K^{1/2}} \text{ (porosity parameter)}$$

and

$$P = -R \frac{\partial p}{\partial x}$$

Solving the Eqn. (2) and using Eqn. (4), we obtain the velocity distribution

$$u = u_0 + \frac{P}{M^2}(\text{Cosh } M - \text{Cosh } My) + \frac{C}{M}(\text{Sinh } My - \text{Sinh } M)$$

$$+ \frac{N_0 P}{M^2}\left((1 - y) + \frac{(\text{Sinh } My - \text{Sinh } M)}{M}\right) \quad (5)$$

where

$$N_0 = \frac{G}{P} \text{ (Buoyancy parameter)}$$

$$C = sa\left(u_B - \frac{P}{a^2}\right)$$

$$u_B = \frac{M}{(M + sa \text{ Sinh } M)}\left(\frac{sP \text{ Sinh } M}{aM} + u_0 + \frac{P(\text{Cosh } M - 1)}{M^2}\right.$$

$$\left.+ \frac{N_0 P}{M^2}\left(1 - \frac{\text{Sinh } M}{M}\right)\right) \quad (6)$$

2.1.1 Mass flow rate

The non-dimensional mass flow rate F per unit width of the channel is

$$F = \int_0^1 u\, dy = f_0 - \frac{P}{M^2}\left(\frac{\text{Sinh } M}{M} + \frac{N_0}{2}\right) + \left(\frac{PN_0}{M^4} + \frac{C}{M^2}\right)$$

$$(\text{Cosh } M - 1) \quad (7)$$

where

$$f_0 = u_0 + \frac{P}{M^2}\left(\text{Cosh } M + N_0\left(1 - \frac{\text{Sinh } M}{M}\right)\right) - \frac{CP \text{ Sinh } M}{M^3}$$

Solving the Eqn. (3) and using Eqn. (4), we obtain the temperature distribution.

2.1.2 *Rate of heat transfer*

The rate of heat transfer at the interface y = 0 is given by

$$q = \left(\frac{dT}{dy}\right)_{y=0} = \beta\, aT_B \quad (8)$$

The rate of heat transfer at the upper plate y = 1 is given by

$$q^* = \left(\frac{dT}{dy}\right)_{y=1} = d \text{ Tanh } d + \frac{\beta a T_B}{\text{Cosh } d} + \frac{P_r Rb}{d^2}\left(f_2 \; d \text{ Tanh } d + \frac{f_3}{\text{Cosh } d}\right.$$

$$+ \frac{P}{M^2}(M \operatorname{Sinh} M + N_0) - f_1 \operatorname{Cosh} M\Big) - \frac{P_r E}{d^2}\Big(f_4 \; d \operatorname{Tanh} d$$

$$+ \frac{f_5}{\operatorname{Cosh} d} - \frac{P^2}{M^4}(M^3 \operatorname{Sinh} 2M + 2N_0 M^2 \operatorname{Cosh} M)$$

$$- f_1^2 \; M \operatorname{Sinh} 2M + \frac{2Pf_1 \operatorname{Sinh} M}{M} - \frac{P^2}{M^3}(M \operatorname{Sinh} 2M$$

$$+ 2N_0 \operatorname{Cosh} M + 2N_0 \; M \operatorname{Sinh} M + 2N_0^2) - f_1^2 \; M \operatorname{Sinh} 2M$$

$$+ 2Pf_0 \; (M \operatorname{Sinh} M + N_0) + 2Pf_1 \operatorname{Cosh} M$$

$$- 2f_0 \, f_1 \, M^2 \operatorname{Cosh} M\Big) \tag{9}$$

where

$$T_B = \frac{d}{(I + \beta a \operatorname{Tanh} d) \operatorname{Cosh} d} + \frac{P_r Rb}{d(1 + \beta a \operatorname{Tanh} d)}\Big(\frac{f_2}{\operatorname{Cosh} d}$$

$$- f_3 \frac{\operatorname{Tanh} d}{d} - f_0 + \frac{P}{M^2}\Big) - \frac{P_r E}{d^2}\Big(\frac{f_4}{\operatorname{Cosh} d} - f_5 \frac{\operatorname{Tanh} d}{d}$$

$$- \frac{P^2 N_0^2}{M'} - f_1^2 + \frac{2Pf_1}{M^2} - f_0^2 M^2 - \frac{P^2}{M^2} + 2Pf_0\Big)$$

$$f_1 = \frac{N_0 P}{M^2} + C$$

$$f_2 = f_0 - \frac{P}{M^2}(\operatorname{Cosh} M + N_0) + \frac{f_1 \operatorname{Sinh} M}{M}$$

$$f_3 = f_1 - \frac{PN_0}{M^2}$$

$$f_4 = \frac{P^2}{M^4}(M^2 \operatorname{Sinh}^2 M + 2N_0 M \operatorname{Sioh} M + N_0^2) + f_1^2 \operatorname{Cosh}^2 M$$

$$\frac{2Pf_1 \operatorname{Cosh} M}{M^2} + M^2 f_0^2 + \frac{P^2}{M^2}(\operatorname{Cosh}^2 M + 2N_0 \operatorname{Cosh} M + N_0^2)$$

$$+ f_1^2 \operatorname{Sinh}^2 M - 2Pf_0 - 2Pf_0(\operatorname{Cosh} + N_0) - 2Pf_1 \operatorname{Sinh} M/M$$

$$+ 2f_0 f_1 \, M \operatorname{Sinh} M$$

$$f_5 = \frac{4N_0 P^2}{M^2} - 2Pfo \; N_0 - 2Pf_1 + 2f_0 f_1 \, M^2$$

2.2 Part B Poiseuille Flow

In this part we consider the buoyancy effect of flow of a viscous incompressible, slightly conducting fluid between a stationary upper plate and a permeable bed at a distance h below the plate, under the influence of a uniform transverse magnetic field. All the results can be obtained from those of Part A by taking $u_0 = 0$.

Thevelocity distribution is

$$u_p = \frac{P}{M^2}\left(\text{Cosh } M - \text{Cosh } My + N_0(1-y) + \frac{(\text{Sinh } My - \text{Sinh } M)}{M}\right) + C\frac{(\text{Sinh } My - \text{Sinh } M)}{M} \quad (10)$$

where

$$C = sa\left(u_{B0} - \frac{P}{a^2}\right)$$

$$u_{B0} = \frac{M}{(M + sa \text{ Sinh } M)}\left[\frac{sP \text{ Sinh } M}{aM} + \frac{P}{M^2}\left(\text{Cosh } M - 1 + N_0\left(1 - \frac{\text{Sinh } M}{M}\right)\right)\right] \quad (11)$$

2.2.1 *Mass flow rate*

The nondimensional mass flow rate F^* per unit width of the channel is given by

$$F^* = \int_0^1 u_p\, dy = \frac{P}{M^2}\left[\text{Cosh } M - \left(\frac{\text{Sinh } M}{M} + 3, + N_0\left(1 - \frac{\text{Sinh } M}{M}\right)\right) + \frac{N_0(\text{Cosh } M - 1)}{M^2}\right] + \frac{C}{M^2}\left\{(\text{Cosh } M - 1) - PM \text{ Sinh } M\right\} \quad (12)$$

Temperature distribution is obtained from the temperature distribution of Part. A by taking $u_0 = 0$

2.2.2 **Rate** of hear *transfer*

The rates of heat transfer q_0 and q_0^* at the interface and upper boundary are given by

$$40 = \left(\frac{dT_p}{dy}\right)_{y=0} = \beta\, aT_{B0} \quad (13)$$

$$q_0^* = \left(\frac{dT_p}{dy}\right)_{y=1} = \left\{ d\,\mathrm{Tanh}\, d + \frac{\beta\, aT_{B0}}{\mathrm{Cosh}\, d} + \frac{P_r Rb}{d^2} \right.$$

$$\left. f_8\, d\,\mathrm{Tanh}\, d + \frac{f_9}{\mathrm{Cosh}\, d} - f_7\, M\mathrm{Cosh}\, M + \frac{P\,(M\,\mathrm{Sinh}\, M + N_0)}{M^2} \right\}$$

$$- \frac{P_r E}{d^2} \left(f_{10}\, d\,\mathrm{Tanh}\, d + \frac{f_{11}}{\mathrm{Cosh}\, d} - M^3 f_7^2\,\mathrm{Sinh}\, 2M - \frac{P^2}{M^4} \right.$$

$$(M^3\,\mathrm{Sinh}\, 2M + 2N_0\, M^2\,\mathrm{Cosh}\, M) + f_7\, P2M\,\mathrm{Cosh}\, M$$

$$+ 2PN_0\, f_7\,\mathrm{Sinh}\, M - f_7^2\,\mathrm{Sinh}\, 2M - \frac{P^2}{M^2}(\mathrm{Sinh}\, 2M + 2N_0^2$$

$$+ 2N_0\,\mathrm{Cosh}\, M + 2N_0\, M\,\mathrm{Sinh}\, M) - 2f_1\, f_7\, M^3\,\mathrm{Cosh}\, M$$

$$+ 2f_6\, P\,(M\,\mathrm{Sinh}\, M + N_0) + 2Mf_7\,\mathrm{Cosh}\, 2M$$

$$\left. + 2f_7\, N_0\, M\mathrm{Cosh}\, M + 2f_7\, N_0\,\mathrm{Sinh}\, M \right) \tag{14}$$

where

$$T_{B0} = \frac{d}{(d + \beta a\,\mathrm{Tanh}\, d)\,\mathrm{Cosh}\, d} + \frac{P_r Rb}{(d + \beta a\,\mathrm{Tanh}\, d)\, d} \left(\frac{f_8}{\mathrm{Cosh}\, d} \right.$$

$$\left. - f_9\,\frac{\mathrm{Tanh}\, d}{d} - f_6 + \frac{P}{M^2} \right) - \frac{P_r E}{(d + a\mathrm{Tanh}\, d)\, d} \left(\frac{f_{10}}{\mathrm{Cosh}\, d} \right.$$

$$- \frac{f_{11}\,\mathrm{Tanh}\, d}{d} - M^2 f_7^2 - \frac{P^2 N_0^2}{M^4} + \frac{2PN_0 f_7}{M} - f_6^2 M^2 - \frac{P^2}{M^2}$$

$$\left. + 2f_6 P \right)$$

$$f_6 = \frac{P}{M^2}\left(\mathrm{Cosh}\, M + N_0 - \frac{N_0\,\mathrm{Sinh}\, M}{M}\right) - \frac{C\,\mathrm{Sinh}\, M}{M}$$

$$f_7 = \frac{C\, N_0 P}{M^3} + \frac{-}{M}$$

$$f_8 = f_6 + f_7\,\mathrm{Sinh}\, M - \frac{P}{M^2}(\mathrm{Cosh}\, M + N_0)$$

$$f_9 = f_7\, M - \frac{PN_0}{M^2}$$

$$f_{10} = M^2 f_7^2\,\mathrm{Cosh}^2 M + \frac{P^2}{M^4}(M^2\,\mathrm{Sinh}^2 M + N_0^2 + 2N_0\, M\,\mathrm{Sinh}\, M)$$

$$- f_7 P \operatorname{Sinh} 2M - \frac{2PN_0 f_7 \operatorname{Cosh} M}{M} + f_6^2 M^2 + f_7^2 M^2 \operatorname{Sinh}^2 M$$

$$+ \frac{P^2}{M^2} (\operatorname{Cosh}^2 M + N_0^2 + 2N_0 \operatorname{Cosh} M) + 2f_6 f_7 M^2 \operatorname{Sinh} M$$

$$- 2f_6 P (\operatorname{Cosh} M + N_0) - 2f_7 \operatorname{Sinh} M (\operatorname{Cosh} M + N_0)$$

$$f_{11} = \frac{4N_0 P^2}{M^2} - 2PMf_7 + 2f_6 f_7 M^3 - 2f_6 PN_0 - 2Mf_7$$

3. Conclusions

Conclusions are drawn on the basis of numerical work done in the cases of slip velocity, velocity, slip temperature, temperature and rate of heat transfer (Fig. 1 to 6).

We have observed that the slip velocity decreases with the increase in magnetic parameter M in the case of Couette flow whereas the slip velocity increases first as M increases and then this trend gets reversed in the case of Poiseuille flow. But Venugopal and Bathaiah[6] have observed that the slip velocities decrease with the increase in magnetic parameter M in both the Couette and Poiseuille flows. Further, it is noticed that the slip velocities decrease with the increase in a in both the Couette and Poiseuille flows. We have noticed that the slip velocities in both the flows decrease with the

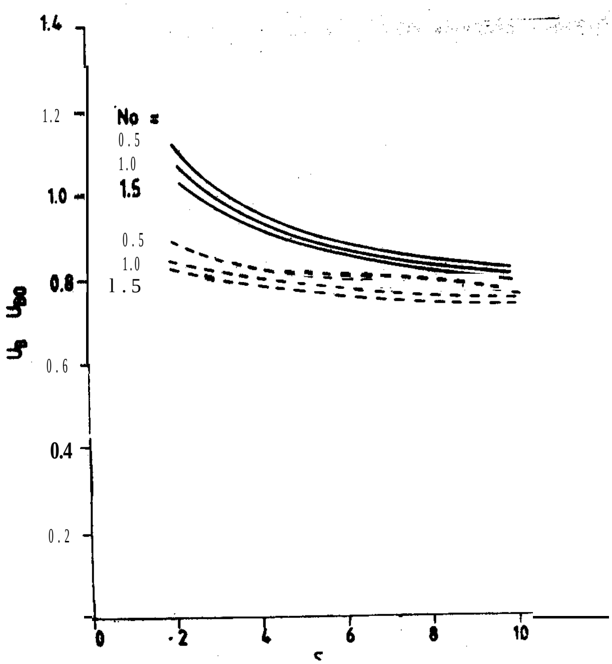

Figure 1. Slip velocities u_B, u_{B0} against s for different values of N_0.

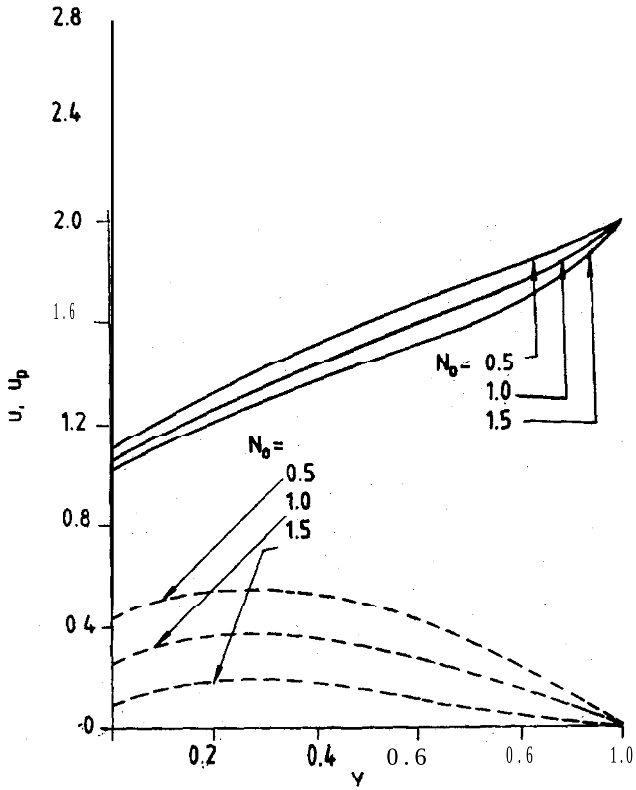

Figure 2. Velocity profiles against v for different values of N_0.

increase in *s* whereas Venugopal and Bathaiah[6] have seen that the slip velocities increase as *s* increases. It is also noticed that the presence of buoyancy force is to decrease the slip velocities in both the flows.

3.1. Velocity distribution

We have observed that the velocity decreases with the increase in *M* in the case of Couette flow whereas it increases in the case of Poiseuille flow. But Venugopal and Bathaiah[6] have obtained that the velocities decrease with the increase in *M* in both the flows under study. It is observed that the velocities decrease with the increase in a in both the flows. The velocities decrease in both the flows with the increase in *s*. But Venugopal and Bathaiah[6] have observed that the velocity increases as *s* increases in the case of Poiseuille flow and definite trend is not followed in the case of Couette flow. The velocities decrease in both the flows with the increase in buoyancy parameter N_0.

3.2 *Slip temperature*

It is observed that the slip temperature decrease with the increase in Min both the flows whereas Venugopal and Bathaiah[6] have found that slip temperature increases in the case of Couette flow, first increases and then decreases in the case of Poiseuille flow with the increase in M. Further, it is noticed that the slip temperature decreases with the increase in a in the case of Couette flow whereas it increases as a increases in the case of Poiseuille flow. But Venugopal and Bathaiah[6] obtained the result that slip temperatures decrease with the increase in a in both the flows. We have seen that the slip temperatures decrease with the increase in buoyancy parameter N_0 in both the flows and the slip temperatures decrease with the increase in s in both the flows. The slip temperatures are found to be decreasing with the increase in P_rE in both the flows whereas Venugopal and Bathaiah[6] have seen that the slip temperatures increase with the increase in P_rE in both the flows under study. The slip temperatures increase with the increase in β in both the flows which differs from the result of Venugopal and Bathaiah[6].

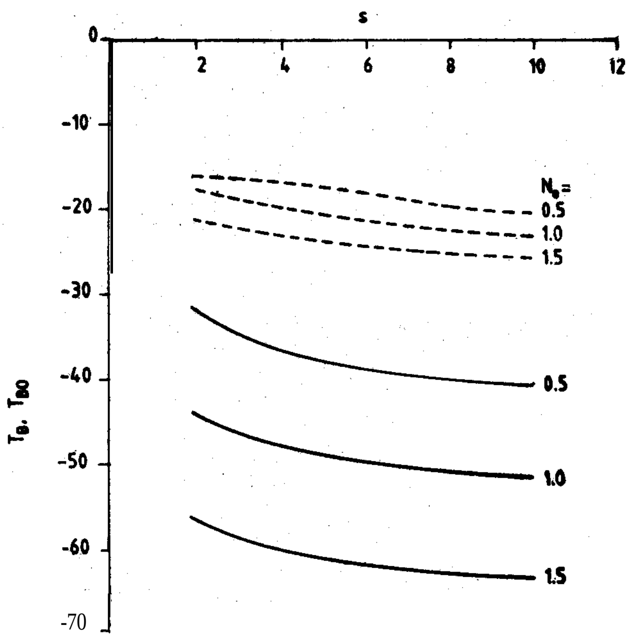

Figure 3. Slip temperature against s for different values of N_0.

3.3 Temperature distribution

It is observed that the temperatures decrease in both the flows with the increase in M. But Venugopal and Bathaiah[6] have noticed that the temperature increases with the increase in M in the case of Couette flow and in the case of Poiseuille flow they have observed that the temperature distribution is not uniform, in that it is neither increasing throughout the field nor decreasing throughout. We have observed that the temperatures increase in both the flows with the increase in a whereas Venugopal and

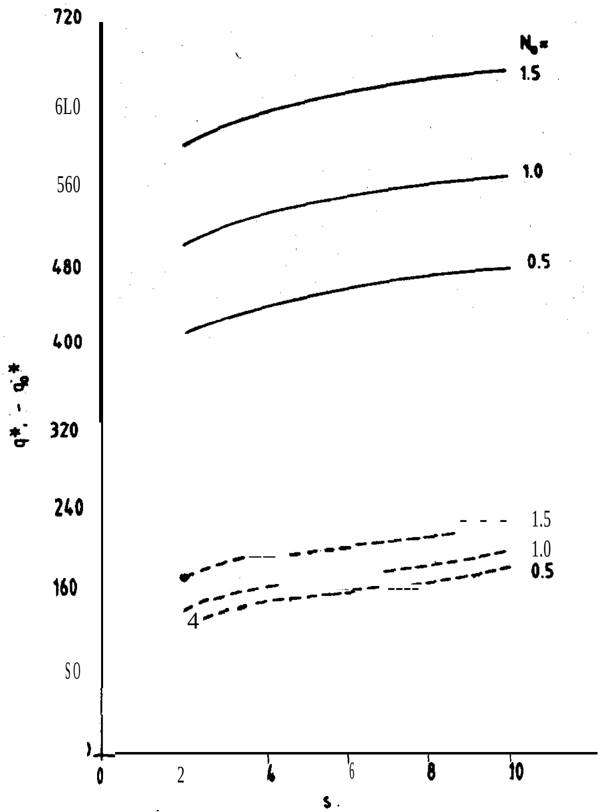

Figure 4. $q^*_1 - q^*_0$ against s for different values of N_0.

Bathaiah[6] have studied that the temperatures decrease with the increase in a in both the flows under study. The temperature decrease with the increase in s in both the flows which vary from the results of Venugopal and Bathaiah[6]. We have seen that the temperatures decrease with the increase in buoyancy parameter N_0 in both the flows. The temperatures decrease with the increase in P_rE in both the flows whereas the results of Venugopal and Bathaiah[6] are different from our results. We have seen that the temperatures with the increase in β in both the flows which are differing from the results of Venugopal and Bathaiah[6].

3.4 Rate of heat transfer

We have observed that the rates of heat transfer coefficients q and q_0 decrease with the increase in M or a which vary from the results obtained by Venugopal and Bathaiah[6]. The rates of heat transfer coefficients q and q_0 decrease with the increase in N_0 or s. But Venugopal and- Bathaiah[6] have shown that q and q_0 increase with the increase in s in both the flows under study. We have seen that the rates of heat transfer coefficients q and q_0 decrease with the increase in P_rE or β. This result is opposite to that of the result obtained by Venugopal and Bathaiah[6]. We have observed that the rates of

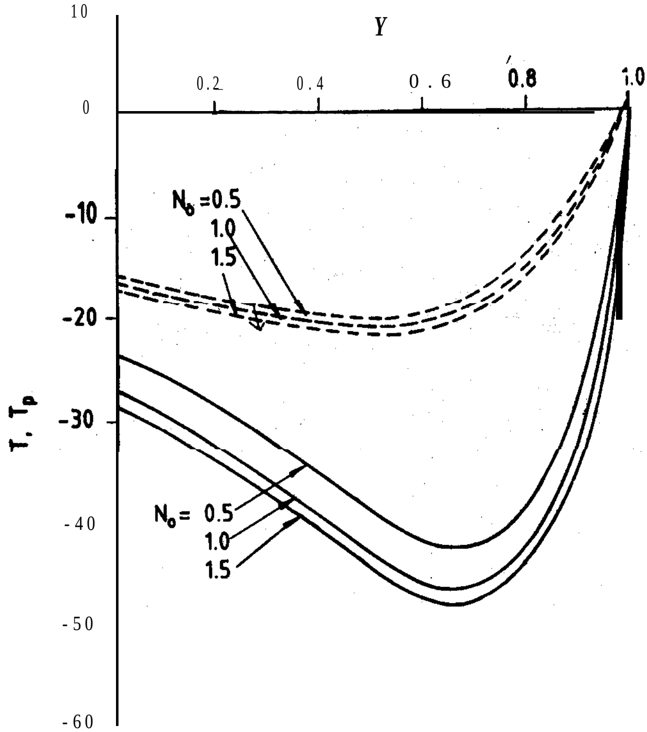

Figure 5. Temperature profiles for different values of No.

heat transfer coefficient q^* increases with the increase in M or a whereas q_0^* decreases with the increase in A4 or a. But Venugopal and Bathaiah[6] have shown that q^* decreases with the increase in M and q_0^* increases as M increases. We have seen that q^* increases with the increase in buoyancy parameter N_0 or s whereas q_0^* decreases with the increase in N_0 or s. We have noticed that q^* increases with the increase in $P_r E$ whreas q_0^* decreases. This result is opposite to that of the result obtained by Venugopal and Bathaiah[6]. Further, it is observed that q^* and q_0^* decrease with the increase in 'β'.

4. Zone-2

The flow region in the permeable bed immediately below the nominal surface. The flow in this region is governed by the Darcy law.

4.1 Part A : Couette flow

The nondimensional equations of momentum and energy are

$$\frac{d^3u}{dy^3} - b_1^2 \frac{du}{dy} = G \qquad (15)$$

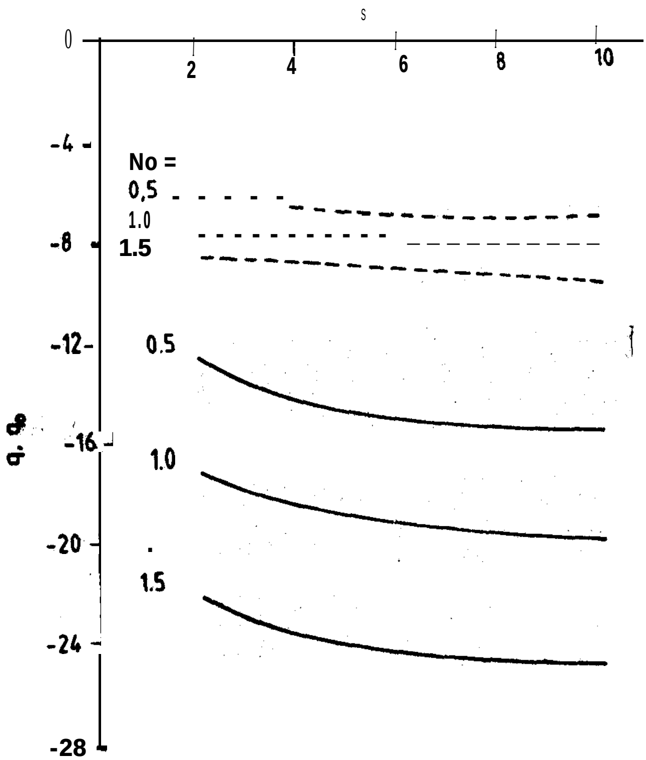

Figure 6. The rate of heat transfer coefficient against *s* for different values of N_0.

$$\frac{d^2T}{dy^2} - d^2T = P_r Rbu - P_r E\left\{\left(\frac{du}{dy}\right)^2 + b_1^2 u^2\right\} \quad (16)$$

where

$$b_1^2 = a^2 + M^2$$

The non-dimensional boundary conditions are

$$u = u_B, \frac{d^2u}{dy^2} - P_= T = T_B \text{ at } Y = 0 \quad (17a)$$

$$u = \frac{P}{a^2}, T = 0 \text{ at } y = -\frac{n}{h} = -n^* \quad (17b)$$

Using the, conditions (17) and solving the Eqn. (15), we obtain the velocity distribution.

$$u = \left(u_B + \frac{P}{b_1^2}\right) + \frac{1}{\text{Sinh}\, b_1 n^*}\left\{F_0 \text{Sinh}\, b_1 y - \frac{P}{b_1^2} \text{Sinh}\, b_1 (y + n^*)\right\}$$
$$- \frac{N_0}{b_1^2} y P \quad (18)$$

where

$$F_0 = u_B + \frac{P}{b_1^2} - \frac{P}{a^2} + \frac{N_0 P}{b_1^2} n^*$$

Using the conditions (17) and solving the Eqn. (16) we obtain the temperature distribution.

4.1.1 *Expression for the boundary layer thickness*

We know that, at the edge of the boundary layer, the shear stress has to be zero. In other words

$$\frac{du}{dy} = 0 \text{ at } y = -n^*$$

Therefore the expression for n^* is given by

$$n^* = \left| \frac{-L_2 \pm (L_2^2 - 4L_1 L_3)^{1/2}}{2L_2} \right| \tag{19}$$

where

$$L_1 = \frac{b_1^2}{2}\left(\frac{P}{a^2} - u_B - \frac{P}{b_1^2}\right)$$

$$L_2 = \frac{N_0 P}{b_1} - N_0 P$$

$$L_3 = -b_1^2\left(u_B - \frac{P}{a^2}\right)$$

Neglecting the order of 0 (n^{*3})

4.2 Part B : *Poiseuille Flow*

The velocity and temperature distributions and the expression for the thickness of the boundary layer are obtained from those of the Couette flow by taking u_0 to be zero. Thus Eqns. (15), (16) and (19) give the velocity, temperature and thickness of the boundary layer respectively for this flow of u_B, T_B and n^* are respectivly replaced by u^*_{B0} , T^*_B, and n^*_0, where n^*_0 is the boundary layer thickness in the flow under consideration.

References

1. Beavers, G. S. & Joseph, D. D, *J. Fluid Mech.*, *3* **(1967)**, 197.
2. Beavers, G. S., Sparrow, E. M., & Mangnuson, R. A., *Basic Engg. Trans., ASME*, *92* **(1970)**, 843.
3. Rajasekhara, B. M., *Ph. D. Thesis, Bangalore University*, (1974).
4. Saffman, P. G. *Appl. Math.*, *50* **(1971)**, 93
5. Rudraiah. N., Rajasekhara. B. M. & Ramaiah, B. K., *J. Math. and Phys. Sci.*, *9* **(1975)**, 49.
6. Venugopal, R., & Bathaiah, D., fief. *Sci. J.*, *33* **(1983)**, *69.*
7. Sparrow, E. M., Beavers, G. S., & Gregg, J. *I., Physics of Fluids 2* **(1959)**, 319.
8. Gill, W. N. & Casol, T. S., *A. I. ch E. J.*, **8 (1962)**, 513.
9. Sparrow. E. M., & Cess, R. D., *Trans. ASME, J., Appl. Mech.*, *29* **(1962)**, A. 181.