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MHD Free Convective Flow Past a Hot Vertical Porous Plate

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ABSTRACT

The influence of the magnetic field on the free convective flow of a viscous fluid past a hot vertical porous plate is analysed under the assumptions that the suction velocity which is normal to the porous wall is constant and the wall temperature is spanwise cosinusoidal. The solutions for the velocity, temperature, skin friction and rate of heat transfer have been obtained in dimensionless form by perturbation technique. The effects of different flow parameters appearing in the solutions have been studied on the above flow quantities.

NOMENCLATURE

(x', y', z')	Coordinates of a point
(u', v', w')	Velocity components at a point
V_o	Suction velocity
l	Wall length
υ	Kinematic coefficient of viscosity
g	Acceleration due to gravity
β	Coefficient of volumetric expansion
θ_o	Constant having dimension of temperature
θ'_w	Wall temperature
θ'	Temperature at any point
ρ	Density of the fluid
σ	Conductivity of the fluid
μ_e	Magnetic permeability
H_o	Magnetic field
C	Specific heat

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K	Thermal conductivity
η_o	Coefficient of viscosity
U_∞	Velocity in the x'– direction at infinity
$U = U'/U_\infty$	Dimensionless velocity
$y = y'/l$ $z = z'/l$	Dimensionless space variables
$\theta = \theta/\theta_o$	Dimensionless temperature
$R = -lV_o/\upsilon$	Suction Reynold's number
$P = \eta_o C/K$	Prandtl number
$G = \nu g\beta\theta_o/U_\infty V^2_o$	Grashoff number
$E = U^2_\infty/C\theta_o$	Eckert number
$M = \mu_e H_o l\sqrt{\sigma/\rho\upsilon}$	Hartmann number
$\tau = (\partial u/\partial y)_{y=0}$	Dimensionless skin friction
$Nu = (\partial\theta/\partial y)_{y=0}$	Dimensionless rate of heat transfer

1. INTRODUCTION

The effect of a transverse magnetic field on free convective flows of an electrically connducting viscous fluid, has been discussed in recent years by several authors, notably by Gupta,[1] Soundalgekar,[2] Mishra and Mudili[3], Mahendra Mohan[4] and Sarojamma and Krishna.[5] Such type of flows have wide range of applications in aeronautics, fluid fuel nuclear reactors and chemical engineering. The problem of free convective boundary layer flow of viscous fluid past a hot vertical wall has been discussed by Schlichting.[6] Gersten and Gross[7] studied the laminar flow off viscous liquid along a porous wall assuming the suction velocity to be spanwise cosinusoidal. Mishra and Mohapatra[8] investigated the unsteady free convective viscous flow with magnetic field assuming that the wall temperature to be a function of time. Datta and Mazumdar[9] studied the Hall effects on MHD free convection past an infinite porous flat plate. Recently Acharya and Padhy[10] discussed the free convective flow of a viscous fluid past a vertical plate with constant porosity assuming the temperature to be spanwise cosinusoidal.

In this paper, the influence of magnetic field on free convective flow of viscous fluid past a vettical plate with constant porosity has been studied.

2. FORMULATION OF THE EQUATIONS AND THEIR SOLUTIONS

Let the wall be the $z'x'$ plane, the y'– axis be normal to the wall and the positive direction of x'– axis be vertically upwards. The wall is uniformly porous and the suction velocity normal to it is V_o. We take (u', v', w') to be the components of velocity of the fluid at any point (x', y', z'). Let θ'_w be the spanwise cosinusoidal wall temperature and be given by

$$\theta'_w = \theta_o\,(1 + \varepsilon \cos(\pi z'/l)) \tag{1}$$

where ε is small positive number. Using Boussinesq approximation, and non-dimensional quantities, the governing equations and boundary conditions become

$$\delta u/\delta y = -1/R\,(\delta^2u/\delta y^2 + \delta^2u/\delta z^2) - GR\theta - M^2/R \quad - u) \tag{2}$$

$$\delta\theta/\delta y = -1/PR\,(\delta^2\theta/\delta y^2 + \delta^2\theta/\delta z^2) - E/R\,|\,(\delta u/\delta y)^2 + (\delta u/\delta z)^2 \tag{3}$$

and
$$y = 0\,;\ u = 0,\ \theta = 1 + \varepsilon\cos(\pi z) \qquad y \to \infty\,;\ u = 1,\ \theta = 0 \tag{4}$$

Let u, θ be perturbed over small values of ε as

$$u = U + \varepsilon V(y)\cos(\pi z) + 0(\varepsilon^2) \qquad \theta = \phi + \varepsilon\psi(y)\cos(\pi z) + 0(\varepsilon^2) \tag{5}$$

where U and ϕ are functions of y only and represented the velocity and temperature fields respectively when the wall temperature is constant ($=\theta_o$)

Using Eqn (5) in Eqn (2) – (4), the equations reduce to

$$U'' + RU' + M^2(1 - U) = -R^2G\phi \qquad \phi'' + PR\phi' = -EPU'^2 \tag{6}$$

with boundary conditions

$$y = 0,\ U = 0,\ \phi = 1 \qquad y \to \infty,\ U = 1,\ \phi = 0 \tag{7}$$

$$V'' + RV' - (\pi^2 + M^2)\,V = -R^2G \qquad \psi'' + PR\psi' - \pi^2\psi = -2EPU'V' \tag{8}$$

subject to the boundary conditions

$$y = 0 : V = 0,\ \psi = 1 \qquad y \to \infty, : V = 0,\ \psi = 0 \tag{9}$$

Now the solutions of the equations in (6) and (8) under the boundary conditions (7) and (9) respectively yield the functions characterizing the velocity and temperature fields. Since the exact solutions of these equations are not possible, we solve them approximately by perturbation technique choosing the Eckert number E to be the perturbation parameter. For this, we assume $E \ll 1$ and represent U, ϕ, V, ψ as

$$U = U_o + EU_1 + 0(E^2)$$
$$\phi = \phi_o + E\phi_1 + 0(E^2)$$
$$V = V_o + EV_1 + 0(E^2)$$
$$\psi = \psi_o + E\psi_1 + 0(E^2)$$

Using Eqn (10) in Eqn (6) – (9), we obtain the $0(E^o)$ and $0(E^1)$ equations and the boundary conditions as

$$U''_o + RU'_o + M^2(1 - U_o) = -R^2 G\phi_o$$
$$\phi''_o + RP\phi'_o = 0 \tag{11}$$
$$V''_o + RV'_o - (\pi^2 + M^2)\, V_o = -R^2 G\psi_o$$
$$\psi''_o + PR\psi'_o - \pi^2\psi_o = 0$$

$$y = 0\,; U_o = V_o = 0,\ \phi_o = \psi_o = 1$$
$$y \to \infty\,; U_o = 1,\ \phi_o = V_o = \psi_o = 0$$

$$U''_1 + RU'_1 + M^2(1 - U_1) = -R^2 G\phi_1$$
$$\phi''_1 + RP\phi'_1 = -PU''^2_o$$
$$V''_1 + RV'_1 - (\pi^2 + M^2)\, V_1 = -R^2 G\psi_1$$
$$\psi''_1 + RP\psi'_1 - \pi^2\psi_1 = -2PU'_o V'_o$$

and

$$y = 0\,; U_1 = \phi_1 = V_1 = \psi_1 = 0$$
$$y \to \infty\,; U_1 = \phi_1 = V_1 = \psi_1 = 0$$

The solutions of the system of Eqns (11) – (14) are given by

$$\phi_o = e^{-PRy} \tag{15}$$
$$U_o = 1 + A_1 e^{-\lambda y} - (1 + A_1)e^{-PRy}$$

$$\psi_o = e^{-\sigma' y} \tag{16}$$
$$V_o = A_2\,(e^{-\sigma y} - e^{-\sigma' y})$$

$$U_1 = 1 + A_8 e^{-\lambda y} - A_4 e^{-PRy} + A_5 e^{-2\lambda y} + A_6 e^{-2PRy} - A_7 e^{-(\lambda + PR)y}$$

$$\phi_1 = A_3\, e^{-PRy} - \{\, PA_1{}^2\lambda /2\,(\,2 - PR\,)\,\}\, e^{-2\lambda y} - \{\, P\,(\,1 + A_1)^2 / 2 \quad e^{-2PRy}$$

$$+ \quad 2P^2RA_1 \qquad + A_1) \qquad \lambda + PR)\,\}\, e^{-(\lambda + PR)\,y} \qquad 17$$

$$\psi_1 = A_{13}e^{-\sigma' y} - A_9 e^{-(\lambda+\sigma)y} - A_{10}e^{-(\lambda+\sigma')y} - A_{11}e^{-(PR+\sigma')y} - A_{12}e^{-(PR+\sigma)y} \tag{18}$$

$$V_1 = A_{19}e^{-\sigma y} - A_{14}e^{-\sigma' y} - A_{15}e^{(\lambda+\sigma)y} - A_{16}e^{-(\lambda\sigma')y} - A_{17}e^{-(PR+\sigma')y} - A_{18}e^{-(PR+\sigma)y}$$

where $\sigma = R/2 + (R^2/4 + \pi^2 + M^2)^{1/2}$
$\sigma' = PR/2 + (P^2R^2/4 + \pi^2)^{1/2}$
$\lambda = R/2 + (R^2/4 + M^2)^{1/2}$

and all the remaining constants A_i are functions of R, G, P, M, λ, σ and σ'

With the help of the velocity and temperature fields, we obtain the skin friction τ and the rate of heat transfer N_u in the dimensionless form as

$$\tau = (\partial u/\partial y)_{y=0} = U'(0) + \varepsilon V'(0)\cos(\pi z) \tag{19}$$

$$N_u = -(\partial\theta/\partial y)_{y=0} = -\phi'(0) + \varepsilon\psi'(0)\cos(\pi z) \tag{20}$$

where

$$U'(0) = -\lambda A_1 + PR(1+A_1) + E[PRA_4 - 2\lambda A_5 - 2PRA_6 + (\lambda+PR)A_7 - \lambda A_8]$$

$$\phi'(0) = -PR + E[-PRA_3 + \{PA_1^2\lambda^2/(2\lambda - PR)\} + P^2R(1+A_1)^2 - 2P^2RA_1(1+A_1)]$$

$$V'(0) = A_2(\sigma'-\sigma) + E[-\sigma A_{19} + \sigma' A_{14} + (\lambda+\sigma)A_{15} + (\lambda+\sigma')A_{16} + (PR+\sigma')A_{17} + (PR+\sigma)A_{18}]$$

$$\psi'(0) = -\sigma' + E[(\lambda+\sigma)A_9 + (\lambda+\sigma')A_{10} + (PR+\sigma')A_{11} + (PR+\sigma)A_{12} - \sigma' A_{13}]$$

3. DISCUSSION

The effects of different flow parameters have been discussed on velocity V, temperature ψ, skin friction $V'(0)$ and the rate of heat transfer $\psi'(0)$ through the graphs. It is observed that the velocity and the skin friction decrease as magnetic

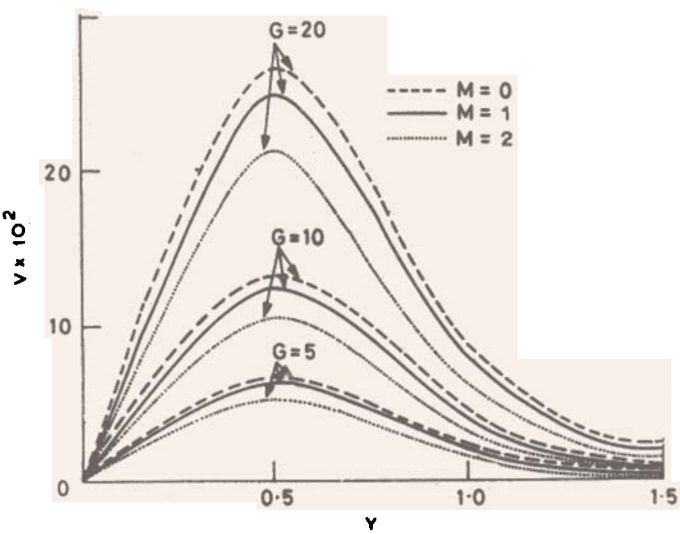

Figure 1. $V(y)$ versus y for P=0.71, E=0.001 and R=1.

parameter M increases and also it is seen that the values of all these flow quantities in magnetic case are less than the values in non magnetic case. As M increases, the temperature ψ decreases for larger $R(>2)$, whereas for small $R(\leq 1)$ it increases and changes its pattern for R=2. It is seen that the velocity V and the skin friction $V'(0)$ increase as R increases. The temperature ψ and the rate transfer $\psi'(0)$ decrease with increasing R.

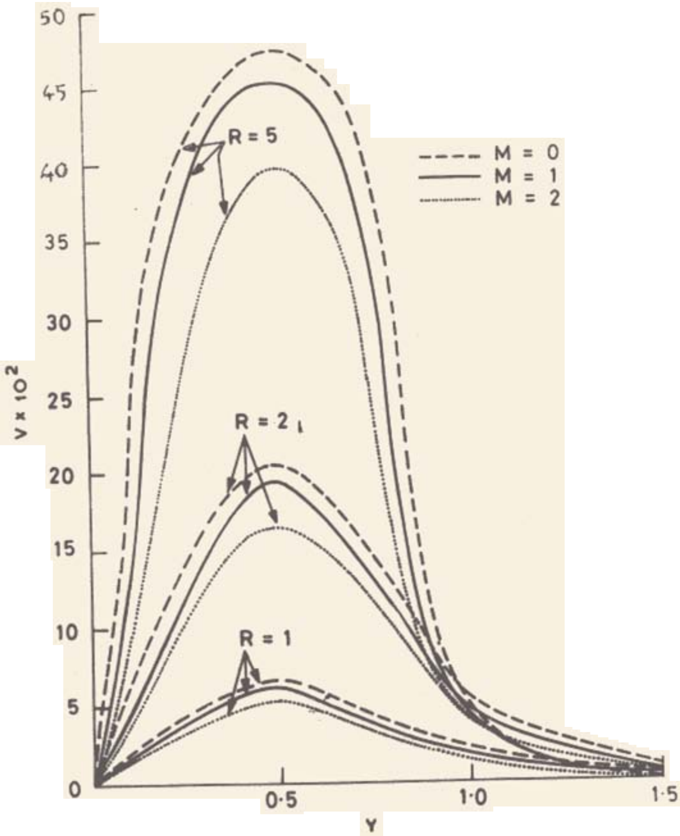

Figure 2. $V(y)$ versus y for P=0.71, E=0.001 and G=5.

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