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Lognormal Distribution of Some Physiological Responses in Young Healthy Indian Males

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Abstract. Evaluation of statistical distribution of physiological responses is of fundamental importance for better statistical interpretation of physiological phenomenon, In this paper, statistical distribution of three important physiological responses viz., maximal aerobic power ($\dot{V}O_2$ max), maximal heart rate (HR max) and maximum voluntary ventilation (MVV) in young healthy Indian males of age ranging from 19 to 22 years have been worked out. It is concluded that these three important physiological responses follow the lognormal distribution.

1. Introduction

The problem of statistical distribution of a variable has been of great interest **to** applied statisticians engaged in biometric research. The distribution of height and weight of human beings has been worked out by Yuan[1] who used the lognormal distribution for weight and normal distribution for height. Camp[2] reported the distribution of height and some other anthropometric measurements to be lognormal. Gaddum[3] pointed out the existence of lognormal distribution in biological data and recommended the use of logarithmic transformation as a routine. Fox *et al*[4] suggested the distribution of initial sweat losses to be lognormal. Recently Verma et *al*[5,6] used lognormal distribution for physical work capacity ($\dot{V}O_2$ max) and some other body measurements. Perhaps, no attempt has yet been made to work out the statistical. distribution of physiological responses during maximal effort like maximal heart rate and maximum voluntary ventilation. This paper deals with the distribution of two important physiological responses. The distribution of $\dot{V}O_2$ max based on the present data of 320 subjects has also been worked out to validate our earlier finding about its distribution based on lesser number of subjects.

2. Material and Methods

A simple random sample consisting of 320 young healthy Indian males (19-22 yrs) of mixed ethnic origin was selected for this study. Mean height and weight of these subjects were 167.1 cm ($\pm$ 4.72) and 52.2 kg. ($\pm$ 3.92) respectively. Maximal aerobic power ($\dot{V}O_2$ max) was estimated by using the continuous method as described by Joseph et al[7]. Maximal heart rate (HR max) was recorded with an ECG using MX lead during maximal effort and maximum voluntary ventilation (MVV) was measured by the standard physiological procedures described by Sen Gupta et al[8].

The normality of the data on these three physiological responses were tested by the standard procedure[9] using the statistics

$$\omega_1 = \pm \left[\frac{\beta_1 (n+1)(n+3)}{6(n-2)}\right]^{\frac{1}{2}}$$ for testing the asymmetry and

$$\omega_2 = \left(\beta_2 - 3 + \frac{6}{n+1}\right)\left[\frac{(n+1)^2 (n+3)(n+5)}{24n(n-2)(n-3)}\right]^{\frac{1}{2}}$$ for testing the departure of kurtosis from normal distribution.

Croxton et al[10] have referred a logarithmic coefficient of skewness as

$$\text{Sk log} = \frac{\log Q_1 + \log Q_3 - 2 \log Q_2}{\log Q_3 - \log Q_1}$$

and have pointed out that a series which yields this logarithmic coefficient of skewness less than 0.15 (or perhaps even 0.20) may tentatively be considered as logarithmically normal.

Following the procedure of Aitchison and Brown" mean (a), variance (β^2), coefficient of skewness γ_1 (x), coefficient of kurtosis γ_2 (x) for lognormal distribution have been calculated from the mean (μ) and variance (σ^2) of the transformed variate $Y = \log x$.

Thus,

$$\alpha = e^{\mu + \frac{1}{2}\sigma^2}$$

$$\beta^2 = e^{2\mu+\sigma^2} (e^{\sigma^2} - 1)$$

$$= \alpha^2 \eta^2$$

The coefficient of variation (η) is given by

$$\eta^2 = (e^{\sigma^2} - 1)$$

Third and fourth moments about the mean are

$$\lambda_3 = \alpha^3 \, (\eta^6 + 3\eta^4)$$

and

$$\lambda_4 = \alpha^4 \, (\eta^{12} + 6\eta^{10} + 15\,\eta^8 + 16\eta^6 + 3\,\eta^4)$$

Coefficient of skewness $\gamma_1(x)$ and coefficient of kurtosis $\gamma_2(x)$ are given by

$$\gamma_1(x) = \lambda_3 \,/\, \beta^3 = \eta^3 + 3\eta$$

$$\gamma_2(x) = \lambda_4 \,|\, \beta^4 - 3 = \eta^8 + 6\eta^6 + 15\eta^4 + 16\eta^2$$

3. Results

After logtransformation of the data on three physiological responses each consisting of 320 observations, the mean (μ) and variance (σ^2) were calculated. The values of α, β^2, η, γ_1, (x) and $\gamma_2\,(x)$ were calculated and presented in Table 1. It was observed that skewness differed significantly and kurtosis did not differ significantly from that of normal distribution in these three physiological responses. The logarithmic coefficient of skewness were obtained to be 0.0348, 0.0439 and 0.0284 for maximal aerobic power, maximal heart rate and maximum voluntary ventilation respectively. All these coefficients were smaller than 0.15 indicating the distribution of three physiological responses to be lognormal.

Table 1. μ, σ^2, α, β^2, η_1, $\gamma_1(x)$, $\gamma_2(x)$ and Sk log for three physiological **responses.**

Statistics	Maximal aerobic power ($\dot{V}O_2$ max) l min^{-1}	Maximal heart rate (HR max) beats min^{-1}	Maximum voluntary ventilation (MVV) l min^{-1}
μ	0.8025	5.2762	4.9190
σ^2	0.0140	0.0013	0.0226
a	2.247	196	138.43
β^2	0.0712	51.3	438.49
η	0.1187	0.0366	0.1513
$\gamma_1(x)$	0.3579	0.1098	0.4573
$\gamma_2(x)$	0.2286	0.0214	0.3740
Sk log	0.0348	0.0439	0.0284

The quality of fit were found to be good for these physiological responses as tested by x^2 test (Tables 2-4). Graphical representation (Figs. 1-3) of the data on these three physiological responses also suggest their distribution to be lognormal.

Table 2. Goodness of fit for the distribution of maximal aerobic power ($\dot{V}O_2$ max)

$\dot{V}O_2$ max (l min^{-1}) class interval	Observed frequency 0	Expected frequency E	
Below 1.7	0	3.424	
1.7—1.8	8	7.808	
1.8-1.9	17	16.576	
1.9-2.0	38	29.408	
2.0-2.1	41	40.384	
2.1-2.2	49	47.104	
2.2-2.3	38	48.128	
2.3-2.4	42	41.536	
2.4-2.5	28	31.712	
2.5-2.6	25	22.400	
2.6-2.7	16	14.336	
2.7-2.8	7	8.416	
2.8-2.9	6	4.544	(χ^2 = 7.4092)
2.9-3.0	3	2.240	(df = 9)
3.0-3.1	2	1.120	
Above 3.1	0	0.864	
Total	320	320	

Table 3. Goodness of tit for the distribution of maximal heart rate (HR max)

HR max (beats min^{-1}) class interval	Observed frequency 0	Expected frequency E	
Below 186	29	26.816	
186-190	45	41.920	
190-194	59	62.144	
194-198	64	70.496	
198-202	64	58.016	
202-206	32	35.712	(χ^2 = 2.4908)
206-210	17	16.512	(df = 5)
210-214	8	6.112	
214-218	2	1.792	
Above 218	0	0.480	
Total	320	320	

Table 4. Goodness of fit for the distribution of maximum voluntary ventilation **(MVV)**

MVV (l min^{-1}) class interval	Observed frequency 0	Expected frequency *E*	
Below 80	0	0.064	
80—90	2	0.768	
90-100	3	5.024	
100-110	24	17.664	
110-120	35	37.984	
120 - 130	50	55.904	
130-140	63	61.664	
140-150	54	54.240	
150-160	42	38.944	
160-170	25	23.776	
170-180	13	12.960	
180-190	6	6.336	(χ^2 = 3.9550)
190-200	1	2.784	(*df* = 7)
200-210	2	1.184	
Above 200	0	0.704	
Total	320	320	

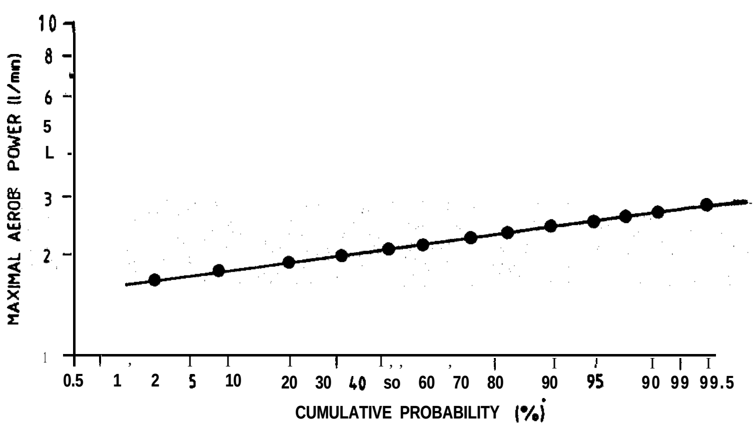

Figure 1. Logarithmic probability graph for maximal aerobic power.

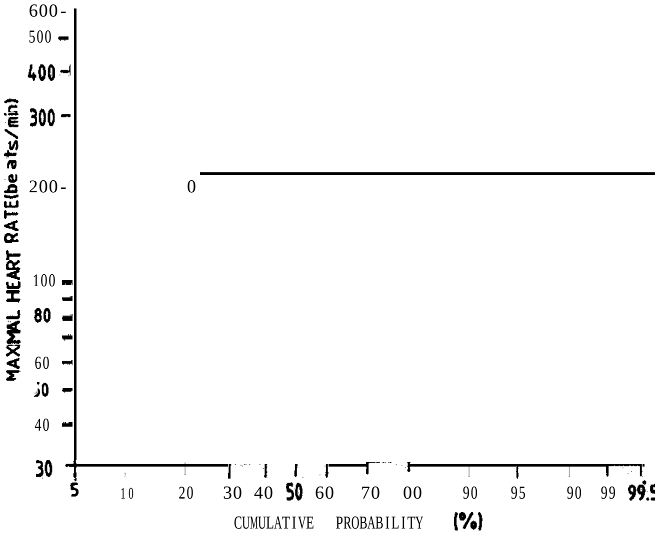

Figure 2. Logarithmic probability graph for maximal heart rate.

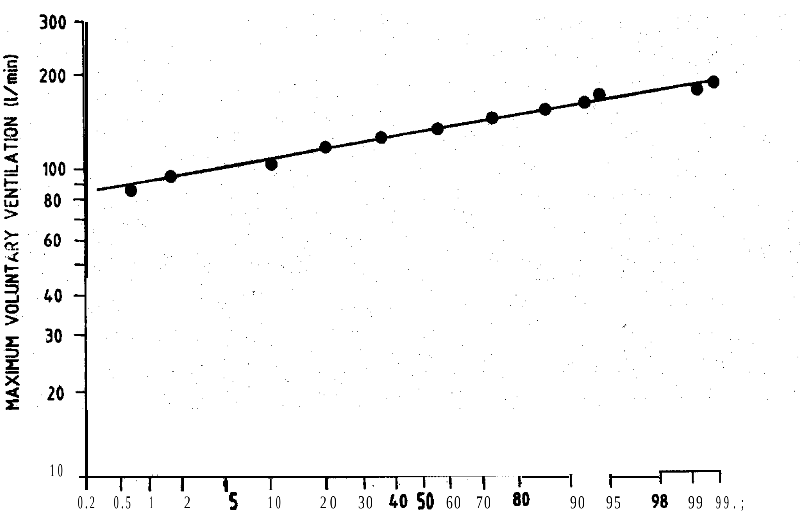

Figure 3. Logarithmic probability graph for maximum voluntary ventilation.

4. Discussion and Conclusion.

Lognormal distribution is a fundamental distribution in statistics as it arises from the theory of elementary errors combined by a multiplicative process, just as the normal distribution arises from a theory of elementary errors combined by addition.

There are many situations in nature where it is more reasonable to suggest that the process underlying change or growth is multiplicative rather than additive. The problem here is formally similar to that of the choice of the geometric or the arithmetic mean as the more appropriate measure. of location. This paper shows that the maximal aerobic power, maxima1 heart rate and maximum voluntary ventilation are multiplicative processes as they follow the lognormal distribution and hence the geometric mean will be more appropriate measure of location for these physiological responses. These responses have significant departure of skewness and insignificant departure of kurtosis from that of normal distribution. These findings are in agreement with the results conjectured by Rao[9] regarding the distribution of biometric measurements. The distribution of $\dot{V}O_2$ max also agrees well with our earlier finding;. Attempts have also been made to evaluate the statistical distribution of some important physiological responses on exposure to heat[12-14]. Lognormal distribution has also been applied in applied biology[15-20] for better interpretation of results. Therefore, for better interpretation of physiological data, a knowledge of its distribution pattern is essential.

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