Def Sci J, Vol 36, No 1, January 1986, pp 45-56

A Model for the Lubrication Mechanism in Knee Joint Replacement

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Received 12 October 1983; revised 18 July 1984; re-revised 23 August 1985

Abstract. Analytical studies are presented for the understanding of the lubrication mechanism occuring in knee joint replacement under restricted motion. The idealised model has been shown to produce results consistent with those in normal situations. Effects of increase in viscoelastic parameter of the lubricant are similar to those of increase in the concentration of hyaluronic acid molecules in synovial fluid. Slip velocity occuring at the poroelastic boundary helps in normal functioning of the joints.

Notations

β	Nondimensional film thickness
a	Nondimensional nominal film thickness
A	Flexibility coefficient $\left(= \dfrac{1 - \nu_0^2}{EL}\right)$
ν_0	Poisson's ratio
E	Youngs modulus of elasticity
L	Length of the bearing material
x, Y	Nondimensional coordinate axes
u, v, $\bar{u}$, $\bar{v}$	Nondimensional velocity components in the fluid film and porous regions respectively
p, $\bar{p}$	Nondimensional pressure distribution in the fluid film and porous regions respectively

ϵ Viscoelastic parameter

t Time

W Load carrying capacity

ω Angular velocity

U_0 Approach velocity

1. Introduction

Friction and lubrication problems are mainly concerned with the flow and deformation of the lubricant between solid bodies as they slide, rotate or roll on each other. Pathological changes either in the synovial fluid or in articular cartilage can lead to a break down of joint lubrication and cause deterioration in parts or the whole joint[1,2]. Joint replacement is the treatment for failed joints. A large number of designs have emerged ranging from hinged to unconstrained configurations. Tn laboratories, attempts have been made to produce data for extrapolation to the human body. At present, most of the joint replacements are carried out at the hip, knee and fingers. It provides us to predict the life time of joint prosthesis and the ability to produce the effects of load or patient activity on the prothesis for example, a ball and socket for hip and a hinge configuration used for knee, elbow and thumb".

An effective joint replacement must give a suitable range of angular movement and allow transmission of force between the relevant skeletal system. The function of the artificial joint should be identical to that of the normal knee joint as closely as possible. The layers of the lubricant slip over each other. Hence, the prothesis must be surrounded by a flexible capsule in which the fluid is contained and the fluid circulates slowly and controls the temperature. Therefore, for smooth movement of the joints, the introduction of lubricant will be much helpful.

Reasons for the low friction and relatively no wear in the normal joints are not yet known. Attempts have also been made to produce data in laboratory for the low friction and wear of the joints by using various lubricants and material combination with a covering 'of porous, elastic or poroelastic layers. So far a life long prosthesis has not yet been devised. After a certain period, the fixation of a metal replacement to living bone show a strong tendency to destroy the bone resulting a degradation in the interface. It is probably the indication of wear and tear, hence it looses the fixation. Such discrepencies are prevented by separating the mating surfaces with a continuous running of lubricant probably under hydrodynamic conditions. But the hydrodynamic conditions may not always be possible in the joint cavity. Still the wear can be kept as low as possible by the use of lubricants and by suitable choice of the materials.

Present investigation is an attempt to derive some conclusions for simplified model for knee joint replacement under restricted motion during the stance or in the supporting phase during walking. The proposed model assumes a two region flow model, that is, the flow of viscous fluid in the poro-elastic matrix and the squeeze film lubrication in between the two arms of joint with viscoelastic fluid[3] as lubricant to represent the synovial fluid. The arms of the metal hinge are connected into the shafts of the femur and the tibia[4]. The lower arms is fixed whereas the upper one moves an angle $\pm 9°$ about the hinge as shown in Fig. 1. During the articulation the rigid arm of femoro-

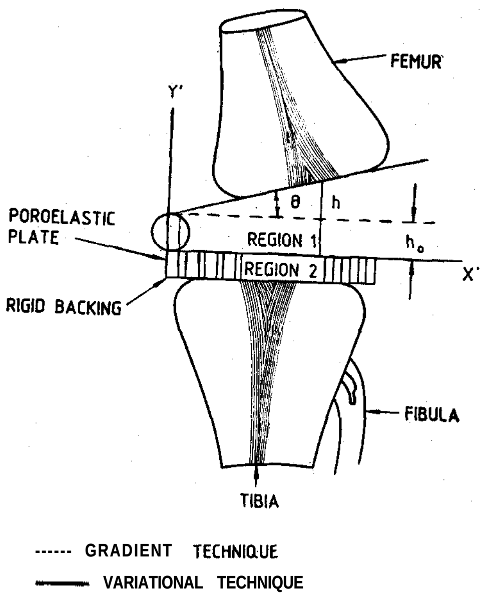

Figure 1. Diagrammatic representation of knee joint.

tibia junction moves through a small angle θ as well as approaches vertically towards the lower arm with a constant velocity $U_0 \left(= \frac{da}{dt} \right)$. The surface of lower arm is covered with a thin layer of poroelastic material, As the two arms approach each other, part of the suspending medium (viscous fluid) naturally enters into the poro-elastic matrix which, in turn, increases the concentration of suspended particles in the fluid film region. During this process, the pressure generated in the fluid would cause a deflexion in the elastic layer, which alters the film thickness of the fluid. Even if the fluid squeezed out fully, the soft elastic layer would act to entrap the lubricant between the mating surfaces. This may be one of the important reasons for low friction and relatively no wear during normal articulation. Thus, the model aims for an analytical study of the lubrication mechanism of a knee joint replacement during a walking cycle. This motivation is supported by the recent observations made by a group of

Japanese Scientists, Tsukamoto[5] et al. They have concluded that the low coefficient of friction in Charnley's prosthesis suggests that the fluid-film lubrication is partly present due to the elasticity of the plastics used for implant purposes. The problem has been analyzed by introducing a slip condition at the porous boundary as suggested by Beavers & Joseph[6]. It also introduces a poro-elastic thin layer on the tibia1 bone, whereas the upper rigid arm movable about the hinge is fixed on a rod inserted in the femur bone. It has been very recently noticed that the ingrowth of bone into the pores of material of the replacement makes it a perfect fixation, rather than getting it loosened in course of time as in the case of metal-on-metal or on rigid plastics'.

2. Formulation of the Problem

Referring to Fig. 1, the proposed model may be considered as two dimensional squeeze film lubrication between two approaching surfaces. The lower one is fixed and poro-elastic, the other is rigid and moveable about the hinge.

The expression for film thickness in a non-dimensional form is given by

$$\beta = a + x \tan\theta + A p = a + x\theta + A p \tag{1}$$

(assuming θ to be very small)

Last term in the right hand side of Eqn. (I) is due to the elastic dellexion of the lower surface*.

3. Governing Equations

3.1 Region-l

Following Oldroyd[8] and introducing the well known lubrication assumptions[9], the governing equations in the fluid film region may be written in non-dimensional form :

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2}\left[1 - 3\epsilon\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{2a}$$

and the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2b}$$

3.2 Region-2

The flow of viscous fluid in a porous matrix is, governed by the nondimensional equation :

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{3}$$

The velocity components $\bar{u}$, $\bar{v}$ are defined as

$$\bar{u} = -\frac{\phi}{\alpha}\frac{\partial \bar{p}}{\partial x} \tag{4}$$

$$\bar{v} = -\frac{\phi}{\alpha}\frac{\partial \bar{p}}{\partial y} \tag{5}$$

Which satisfies the Laplace equation :

$$\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} = 0 \tag{5a}$$

3.3 Boundary Conditions

3.3.1 *Fluid film region*

$$\begin{aligned} u &= -\sigma \frac{\partial u}{\partial y} && \text{at } y = 0 \\ u &= x\,\omega\,\theta && \text{at } y = \beta \\ v &= \frac{d\beta}{dt} && \text{at } y = \beta \\ v &= -\frac{\phi}{\alpha}\frac{\partial \bar{p}}{\partial y} && \text{at } y = 0 \\ p &= 0 && \text{at } x = 0 \text{ and } 1 \end{aligned} \tag{6}$$

3.3.2 *Porous region*

$$\bar{p} = 0 \qquad \text{at } x = 0 \text{ and } 1$$

and

$$\left.\frac{\partial \bar{p}}{\partial y}\right|_{y=-\alpha} = 0 \tag{7}$$

and the matching condition

$$p(x) = \bar{p}(x, 0) \tag{7a}$$

3.4 *Analysis*

The first integral of Eqn. (2) with respect to y is given by :

$$\frac{\partial p}{\partial x} y + C = \frac{\partial u}{\partial y}\left[1 - \epsilon\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{8}$$

where C is a dimensionless constant of integration,

To obtain solutions for the fluid film region, perturbation technique is applied, which is based on the following assumptions :

(i) Restricted the solution for small values of ϵ.

(ii) In the limit of $\epsilon \to 0$, we expect to obtain the corresponding solution for viscous lubricants. Hence, for an approximate solution, the variables are assumed in a sequence of the functions :

$$u = u_0 + \sum_{k=1}^{\infty} \epsilon^k U_k \tag{9}$$

where u_0 is the limiting solution for viscous fluids as $\epsilon \to 0$.

To carry out the perturbations, we introduce the variables u, v, p, c in the above form in Eqn. (8) and collect the coefficients of zeroth and first in order in ϵ. We obtain the following differential equations :

$$\frac{\partial p^{(0)}}{\partial x} y + c^{(0)} = \frac{\partial u^{(0)}}{\partial y} \tag{10}$$

$$\frac{\partial p^{(1)}}{\partial x} y + c^{(1)} = \left[\frac{\partial u^{(1)}}{\partial y} - \left(\frac{\partial u^{(0)}}{\partial y}\right)^3\right] \tag{11}$$

Since, the boundary conditions (6) must hold for all values of ϵ, if follows that

$$u^{(0)} = -\sigma \frac{\partial u^{(0)}}{\partial y} \quad \text{at } y = 0$$

$$u^{(0)} = x\,\omega\,\theta \quad \text{at } y = \beta$$

$$u^{(1)} = u^{(2)} = u^{(3)} \ldots = 0 \quad \text{at } y = 0 \text{ and } \beta$$

$$v^{(0)} = -\frac{\partial \beta}{\partial t} \quad \text{at } y = \beta$$

$$v^{(0)} = \frac{\phi}{\alpha} \frac{\partial \bar{p}^{(0)}}{\partial y} \quad \text{at } y = 0 \tag{12}$$

$$v^{(1)} = v^{(2)} = v^{(3)} \ldots = 0 \qquad \text{at } y = 0 \text{ and } \beta$$

$$p^{(0)} = p^{(1)} = p^{(2)} \ldots = 0 \qquad \text{at } x = 0 \text{ and } 1$$

Zeroth and first order velocity distributions are

$$u^{(0)} = \frac{1}{2} \frac{\partial \bar{p}^{(0)}}{\partial x} \left[y^2 + \frac{\beta^2 (y - \sigma)}{(\sigma - \beta)} \right] - \frac{x \omega \theta (y - \sigma)}{(\sigma - \beta)} \tag{13}$$

$$u^{(1)} = \frac{1}{2} \frac{\partial p^{(1)}}{\partial x} y (y - \beta) - \left[\frac{y}{\beta} (I_{y=\beta} - I_{y=0}) + I_{y=0} - I \right] \tag{14}$$

where

$$I = \int \left(\frac{\partial u^{(0)}}{\partial y} \right)^3 dy$$

Now, substituting $u^{(0)}$ and $u^{(1)}$ in the zeroth and first order continuity equation, we get the following differential equation :

$$\frac{da}{dt} + \frac{\phi}{\alpha} \frac{\partial \bar{p}^{(0)}}{\partial y} \Big|_{y=0} + \omega \theta = - \frac{\partial}{\partial x}$$

$$\left[\frac{\partial p^{(0)}}{\partial x} \left(\frac{\beta^3}{3} \right) \left(1 - 3 \frac{2\beta - 2\sigma}{\sigma - \beta} \right) \right] - \left[\frac{(\beta^2 - 2\sigma\beta)}{(\sigma - \beta)} \omega \theta x \right] \tag{15}$$

and

$$\frac{\partial}{\partial x} \left[\int \left\{ \frac{1}{2} \frac{\partial p^{(1)}}{\partial x} y (y - \beta) \left(\frac{y}{\beta} (I_{y=\beta} - I_{y=0}) \right. \right. \right.$$

$$\left. \left. \left. + I_{y=0} - I \right) \right\} dy \right] = 0 \tag{16}$$

Assuming, the poro-elastic bearing thickness is very small compared to the length of the bearing, it is reasonable to assume that $\frac{\partial \bar{p}}{\partial y}$ is linear across the bearing matrix[10].

Thus, we represent

$$\frac{\partial p}{\partial y} = G y + M \tag{17}$$

Where G = G (x) and $M = M(x)$.

Substituting this in Eqn. (5), we get

$$\bar{v}_{y=0} = -\frac{\phi}{\alpha}\frac{\partial \bar{p}}{\partial y}\bigg|_{y=0} \tag{18}$$

From Eqns. (5a), (18) and (7a), we get

$$\bar{v}\bigg|_{y=0} = +\frac{\phi}{\alpha}\frac{\partial^2 p^{(0)}}{\partial x^2}\bigg|_{y=0} \tag{19}$$

Substituting $\bar{v}$ from Eqn. (19) in Eqn. (15), we get the following equation for $\bar{p}^{(0)}$,

$$\frac{\partial}{\partial x}\left[\frac{\partial p^{(0)}}{\partial x}\left(\frac{\beta^3}{3}\right)\left\{1-\frac{3(\beta-2\theta)}{(\sigma-\beta)}+\frac{\phi}{\alpha}\right\}\right.$$
$$\left.-\frac{(\beta^2-2\sigma\beta)}{(\sigma-\beta)}\omega\,\theta\,x\right]+\frac{da}{dt}+\omega\,\theta=0 \tag{20}$$

The pressure distribution for the zeroth order in the fluid film region is obtained in the following manner :

The lubrication process is divised into two stages : (a) initial stage, and (b) elastic deformation stage. In the initial stage both surfaces are rigid and hence the film thickness does not vary with position. In the elastic deformation stage, the lubricant film thickness vary with elastic deflection of the surfaces.

An approximate solution of the Eqn. (19) is 'obtained by improving the solution by iteration procedure. Following the similar technique for the first order pressure distribution $[p^{(1)}]$, we get the solution for pressure distribution.

$$p = p^{(0)} + \epsilon\, p^{(1)} \tag{21}$$

3.5 Load Capacity

The load capacity of the bearing material is found by integrating the fluid film pressure. Then W is

$$W = \int_0^1 (p^{(0)} + \epsilon\, p^{(1)})\, dx \tag{22}$$

3.6 Closure Time

For a particular load W, the expression for time of approach for the angle is given by

$$t = \frac{1}{\bar{W}}\int_{-\theta_1}^{\theta} W d\theta$$

Where

$$\theta_1 = 9° = \pi/20 \text{ radians.}$$

The results of the analysis are given in Tables 1 to 3 and in Figs. 2 to 3.

4. Results and Discussions

Table 1 to 3 present the variation of load carrying capacity for various values of slip parameter (σ), porosity (ϕ), flexibility coefficient (A), femoro-tibial angle (θ), minimum film thickness (a), and the angular velocity (ω).

Table 1 depicts that the load carrying capacity of the tibial replacement increases as the femoro-tibial angle decreases from +9° to -9°. These results are in good

Table 1. Variation of load capacity W with σ (slip parameter)
For $a = 0.2$, $\phi = 0.00001$, $\omega = 0.01$, $\epsilon = 0.01$, $A = 10^{-8}$

σ/θ	-0.157	-0.104	-0.051	0	0.01	0.05	0.157
0.5	1.63812	1.39712	1.01812	0.88721	0.84143	0.74216	0.62107
1.0	1.66812	1.48291	1.23817	0.97372	0.92832	0.85721	0.69728
1.5	1.70121	1.59281	1.38927	1.09428	0.99281	0.89138	0.78123
2.0	1.80128	1.72171	1.53281	1.2937	1.01372	0.97250	0.86738

Table 2. Variation of load capacity with ω and σ
For $\epsilon = 0.05$, $\theta = -0.051$, $A = 10^{-8}$, $\phi = 0.00001$, $a = 0.3$

σ/ω	0.1	0.5	0.9
0.5	1.02312	1.09671	1.20127
1.0	1.09261	1.16821	1.24587
1.5	1.20182	1.26317	1.33272
2.0	1.28617	1.35272	1.42172

Table 3, Time of approach for the angle ($d\theta = 0.053$)
For $a = 0.2$, $\phi = 0.00001$, $\sigma = 0.5$, $A = 10^{-8}$

ϵ/θ	0.055 to 0.002	0.002 to 0.051	0.051 to 0.104	-0.104 to 0.157
0.02	2.1812	2.9913	3.48188	3.94183
0.03	4.0821	5.0123	5.9823	6.4823
0.04	6.0128	7.0012	7.9932	8.2867

agreement with that of normal situation in the knee joint'. We observe from Table 1 that as femoro-tibia1 angle decreases, load capacity increases. Further, we also observe that as the slip velocity at the porous boundary increases, the load capacity increases, This is in good agreemedt with Beavers et al[6]. Thus, we may conclude that the slip velocity existing at the boundary is a contributing factor for the easy functioning of human joints.

Table 2 shows that the load carrying capacity also increases with the angular velocity w. Table 3 describes that the closure time increases with the viscoelastic parameter ϵ and we also notice that the time taken for the closure of a particular angle increases as the gap decreases.

Figure 2 depicts the variation of load capacity with the flexibility coefficient and porosity. We can observe from the figure **that** the **load capacity** increases as the flexibility coefficient increases and porosity decreases. It may be concluded that during the normal functioning of the joint, only very fine particles can get into pores of the cartilage, leaving the suspension of enriched large molecules of hyaluronic acid. These molecules, in turn, increase the concentration and thus increase the apparent viscosity of synovial fluid. ***So*** it can sustain more load.

Figure 3 shows that the effect of viscoelastic pararmeter $\epsilon > 0$ for a particular gap is to increase the load capacity indicating a positive effective of the increase in **concen-**

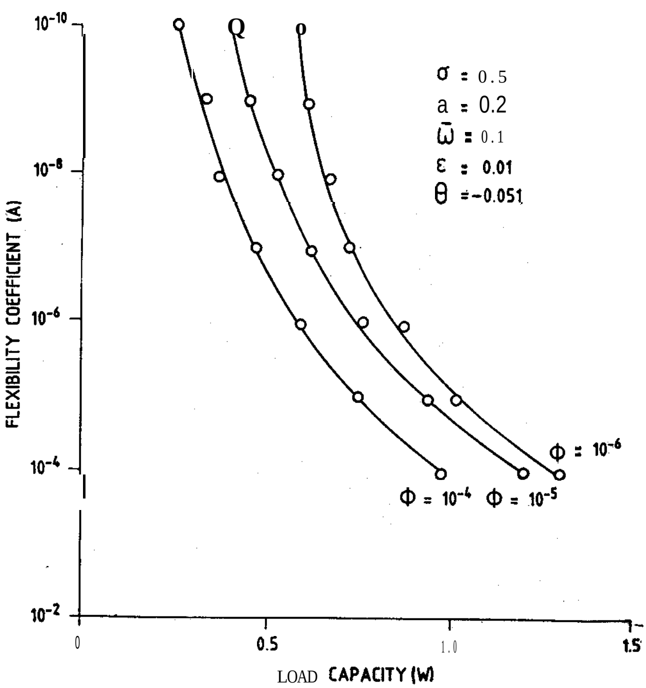

Figure 2. Variation of load capacity W with flexibility coefficient A & ϕ.

tration of suspended particles in the lubricant consistant to the observations of Dowson et al[11]. The increase in the viscoelasticity of the lubricant depends directly on the increase in concentration of the suspended particles.

Thus, it may be concluded that the elasticity of the cartilage plays a dominant role in maintaining a soft surface layer in between the matrix surfaces of the joints. It is also concluded that at the closing end of the walking cycle, the slip velocity existing at the poro-elastic boundary plays an important role of self adjusting nature of human joints.

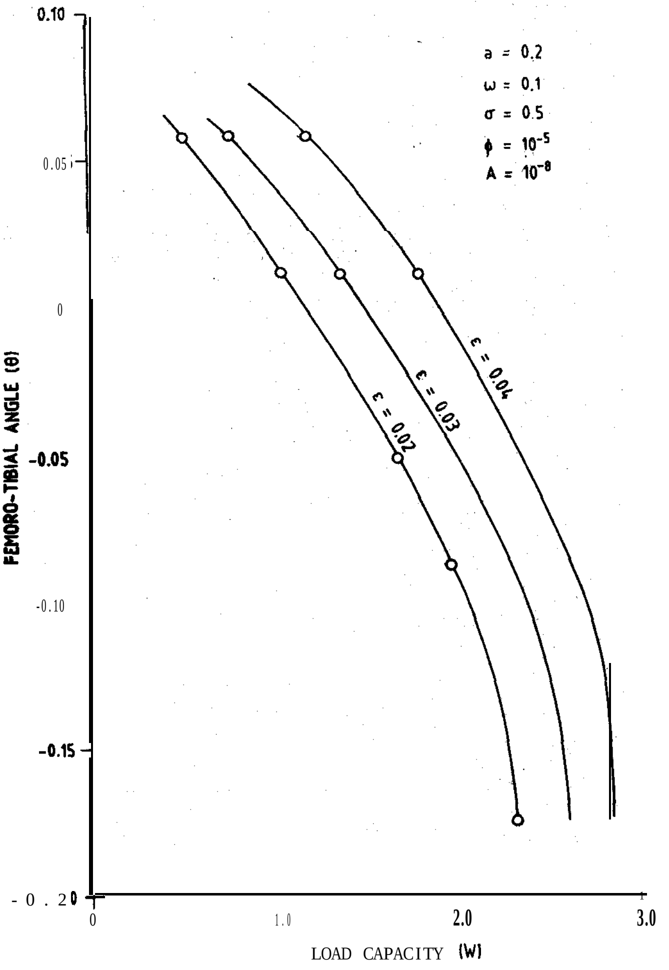

Figure 3. Variation of load capacity with θ & ε.

Acknowledgement

Authors are grateful to the Indian Council of Medical Research for the assistance at the final stage of this work.

References

1. Dintenfass, L., *J.* Bone ft. *Surg. Am.*, **45 (1963)**, 1241.
2. Adams, **J.** C., 'Outline of Qrthopaedics' (English Language Book Society, Edinburg), 1967.
3. **Oldroyd,** J. G., *Proc.* Roy. *Soc.* London, Ser A, 4 **(1958)**, 1278.
4. **Morrison,** J. **B.**, *J. Biomeck.*, *3* **(1970)**, 5.
5. Tsukamoto, Y., Yamemoto, M., Maberchi, K., Morita, M. & Sasadana, T., *J. Jap. Orthop. Association*, *57* **(1983)**, 91-98.
6. Beavers, G. S. & Joseph, D., *J. Fluid Mech.*, **30 (1967)**, 187.
7. **Schaldach,** M. & Hohmann, D., 'Advances in Knee and Hip Joint Technology' (Springer **Verlag)**, 1976, 422-433.
8. Higginson, G. R. & Norman, R., *J. Mech. Eng. Sci.*, *16* **(1974)**, 250.
9. **Pinkus,** O., 'Theory of Hydrodynamic Lubrication' (McGraw Hill, New York), 1961.
10. Conway, H. D. & Lee, H. C., Trans. *ASME, Jr. Lub. Tech.*, **99F (1977)**, 376.
11. **Dowson,** D., Unsworth, A. & Wright, V., *J. Mech. Eng. Sci.*, *12* **(1978)**, 364.