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Penetration Dynamics of Earth Penetration Warhead into Composite Target Media

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ABSTRACT

Attempts have been made to develop a suitable computer code that can find solutions to the axi-symmetric penetration of an Earth Penetrating Warhead yielding complete space-time histories of the resistive force offered by the target medium. The consequent warhead deceleration and velocity reduction, the resulting axial compressive stress developed in warhead casing as the penetration process progresses into the composite target media consisting of hard concrete of specified thickness followed by earth soil have been discussed.

1. INTRODUCTION

The problem of warhead penetration into the concrete or earth soil or concrete backed by earth-soil has attracted the attention of the weapon designers for a long time. The design and deployment of Earth Penetrating Warhead (EPW) requires study and investigation of various aspects of warhead and target interaction. The information on the space-time histories of the resistive force offered by target medium, consequent warhead deceleration and velocity profile and also the resulting axial compressive stresses developed in EPW casing during the penetration process into the composite target media will be of immense help in designing EPWs and their fuzing systems. This paper describes penetration dynamics code to find solutions to the axi-symmetric penetration of an EPW yielding the above information.

2. ASSUMPTIONS

The following assumptions are made in the penetration dynamics code (PEN) reported in this paper:

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(a) Impact angle of warhead is 90 degrees with respect to the horizontal, i.e. normal impact.

(b) Resistive pressure of the target medium causes all points under warhead to suffer an equal deceleration at any given instant, i.e. the warhead is considered as a rigid body comprising of metallic casing which is ogival from the nose to shoulder and cylindrical from the sholuder to the base. (Fig. 1)

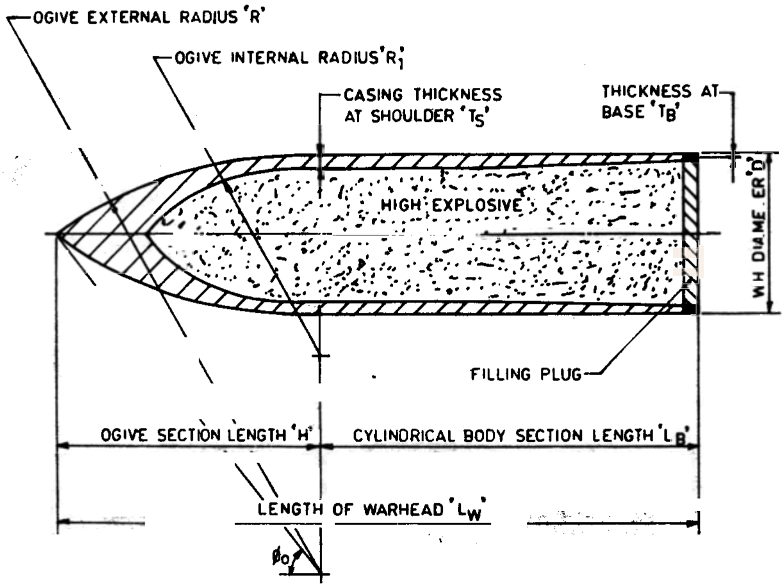

Figure 1. Elements of a typical earth penetrating warhead(EPW).

3. PENETRATION INTO TARGET MEDIUM

On impact, a warhead imparts load to the target medium (i.e. concrete/earth), thus leading to the overall response of the target medium. High resistive contact pressure is generated at the interface between the warhead and the penetrating target medium. If the warhead body is capable of withstanding this pressure, it advances into the target. The warhead comes to rest when its kinetic energy is totally absorbed in overcoming the resistive pressure of the target medium.

3.1 Estimation of Resistive Pressure of Target Medium

Resistive pressure offered by the target medium during the penetration of warhead an be estimated by means of an equation derived by differentiating the penetration function. The review study of various empirical formulae available in published literature[1-3] for predicting penetration depth into concrete medium, the following formula[3] is considered to be most suitable.

$$Z_c = \frac{4\ N_c\ W}{\pi\ \rho_c\ D^2}\left[\ \frac{1}{3} V_o \sqrt{\frac{\rho_c / g}{S_{cm}}}\ -\frac{4}{9} ln\left(1 + \frac{3}{4} V_o \sqrt{\frac{\rho_c / g}{S_{cm}}}\right)\right] \qquad (1)$$

Wherein

Z_c	=	total penetration depth into concrete (cm/s^2)
W	=	warhead weight (kgf)
ρ_c	=	density of concrete (=2.3 × 10^{-3} kg/cm^3)
D	=	warhead diameter (cm)
V_o	=	striking velocity of warhead (cm/s^2)
g	=	acceleration due to gravity (cm/s^2)
N_c	=	nose shape factor for concrete
S_{cm}	=	modified concrete strength (kgf/cm^2)

in which

$$N_c = 0.863\left[\frac{4(\ CRH)^2}{4(CRH\ )-1}\right]^{0.25} \quad \text{and} \quad S_{cm} = S_c\left(\frac{3c}{D}\right)^{0.2}$$

Where in

S_c = cube compressive strength of concrete
350–450 (kgf/cm^2)

C = maximum size of aggregate in concrete (2 cm)

At any instant during penetration process, the distance Z that the warhead has already travelled inside the concrete target is related to its instantaneous velocity V at that point by the relation :

$$Z = A\left[\ \frac{1}{3} VB\ -\frac{4}{9} ln\left(1 + \frac{3}{4} VB\right)\right] \qquad (2)$$

wherein

$$A = \frac{4\ N_c\ W}{\pi\ \rho_c D^2} \quad \text{and} \quad B = \sqrt{\frac{\rho_c / g}{S_{cm}}}$$

As Z increases, V decreases. In order to obtain the deceleration of the warhead as it penetrates more and more into the target, the above Eqn. (2) is differentiated with respect to time t.

Thus,

$$\frac{dZ}{dt} = A\left[\ \frac{1}{3} B \frac{dV}{dt} - \frac{4}{9}\ \frac{\frac{3}{4} B \frac{dV}{dt}}{1 + \frac{3}{4} V B}\right]$$

or

$$\frac{dZ}{dt} = \frac{dV}{dt} A\, B^2 \left[\frac{V}{4 \left(1 + \quad V B \right)} \right] \tag{3}$$

Since $\frac{dZ}{dt}$ is the velocity V and $\frac{dV}{dt}$ is the deceleration, the above Eqn. (3) can be written as

$$\frac{dV}{dt} = \frac{4}{A\, B^2} \left(1 + \frac{3}{4} V B \right) \tag{4}$$

Thus for a warhead of weight W penetrating into concrete at an instantaneous velocity V the momentary resistive force F_r is given by $\frac{W}{g}\frac{dV}{dt}$ which becomes

$$F_r = \frac{W}{g}\frac{dV}{dt} = \frac{W}{g}\frac{4}{A\, B^2} \left(\quad + \frac{3}{4} V B \right)$$

Substituting for A and B

$$F_r = \frac{\pi\, D^2}{4 N_c} \left[4 S_{cm} + 3 V \quad \sqrt{\frac{\rho_c S_{cm}}{g}} \right]$$

This resistive force is maximum when instantaneous velocity V is equal to the striking velocity V_o. At any instant during the penetration process, the momentary resistive pressure acting on the warhead will be given by

$$P_c = \frac{F_r}{A_i} = \frac{4 S_{cm} + 3 V \sqrt{\rho_c S_{cm} / g}}{N_c} \tag{5}$$

Where A_i is the instantaneous area of the warhead section at interface. If V is expressed in m/s and P_c in kgf/cm then

$$P_c = \frac{4 S_{cm} + 300 V \sqrt{\rho_c S_{cm} / g}}{N_c} \tag{6}$$

Similarly, by differentiating the empirical formula predicting depth into the earth soil[4], the resistive pressure of the soil (Z_s) acting on the warhead surface can be btained as

$$P_s = 21.49 \frac{S_s 0.5\, V^{0.75}}{N_s D^{0.69}} \tag{7}$$

in which N_s is the nose shape factor for soil and is given by

$$N_s = 0.72 + \left[(CRH)^{2.72} / 1000 \right] \tag{8}$$

S_s = Compressive strength of soil (kgf/cm)

Thus, the resistive pressure offered by target media (concrete or soil) depends on striking velocity of the warhead, the compressive strength of target medium, and the warhead calibre but it is independent of the warhead weight.

4. INSTANTANEOUS VELOCITY AND DECELERATION OF WARHEAD

Consider a warhead of diameter D with a body length L and a ogive radius R (as shown in Fig. 1) has penetrated upto a depth z into the target medium (Fig. 2). The resistive pressure P offered by the target medium is acting hydrostatically over the part of the ogive BAC, is defined by the angle ϕ, i.e. BC represents the surface of target at the particular instant under consideration. If r is the radius of the warhead at that particular section BC and if the nose of the warhead be defined by the angle ϕ_o, then by geometry (with reference to Fig. 2) we get

$$D/2 = R\,(\quad - \mathrm{Sin}\,\phi_o) \tag{9}$$

and

$$r = R\,(\mathrm{Sin}\,\phi - \mathrm{Sin}\,\phi_o)$$

As the warhead goes on penetrating into the target medium, it gradually looses its velocity. The resistive pressure of the target media exerts a net axial force on the warhead and hence a net acceleration G in the direction opposite to the direction of motion of the warhead into the target medium. To convert the problem into quasi-static one, a gravitation field producing an equal and opposite acceleration must be superimposed when the problem reduces to that of the warhead equilibrium under a net upward force produced by the target resistive pressure ($P \times A$) and a downward gravitation force ($W \times G$) acting on the warhead.

The magnitude of the deceleration G may be obtained from the following force equation

$$W\,G - P\,A_i = 0$$

or

$$G = \frac{PA_i}{W}$$

wherein

A_i = area of the warhead section under consideration at the interface.

G = deceleration (in terms of g) of the warhead at the instant when the warhead has penetrated upto the section at a distance Z from the warhead nose.

For any penetration depth upto the ogive length, instantaneous area A_i is given by

$$A_i = \pi r^2 = \pi R^2 (\operatorname{Sin} \phi - \operatorname{Sin} \phi_o)^2$$

and for penetration depth beyond ogive length,

$$A_i = \frac{\pi}{4} D^2$$

The instantaneous velocity V of the warhead at that point will be

$$V = \sqrt{V_o^2 - 2\, G\, g\, z}$$

wherein

V_o = striking velocity of the warhead.

Time taken t to penetrate the depth upto that section is given from the first principle of dynamics as

$$t = \frac{V - V_o}{G\, g} \tag{15}$$

5. AXIAL COMPRESSIVE STRESS DEVELOPED IN WARHEAD CASING

Let us now consider the free body diagram of the penetrating warhead at the instant as shown in Fig. 2.

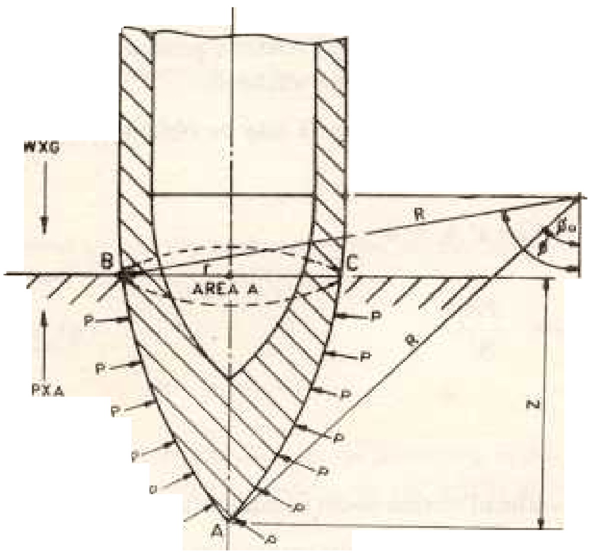

Figure 2. Part penetration of warhead into target medium.

The forces acting on the free body BAC are shown in Fig. 3. They are

Inertia force, $F_i = W_i\, G$

Resistive force, $F_r = P\, A$

Compressive force, $F_c = \sigma_c\, A_c$

where

σ_c = Compressive stress developed in the warhead casing at the section under consideration

A_c = Cross-sectional area at that section

W_f = Mass ahead of the section, i.e. the mass of ogive part BAC

For equilibrium condition, $F_i + F_r - F_c = 0$ for which the compressive stress in the casing at the section under consideration can be calculated as

$$\sigma_c = W_r^{\bullet} G / A_c$$

where

W_r = remaining mass of the warhead behind the section under consideration, ($W_r = W - W_f$)

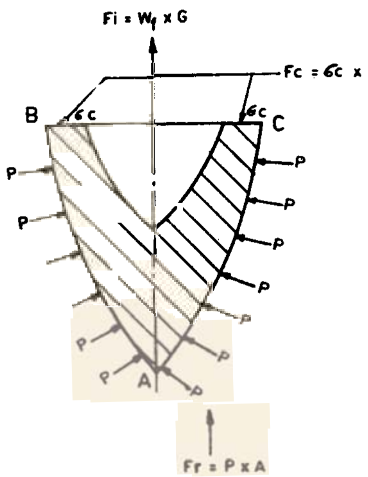

Figure 3. Free body diagram of penetrated part of the warhead.

6. COMPUTER CODE FOR ANALYSIS OF PENETRATION DYNAMICS

A representative sketch of the penetration dynamics of earth penetrating warhead into the composite target media (concrete followed by earth soil) is depicted in Fig. 4 along with the formulae for associated parameters of the penetration process shown in Table 1. Based on this and the earlier discussion a suitable computer code (PEN) has been evolved. The system flow chart of the code is given in Fig. 5. A typical Computer Print out is shown in Table 2. Space-Time histories of the warhead penetration, velocity and deceleration as well as the axial compressive stress developed in the casing are shown from Fig. 6 to 9.

Table 1. Penetration dynamics (Ref. Fig. 4)

Time	T_o	T_1	T_2	T_3	T_4
Penetration depth	0	Z_1	Z_2	Z_3	Z_4
Resistive pressure		$P_1 = \dfrac{4\,S_{cm} + 300\,V_o \sqrt{\frac{\rho_c}{g}\,S_{cm}}}{N_c}$	$P_2 = \dfrac{P_{2c}(A_{2c} - A_{2s}) + P_{2s}\,A_{2s}}{A_{2s}}$	$P_3 = \dfrac{21.49\,S_s^{0.5}\,V_2^{0.75}}{N_s\,D^{0.69}}$	$P_4 = \dfrac{21.49\,S_s^{0.5}\,V_3^{0.75}}{N_s\,D^{0.69}}$
Resistive force		$F_1 = P_1 \times A_1$	$F_2 = P_2 \times A_{2s}$	$F_3 = P_3 \times A_{3s}$	$F_4 = P_4 \times A_{3s}\,S$
Warhead deceleration		$G_1 = F_1 / M$	$G_2 = F_2 / M$	$G_3 = F_3 / M$	$G_4 = F_4 / M$
Compressive stress in warhead body		$Y_1 = \dfrac{G_1 \cdot M_{R1}}{\pi\,(D_1 - t_1)\,t_1}$	$Y_2 = \dfrac{G_2\,M_{R2}}{\pi\,(D_2 - t_2)\,t_2}$	$Y_3 = \dfrac{G_3\,M_{R3}}{\pi\,(D_3 - t_3)\,t_3}$	$Y_4 = \dfrac{G_4\,M_{R4}}{\pi\,(D_4 - t_4)\,t_4}$
Warhead velocity	V_o	$V_1 = \sqrt{V_o^2 - 2\,G_1\,g\,z_1}$	$V_2 = \sqrt{V_1^2 - 2\,G_2\,g \cdot z_2}$	$V_3 = \sqrt{V_2^2 - 2\,G_3\,g \cdot z_3}$	$V_4 = \sqrt{V_3^2 - 2\,G_4\,g\,z_4}$
Time to reach penetration depth	0	$T_1 = (V_1 - V_o) / G_1\,g$	$T_2 = (V_2 - V_1) / G_2\,g$	$T_3 = (V_3 - V_2) / G_3\,g$	$T_4 = (V_4 - V_3) / G_4\,g$

Note : M_R = Remaining mass of warhead behind the section considered.

$D_{1,2,3,4}$ = Max. diameter ofembeddedsection of warhead at time $T_{1,2,3,4}$ respectively.

$t_{1,2,3,4}$ = Casing thickness at respective sections.

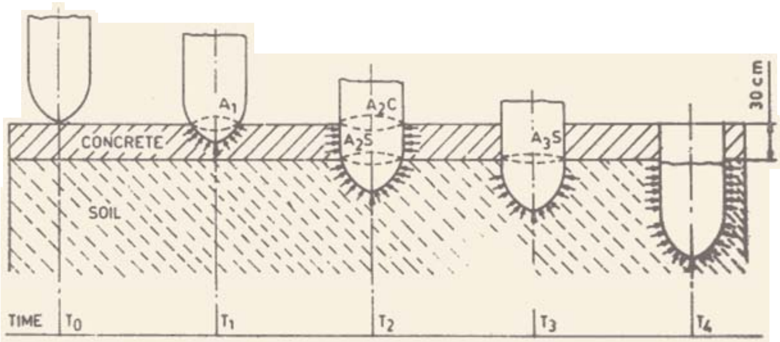

Figure 4. Representative sketch of the warhead penetration dynamics.

Table 2. An extract from a typical computer printout

Input parameters

D = 28.43 cm, $RBYD$ = 2.0, $RBYDI$ = 1.5, XLP = 85.29 cm, XLT = 85.29 cm

XL = 208.2 cm, $VNOT$ = 350 m/sec, WE = 143.25 kg, SC = 450 kg/sq cm

$XINT$ = 20, TB = 1.42 cm, WW = 400 kg, TS = 1.84 cm, C = 2 cm

$RC = 2.28 \times 10^{-3}$ kg/cc, DLC = 30 cm, SS = 5.32 kg/sq cm, RS = 7.8 g/cc

RE = 1.6 g/cc, YS = 12000 kg/cm^2

(* For descriptions of the symbols, refer to appendix 'A')

6.1 Results of Analysis

Profile of penetration dynamics parameters

Step No.	Penetration (cm)	Time (msec)	Casing thick-ness (cm)	Deceleration (g)	Velocity (m/sec)	Resistive pressure (kg/sq cm)	Compressive stress (kg/sq cm)
1	1.6	0.04	1.33	55.40	350.0	3982	3982
2	3.2	0.09	2.60	212.15	349.9	3981	3980
6	10.1	0.29	6.96	1569.31	348.0	3966	3928
9	15.6	0.45	4.63	2966.75	344.0	3935	5275
10	17.5	0.50	4.08	3435.50	342.2	3921	6035
19	35.6	1.05	1.86	5687.47	314.5	3705	11970
20	37.6	1.12	1.84	5410.56	311.0	3677	11293
27	67.5	2.20	1.84	4579.98	258.2	3264	8725
28	71.7	3.33	1.84	677.20	250.7	427	1253
60	208.2	9.24	1.84	600.91	213.7	379	62
61	212.5	9.44	1.84	598.42	212.6	377	29
71	542.3	34.08	–	358.73	107.4	226	–
72	642.3	50.38	–	252.13	67.1	159	–

Total penetration in composite medium 6.42 m
Total travel time = 50.37 msec.

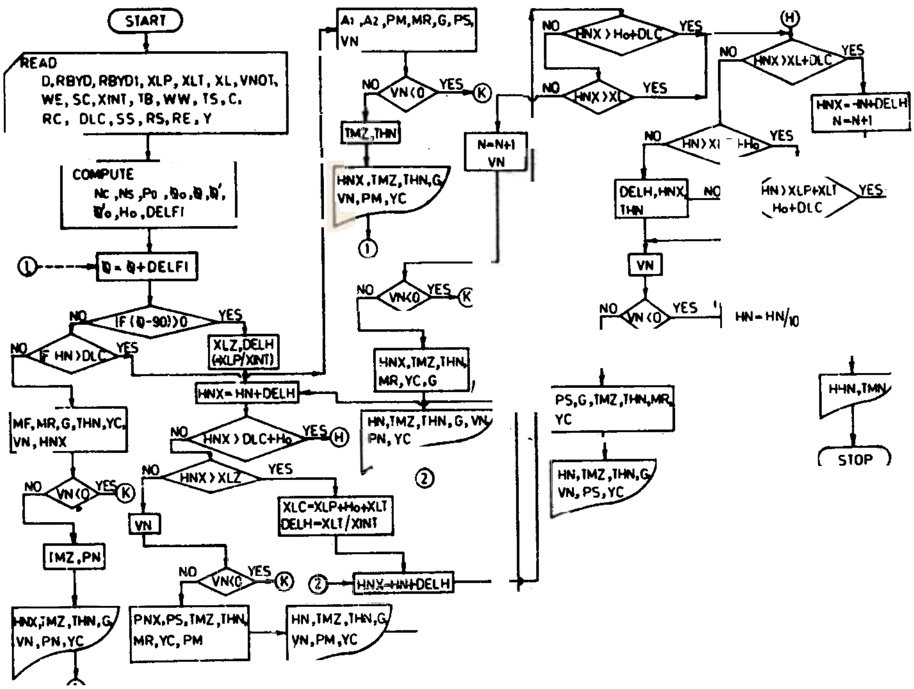

Figure 5. Abridged computer flow-chart for penetration dynamics (Code : PEN).

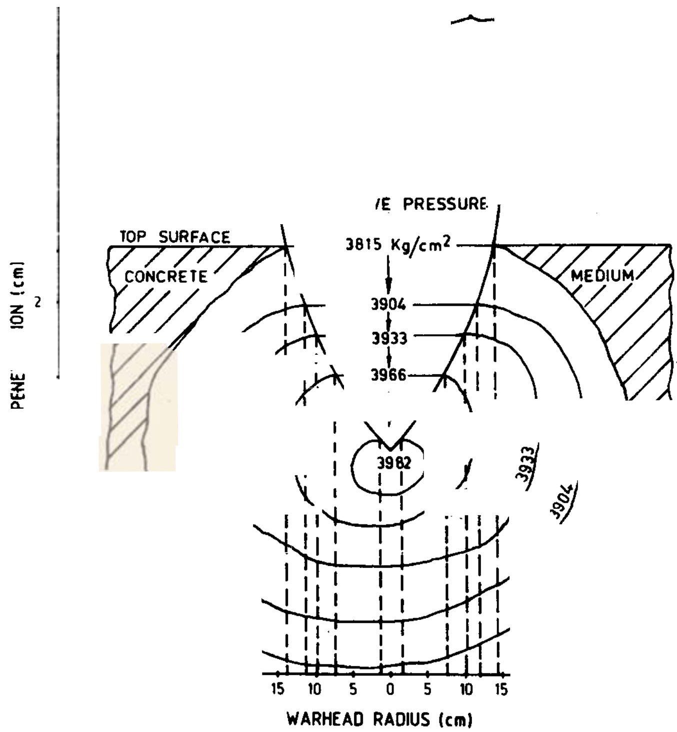

Figure 6. Profile of estimated resistive pressure of concrete (Code : PEN)

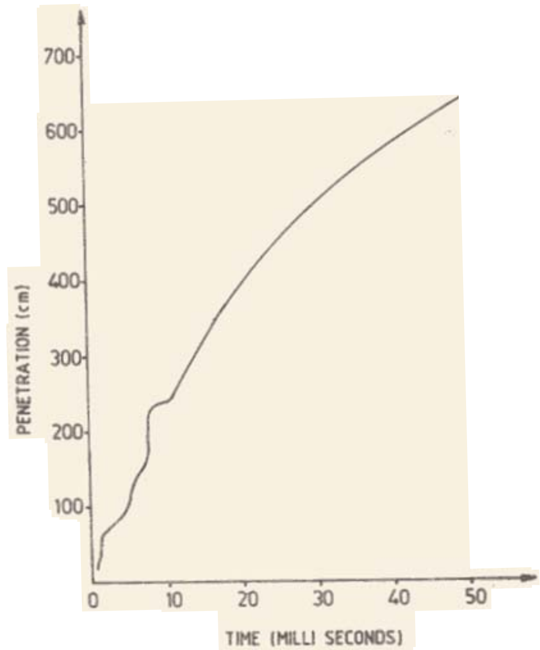

Figure 7. Warhead penetration on a time scale (Code : PEN)

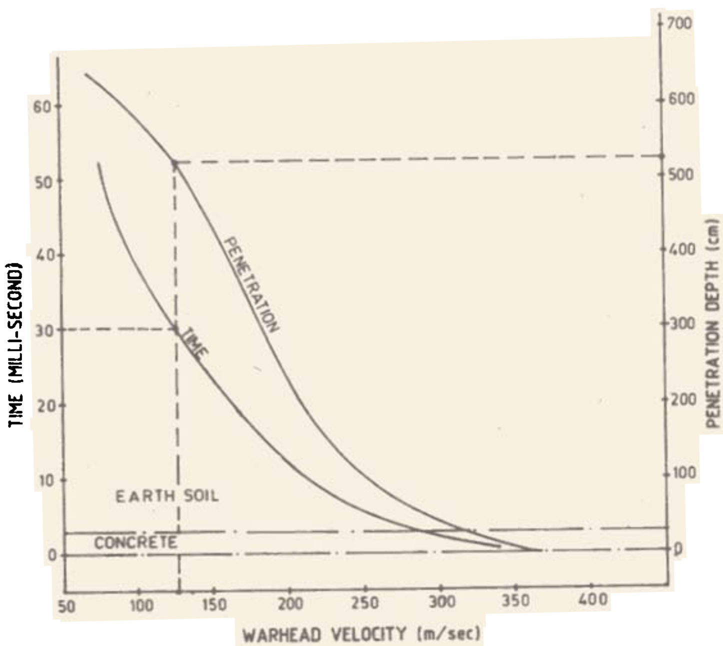

Figure 8. Space-time histories of warhead velocity during penetration process (Code : PEN).

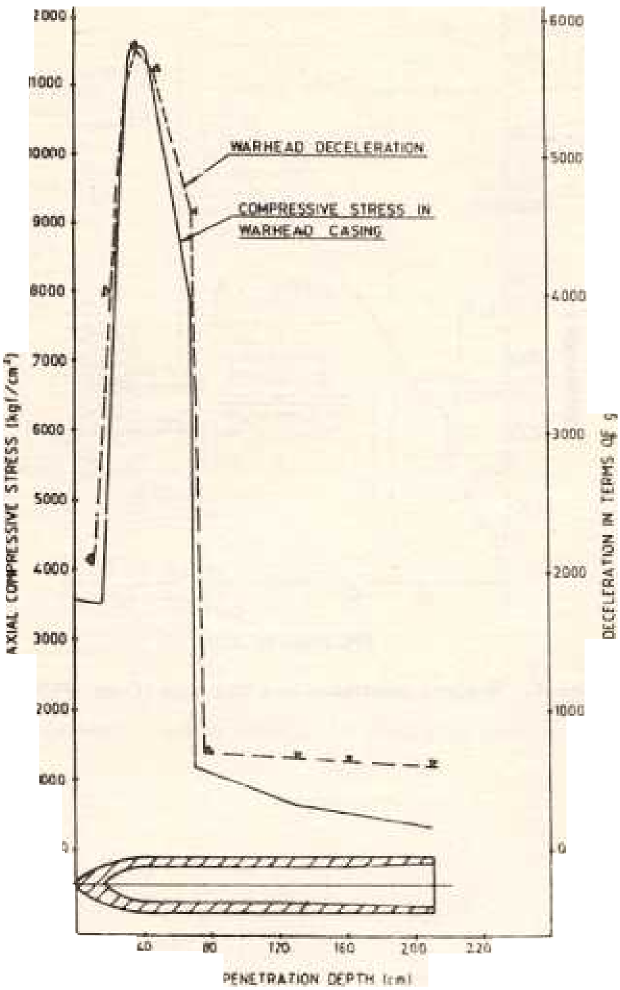

Figure 9. Warhead deceleration and compressive stress developed in warhead casing during penetration (Code : PEN).

7. CONCLUSIONS

The computer code, developed takes into account the target as well as penetrater parameters, yields detailed information about various characteristics of the warhead penetration process into the target medium. The above code will help the designer in design and development of the Earth Penetrating Warhead including its Fuzing System.

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APPENDIX

Table of symbols used in computer flow-chart and printout

Symbol		Description
D	=	Warhead calibre
RBYD	=	External CRH
RBYDI	=	Internal CRH
HNOT	=	Length of ogive section
XLP	=	Parallel portion length of body section
XLT	=	Taper portion length of body section
XL	=	Total length of warhead (body + ogive section)
	=	Striking velocity
WE	=	Explosive weight
SC	=	Compressive strength of concrete
	=	No. of sections considered for analysis in body portion
TB	=	Casing thickness at base end
WW	=	Total warhead weight
TS	=	Casing thickness at shoulder
C	=	Max size of aggregate in concrete
RC	=	Concrete density
DLC	=	Thickness of concrete target
SS	=	Soil strength
RS	=	Casing material density
RE	=	Explosive density

YS	= Dynamic compressive yield strength of casing material
NC	= Nose shape factor for concrete
NS	= Nose shape factor for soil
P_r	= Max resistive pressure of concrete at the instant of warhead striking it
	= Angles as defined in Fig. 2
ϕ_o' , ϕ'	= Similar angles for internal ogive profile
DELFI	= Increment in angle ϕ
HN	= Instantaneous penetration depth of ogive section
MF	= Forward mass of the penetrated part of warhead
MR	= Remaining mass of unpenetrated part
TMZ	= Time to penetrate the depth HN
	= Instantaneous casing thickness
G	= Instantaneous warhead deceleration
VN	= Instantaneous warhead velocity
YC	= Compressive stress developed in casing
	= Instantaneous penetration depth of body section
XLZ	= *HNOT + XLP*
DELH	= Increment in penetration depth
PS	= Resistive pressure of soil
A_1	= Projected area of warhead in concrete
A_2	= Projected area of warhead in soil
PM	= Mean resistive pressure of composite target media
TMN	= Total time for total warhead travel
HHN	= Total warhead travel in composite target media