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Sound-Speed Prediction as a Function of Temperature at Discrete Depths in the Bay of Bengal

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ABSTRACT

Through regression analysis, temperature dependent relationships are developed to predict sound-speed at discrete depths in the Bay of Bengal, thereby demonstrating the feasibility of sound-speed prediction through polynomial expressions in temperature disregarding salinity variations. A separate regression equation is developed for the historical sound-speed and temperature data at the standard depths up to 250 m. At specific depths and in the given geographic area in the Bay of Bengal (5–8° N, 90–93° E) polynomials of temperature are proved to be a precise way to predict sound-speed.

1. INTRODUCTION

The most important acoustical parameter associated with the ocean is the speed of the sound[1]. The traditional means of obtaining the vertical sound-speed profile in the ocean are direct measurements with a velocimeter and computation from temperature salinity and pressure triplets obtained from shipboard measurements. The basic need common to both these means is the necessity for salinity estimates. It is possible for the near-surface water column to be statically unstable over the time period required for a BT cast. In the near-surface layers T–S relationships are generally not well defined and the vertical distribution of salinity could be varying[2]. Propagation of sound in the ocean is complicated by several factors. It strongly depends on temperature, pressure and to a lesser extent up on salinity[3]. Hence to consider sound-speed as a function of temperature alone is to ignore the other two sources of variations. Variability due to differences in pressure can be minimised by establishing sound-speed relationships only at discrete ocean depths. The variability caused by salinity at these depths can be taken as a random variability for the purpose of fitting sound-speed to temperature.

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Theoretically sound-speed in sea-water is given by

$$c = [\gamma/\rho k]^{1/2}$$

where c is the sound-speed, γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume for sea-water, ρ is the density of sea-watèr and k is the true compressibility coefficient. But ρ; γ, and k-are functions of temperature, pressure and salinity and these functional relationships are known only empirically[4]. In this study, at specific depths in a three degree square in the Bay of Bengal, (5–8° N, 90–93° E) empirical relationships of the form $c = f(t)$ based on regression analysis are derived, employing the data archived at the Australian Oceanographic Data Centre, New South Wales. As the $c = f(t)$ relationship has no natural origin or scale, a ploynomial in temperature is proposed as the equation form[5]. In the study area, it is assumed that a no covariation relationship exists for salinity and temperature, since salinity and pressure can't be held constant in a sound-speed equation such as Wilson's[6] and the resulting polynomial espression in temperature for a particular depth. While considering the relative ease with which a polynomial expression can be fitted to the sound-speed and temperature data, this becomes an unnecessarily restrictive assumption. The method of fitting the least squares in regression analysis allows the coefficients of the polynomials to reflect systematic temperature-salinity covariation as far as the equation form accommodates. The portion of the systematic covariation that the equation may be too inflexible will be met in the residuals.

2. METHODS

The archived data used in this study is obtained from the Australian Oceanographic Data Centre, New South Wales. The initial data set was divided into ten subsets corresponding to one of the following standard oceanographic depths: 10, 20, 30, 50, 75, 100, 125, 150, 200 and 250 m. Within each depth the sound-speed observations are considered uncorrelated and replicated with respect to temperature.

2.1 Curve Fitting

Five polynomials, first through fifth order, were fitted to the ten sets of sound-speed and temperature data in the linear regression equation,

$$Y_i = A(0) + A(1)X_i$$

where Y_i is the predicted sound-speed, and X_i is the temperature in the station; $A(0)$, $A(1)$ are the regression coefficients obtained using the principle of least squares[7].

$$A(0) = \bar{Y}_i - A(1)\bar{X}_i$$

$$A(1) = \frac{S_{Xy}}{S_{Xx}} \qquad \text{and}$$

where

$$S_{Xx} = \Sigma X_i^2 - n\bar{X}_i^2$$

$$S_{Xy} = \Sigma X_i Y_i - n\,\bar{X}_i \bar{Y}_i$$

The linear regression is tested for significance using the ANOVA technique[7] and the sequential – *F*–ratio statistic[8].

The test statistic, standard error of estimate calculated from an unbiased estimate of the error variance is calculated from

$$S_{y.x} = \sqrt{\frac{\Sigma(Y_i - Y_{iest})^2}{n}}$$

where Y_{iest} is the estimated value of Y_i for a given value of X_i, as obtained from the regression curve. Examination of $S_{y.x}$ will indicate the preciseness of the prediction equation[7].

The situation is generalized to more variables in second, third, fourth and fifth demensional spaces.

Since the Y_i's are random variables, any function of them is also random; two particular functions are *Ms.Reg*, the mean square due to regression and S^2, the mean square due to residual variation, which arise in the analysis of variance. These functions then have their own distribution, mean and variance. The ratio $Ms.Reg/S^2 = F$ follows an *F*-distribution with *p* and *n–p* degrees of freedom. Examining the probabilities of the sequential *F*-ratios, the polynomial which appears adequate to fit the data sets can be selected as the prediction equation.

2.2 Residuals Analysis

The difference between the predicted sound-speed c_i at a temperature decided by the data and the sound-speed *c*, as provided by the data is computed for each datum, in the data sets. The locus of the appropriate confidence-interval points for each equation is computed as

$$\pm\, sT\left[\,(\overline{n-p-1}\,),\, m\,\right] \quad 1 + \frac{1}{n} + \sum_{i=1}^{p} \sum_{j=1}^{p} (t_{oi} - \bar{t}_i)\,(t_{oj} - \bar{t}_j)\, d_{ij} \;\}^{1/2}$$

where *s* is the standard error of estimate for the prediction equation; $T[(\overline{n-p-1}), m]$ is the student's *t*–distribution with $(n–p–1)$ degrees of freedom, *n* is the sample size; *p* the order of the respective polynomial, t_{oi} is the power of some specific temperature

$$t_o,\ \bar{t}_i = \frac{1}{n}\sum_{k=1}^{n} t_{ki}$$

where each t_k is an original temperature datum; and d_{ij} is the *ij*th element of the inverse matrix formed by it's transpose[8].

3. DISCUSSION

The curve-fit statistics is presented in Table 1. The probabilities associated with the sequential *F*-ratios and the standard error of estimate of the polynomials is given A notation such as $(b_2/b_0, b_1)$ means that the temperature squared for quadratic term for

Table 1. Curve-fit statistics

	Depth (m)									
Statistical variable	10	20	30	50	75	100	125	150	200	250
	Sample size									
	45	51	61	72	84	77	73	68	53	46
Probabilities associated with sequential F-ratios										
b_1/b_0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$b_2/b_0, b_1$	1.000	1.000	1.000	0.996	0.940	0.999	0.929	1.000	1.000	1.000
$b_3/b_0, b_1, b_2$	0.351	0.041	0.334	0.731	0.584	0.969	0.931	0.991	0.948	0.940
$b_4/b_0, b_1, b_2, b_3$	0.724	0.213	0.839	0.799	0.721	0.784	0.694	0.831	0.597	0.697
$b_5/b_0, b_1, b_2, b_3, b_4$	0.821	0.831	0.677	0.803	0.807	0.791	0.856	0.933	0.782	0.714
Standard error of estimate										
Linear	0.727	0.849	0.701	0.561	0.583	0.231	0.527	0.843	0.541	0.916
Quadratic	0.201	0.364	0.443	0.382	0.493	0.231	0.508	0.831	0.538	0.916
Cubic	0.238	0.354	0.451	0.380	0.493	0.206	0.329	0.701	0.127	0.512
Quartic	0.237	0.354	0.451	0.383	0.493	0.213	0.513	0.731	0.121	0.508
Pentic	0.236	0.354	0.451	0.380	0.493	0.213	0.513	0.731	0.121	0.508
Order of the selected polynomial	2	2	2	2	2	3	3	3	3	3

which b_2 is the coefficient was added to the polynomial after the constant and linear terms, whose coefficients are b_0 and b_1. The tabulated value, therefore indicates the necessity for including that term in the prediction equation. Higher the tabulated value, the more desirable would be the inclusion. A minimum standard error of estimate calculated from an unbiased estimate of error variance is an indicator that the associated polynomial is adequate for prediction purposes. The smaller the standared error of estimate, the more precise would be the prediction.

Table 2 lists the coefficients and standard error of estimate for the selected equations. For example, at 150 m depth, the appropriate prediction equation is

$$c_i = 1496.3 + {}^{-}3.7\, t_i + 0.4368\, t_i + {}^{-}0.01047\, t_i^3$$

where c_i is the predicted sound-speed in meter per second and t_i is the temperature in degree celcius.

The prediction confidence-interval width and the distribution of residuals would have to be the basis for deciding whether a prediction equation is useful. A prediction confidence-interval pertains to the prediction of an event. With a locus of end points

Table 2. Coefficients and standard error of estimates for the sound-speed equations

Depth (m)	$c_i = A(0) + A(1)\, t_i + A(2)\, t_i^2 + A(3)\, t_i^3 + A(4)\, t_i^4 + A(5)\, t_i^5$						Sample standard error of estimate
	A(0)	A(1)	A(2)	A(3)	A(4)	A(5)	
10	1660.9						0.201
20	1637.06						0.364
30	1614.65						0.443
50	1404.75						0.382
75	1563.108						0.493
100	2117.2						0.206
125	1662.9						0.329
150	1496.3						0.281
200	1366.7						0.127
250	1473.5						0.512

for the 95 per cent prediction confidence-intervals. Since the data sets are sets of events, like those to be predicted, the locus of end points for the 95 per cent prediction confidence-intervals should generally encompass 95 per cent of the data used to obtain the regression equation coefficient. For each subset of the data, the confidence-interval width are approximately.

±0.40 m/s	for the	10 m	prediction equation
±0.73 m/s	"	20 m	
±0.89 m/s		30 m	
±0.76 m/s		50 m	
±0.99 m/s		75 m	
±0.41 m/s		100 m	
±0.66 m/s		125 m	
±0.56 m/s		150 m	
±0.25 m/s		200 m	
±1.02 m/s		250 m	

4. CONCLUSION

In the specific geographic area in the Bay of Bengal, at specific depths, polynomials in temperature have been shown to be a simple and precise way to predict the sound-speed. Hence bathythermographic data which are more easily acquired may be readily converted to a sound-speed profile. These equations are of much use to the observational Navy, as this may help to optimise the SONAR performance.

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