Passive Source Localization Using Compressively Sensed Towed Array

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ABSTRACT

The objective of this work is to estimate the sparse angular power spectrum using a towed acoustic pressure sensor (APS) array. In a passive towed array sonar, any reduction in the analog sensor signal conditioning receiver hardware housed inside the array tube, significantly improves the signal integrity and hence the localization performance. In this paper, a novel sparse acoustic pressure sensor (SAPS) array architecture is proposed to estimate the direction of arrival (DOA) of multiple acoustic sources. Bearing localization is effectively achieved by customizing the Capons spatial filter algorithm to suit the SAPS array architecture. Apart from the Monte Carlo simulations, the acoustic performance of the SAPS array with compressively sensed minimum variance distortionless response (CS-MVDR) filter is demonstrated using a real passive towed array data. The proposed sparse towed array architecture promises a significant reduction in the analog signal acquisition receiver hardware, transmission data rate, number of snapshots and software complexity.

Keywords: Compressive beamforming, compressive sensing, DOA estimation, minimum variance distortionless response.

NOMENCLATURE

Ν	Number of	of acoustic	sensors	used	in APS	array
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- *d* Inter sensor spacing
- ρ Density of water
- *c* Sound speed in water
- *J* Number of acoustic sources
- θ_{j} Azimuth bearing of j^{th} source with reference to the array axis.
- $\mathbf{a}(\theta_{j})$ Array steering vector corresponding to the source direction θ_{j}
- A Aarray steering matrix corresponding to the J sources
- λ Wavelength of the received signal
- k Wave number; $k = \frac{2\pi}{k}$
- \mathbb{C} Complex vector space
- $\eta_j(t)$ Slowly varying complex amplitude of the signal from the j^{th} source at time t
- σ_{i}^{2} Variance of j^{th} acoustic source
- $\mathbf{w}(t)$ i.i.d circular complex gaussian random vector at time t
- SNR Signal to noise ratio
- σ^2 Noise variance
- \mathbf{Z}^{H} Conjugate transpose (Hermitian) of \mathbf{Z}

 \mathbf{R}_{N} Conventional spatial correlation matrix $\in \mathbb{C}^{N \times N}$

- E[.] Expectation operator
- *L* Number of snapshots or time samples
- **x** Received array data vector $\in \mathbb{C}^N$
- *K* Sparsity level
- *M* Number of non-adaptive linear projection measurements used for compressive sensing

\mathcal{O}	order of
S	Sparsity pattern vector $\in \mathbb{C}^{N}$
Ψ	Sparsifying basis matrix $\in \mathbb{C}^{N \times N}$
ø	Sensing or measurement matrix $\in \mathbb{C}^{N \times M}$
У	Compressed array data vector $\in \mathbb{C}^{M}$
$\left\ . \right\ _{p}$	p^{th} frobenius norm
F_{s}^{r}	Sampling frequency in Hz
Ğ	Number of bits/sample
f	Acoustic source frequency in Hz

1. INTRODUCTION

Passive source localization is a problem of great interest in underwater scenario to localize the acoustic sources using the radiated or emitted signatures from the sources. Passive surveillance systems enjoy the important strategic advantage, especially in a military/battlefield scenario, of providing vital information about the location and trajectory of potential targets without revealing their own presence on the scene. A towed horizontal linear array is used to estimate the location of acoustic sources by spatially sampling the propagating pressure field radiated from the acoustic sources. A passive towed array receiver consists of large number of acoustic pressure sensors configured in a linear fashion, housed in a long tube, to capture the low frequency components emanating from the subsurface targets. In a modern compact towed array sonar system, the onboard electronics hardware positioned in the tow ship receives the digital data from an ultra thin line array towed behind the ship¹. The complete data acquisition circuitry is integrated inside the array tube. The diameter of the array tube must be as small as possible to design a compact

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onboard winch and array handler system for easier deployment and retrieval of the array. The major design challenges to be addressed in the passive towed receiver array design are:

- (a) Number of acoustic sensors to be used and its optimal configuration inside the tube for long range detection, localization, tracking and classification of subsurface targets;
- (b) Packaging of the signal conditioning hardware inside the small diameter tube;
- (c) Ensuring the signal integrity up to the onboard digital signal processor hardware with minimum inter-signal interferences or cross talk between the adjacent channels and,
- (d) Reliability of the data communication link between the towed array and the onboard tow ship.

In this paper, compressively sensed towed receiver array architecture is proposed which significantly reduces the hardware complexity and addresses these design issues with superior strategic performance. The proposed system utilizes a compressively sensed array along with the high resolution angular power spectral estimators customized for sparse array processing.

Compressive sensing or sampling is a new signal acquisition technique which requires fewer measurements or front-end signal conditioning hardware chain to represent or reconstruct the signal, which are sparse in some basis vectors. In the present context, the array signal vector is sparse in angular spectral basis. The applications of compressive sampling were initiated by Donoho and Candes^{2,3} *et al.* A good review of the theory behind compressive sensing is given in⁴.

In this study, authors estimates the sparse angular power spectrum using compressive beamforming via CS-MVDR. Apart from the Monte Carlo simulations, the source localization performance of compressive beamforming is compared with the conventional Capon beamformer using a real passive towed array sonar data. The superior strategic performance of the SAPS based towed array architecture is demonstrated using minimum number of snapshots or time samples. Using compressive measurements, the measurement vector dimension is reduced to a large extent leading to deal with smaller sized spatial covariance matrix.

2. APS ARRAY DATA MODEL

Consider a uniform horizontal APS array of N sensors spaced equally by a distance d. The scenario under consideration is shown in Fig. 1. ρ is the density of water and c is the sound speed in water.

Let there be *J* sources at azimuth angles θ_j , j = 1, 2, ..., JThe received signal at the sensor array at time *t* can be expressed as an $N \times 1$ vector $\mathbf{x}(t)$ of the form

$$\mathbf{x}(t) = \mathbf{A}\eta(t) + \mathbf{w}(t) \tag{1}$$

where $\mathbf{A}, \eta(t), \mathbf{w}(t)$ are given by

$$\mathbf{A} = [\mathbf{a}(\theta_1)\mathbf{a}(\theta_2)\dots\mathbf{a}(\theta_J)]$$
(2)

 $\mathbf{a}(\boldsymbol{\theta}_{j})$ is the steering vector corresponding to the source direction $\boldsymbol{\theta}_{j}$.

$$\mathbf{a}(\boldsymbol{\theta}_{j}) = [1, e^{ikd\cos\theta_{j}}, \dots, e^{i(N-1)kd\cos\theta_{j}}]^{T}, \qquad (3)$$



Figure 1. Geometry of source and receiver array.

where
$$k = \frac{2\pi}{\lambda}$$
, λ being the wavelength of the received signal.
 $n(t) = [n(t) n(t) - n(t)]$ (4)

$$\eta(t) = [\eta_1(t), \eta_2(t), \dots, \eta_J(t)]$$
(4)

where $\eta_i(t)$ is the slowly varying complex amplitude of the signal from the *j*th source at time *t*, modeled as jointly stationary and uncorrelated circular complex narrowband Gaussian random processes with mean zero and variance, $\sigma_i^2 = E[|\eta_i(t)|^2]$

$$\mathbf{w}(t) = [\mathbf{w}_1(t), \dots, \mathbf{w}_N(t)]^T$$
(5)

where $W_1(t), ..., W_N(t)$ are the i.i.d circular complex Gaussian random variables with variance σ^2 .

The SNR for the j^{th} source is defined as

$$(SNR)_{j} = \frac{\sigma_{j}^{2}}{\sigma^{2}}$$
(6)

The spatial correlation matrix of the array data vector $\mathbf{x}(t) \in \mathbb{C}^{N \times 1}$ is defined as

$$\mathbf{R}_{N} = E[\mathbf{x}(t)\mathbf{x}^{H}(t)] \tag{7}$$

In practical calculations, considering the received data is finite, the true correlation matrix can be estimated as the following sample correlation matrix

$$\widehat{\mathbf{R}}_{N} = \frac{1}{L} \sum_{t=1}^{L} \mathbf{x}(t) \mathbf{x}^{H}(t) , \qquad (8)$$

where L is the number of snapshots or time samples.

3. COMPRESSIVE SENSING

Compressive sensing (CS) is an emerging sampling technique which uses few linear measurements in comparison with the conventional Nyquist sampling theory together with a non-linear recovery process. Compressive sensing enables sparse or compressible signals to be captured and stored at a rate much below the Nyquist rate. The reconstruction of the original signal from its random projections is possible by means of an optimization process as long as the measurements satisfy reasonable conditions such as incoherence and restricted isometry property (RIP)⁵.

Consider a signal vector $\mathbf{x} \in \mathbb{C}^{N \times N}$. If \mathbf{x} is *K*-sparse in some orthonormal basis, then \mathbf{x} can be represented as

$$\mathbf{x} = \Psi_{\mathbf{S}} \,, \tag{9}$$

where $\Psi \in \mathbb{C}^{N \times N}$ is the sparsity basis matrix and $\mathbf{s} \in \mathbb{C}^{N \times 1}$ is sparsity pattern vector with $K \ll N$ nonzero entries. The CS theory states that \mathbf{x} can be recovered using $M = K\mathcal{O}(\log N)$ non-adaptive linear projection measurements on to a random matrix namely $\Phi \in \mathbb{C}^{N \times M}$ sensing or measurement matrix. The compressed array data vector **y** can be written as

$$\sqrt{\Phi^{H}} \mathbf{x} = \Phi^{H} \Psi \mathbf{y} = \Omega \mathbf{y}$$
(10)

where $\Omega = \Phi^H \Psi \hat{I} C^{M \times N}$. The choice of measurement matrix Φ is important since it decides the stability and reliability of the compressive sensing process. The measurements should satisfy incoherence with respect to the original basis and RIP for the recovery of the original signal from its randomized projections by means of an optimization process. Both the RIP and incoherence can be achieved with high probability simply by selecting Φ as a random matrix.

3.1 Benefits of Compressive Sensing on Towed Array Processing

In towed array sonar, the data communication link between the towed array and the onboard tow ship is mission critical and any failure in this link leads to catastrophic system failure. It is essential that, the system must be designed with robust and reliable array onboard data link. It is noteworthy that, the reliability of the data communication link increases with decrease in data transmission rate though the link for a given set of system parameters. The data rate of conventional towed array is given by

Transmission-rate_{APS} = NF_sG bits/second (11) where *N* is the number of sensors, is the sampling frequency in Hz, *G* is the number of bits/sample. Generally 16-bit or 24bit sigma delta converters are used for digitization. The above issues are efficiently tackled by the proposed compressive sampling or sensing receiver architecture. The proposed architecture uses only around $J\log(N)$ analog front-end signal conditioning hardware and two bits of phase precision⁶ instead of 16 or 24 bits. The data rate of the compressively sampled towed array is given by

Transmission-rate_{SAPS} = $2F_s J \log(N)$ bits/second (12)

where J is the maximum number of expected acoustic sources.

CS array has the effect of compressing a large sized array into a smaller sized array. This in turn reduces the hardware complexity on account of the much smaller number of front-end circuit chains. Also, due to the smaller dimension of the array data vector, the software complexity is greatly reduced. In general, the high resolution DOA estimation algorithms use the inverse of the spatial correlation matrix with computational complexity $O(n^3)$ for an $n \times n$ matrix. The CS array architecture with CS spatial filters needs to work with the correlation matrix $\hat{\mathbf{R}}_{y}$ of size $M \times M$ only, where $M \ll N$, while the conventional APS array needs to work with $\hat{\mathbf{R}}_{N}$ of size $N \times N$. In the proposed SAPS array architecture, the compressed array data vector $\mathbf{y}(t)$ is M-dimensional and the covariance matrix estimate is given by

$$\hat{\mathbf{R}}_{\mathbf{y}} = \frac{1}{L} \sum_{t=1}^{L} \mathbf{y}(t) \mathbf{y}^{H}(t), \qquad (13)$$

where L is the number of time samples or snapshots.

Table 1 summarizes the hardware requirement of the proposed SAPS array architecture and compares it with that of the conventional scalar sensor array. It is clear that, the

hardware and software complexity of the proposed architecture is significantly less in comparison with the conventional array for large values of *N*.

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Parameter	APS Array	SAPS Array
No. of channels	Ν	$J \log(N)$
No. of signal conditioning hardware	Ν	$J \log(N)$
Data rate (bits/sec)	NF _s K	$J \log(N) F_s K$
Spatial correlation matrix	$N \times N$	$J \log(N) \times J \log(N)$
Minimum number of snapshots	Ν	$J \log(N)$

 Table 1.
 Comparison of conventional and compressed sensing array architectures

Note : The hardware requirement for the SAPS array is greatly reduced for large values of *N* and *K*.

4. DOA ESTIMATION USING SAPS ARRAY

The acoustic sensors distributed in the array spatially sample the propagating acoustic wave field. The objective is to estimate the direction of arrival of acoustic sources from the received signal vector in the presence of noise and interfering signals. The angular signal spectrum can be viewed as a linear combination of discrete set of spatially distributed farfield sources with few of them being of high acoustic power. Motivated by this physical structure of the spatial signal spectrum, it can be viewed as a sparse signal in the angular spectral domain. This representation allows us to explore the compressive sensing framework in the DOA estimation problem. Using CS technique, a large sized array is compressed or transformed into an array with few elements.

We estimate the angle pseudospectrum by modifying MVDR⁷ to suit the SAPS array. In compressive beamforming, we use $\mathbf{v}(\theta)$ instead of $\mathbf{a}(\theta)$ as given in Eqn. 3, where

$$\mathbf{w}(\theta) = \Phi^{H} \mathbf{a}(\theta) , \qquad (14)$$

is the compressed array manifold vector.

The pseudospectrum of the SAPS array using CS-MVDR is given by

$$P_{CS-MVDR}(\theta) = \frac{1}{\mathbf{v}^{H}(\theta)\widehat{\mathbf{R}}_{\mathbf{y}}^{-1}\mathbf{v}(\theta)},$$
(15)

where $\widehat{\mathbf{R}}_{y}$ is the estimated spatial correlation matrix of the compressed signal **y** given by Eqn. 13. The peaks in the MVDR spectrum occur whenever the steering vector is orthogonal to the noise subspace.

5. SIMULATION RESULTS AND DISCUSSIONS

In this section, the simulation results of compressive beamforming using SAPS array are presented and compared with the Capons spatial filter using the conventional APS array. We simulate an uniformly spaced horizontal linear array of 36 sensors with inter-sensor spacing 9.6 m, receives signal in the form of plane waves from **J** narrowband uncorrelated acoustic sources with known centre frequency 78Hz and azimuth bearing $\theta_1, \theta_2, \dots, \theta_J$. We model the ambient noise as statistically independent circular complex Gaussian random process. All the results have been obtained by averaging over 250 Monte Carlo simulations, each uses 75 snapshots for conventional array processing and 25 snapshots for compressed sensing array processing unless otherwise stated, to demonstrate the superior strategic performance of the proposed SAPS array processor. It is noteworthy that the array processing algorithms which utilizes the inverse of the spatial covariance matrix to estimate the DOA of acoustic sources, requires a minimum of 36 snapshots for a 36 element APS array to achieve a nonsingular covariance matrix. The measurement or sensing matrix Φ consists of *M* column vectors that has entries drawn from an i.i.d Gaussian random process with mean, $\mu = 0$ and variance $\sigma^2 = \frac{1}{M}$.

First author compare the pseudospectrum performance of the Capons spatial filter for a 10-element conventional array with SAPS array (N = 36, M = 10) using CS-MVDR. We consider two acoustic sources with -5 dB SNR at 87° and 97° with respect to the array end-fire direction. The pseudospectrum response of MVDR and CS-MVDR angular spectral estimators is shown in Fig. 2. It is observed that, though conventional and sparse array uses the same number



Figure 2. Pseudospectrum of MVDR and CS-MVDR algorithm on the conventional APS array (N = 10) and the SAPS array (N = 36, M = 10). Two sources at 87° and 92° . SNR = -5 dB, f =78 Hz.



Figure 3. Bias vs. source bearing of MVDR and CS-MVDR algorithms on the conventional APS array (N = 10) and the SAPS array (N = 36, M = 10). SNR = -5 dB, f = 78 Hz.

of measurements or dimension of the spatial correlation matrix remains same, the bearing resolution performance of CS-MVDR is much superior in comparison with the conventional MVDR. Figs. 3 and 4 show, respectively, the variation of bias and RMSE with source bearing of MVDR and CS-MVDR on conventional APS array and the SAPS array. The Cramer-Rao lower bound (CRLB)⁸ for a 10-element APS array is also shown in Fig. 4. The variation of bias and RMSE with number of snapshots of CS-MVDR algorithm for SAPS array is shown in Fig. 5. It is seen that both the bias and RMSE approaches zero asymptotically with increase in number of snapshots.



Figure 4. RMSE vs. source bearing of MVDR and CS-MVDR algorithms on the conventional APS array (N = 10) and the SAPS array (N = 36, M = 10). SNR= -5 dB, f = 78 Hz.



Figure 5. Bias and RMSE vs. number of snapshots of CS-MVDR algorithm on the SAPS array (N = 36, M = 10). SNR= -5 dB, f = 78 Hz, source bearing, $\theta = 60^{\circ}$.

6. EXPERIMENTAL VALIDATION

6.1 Data Acquisition

A passive towed array sonar was used to capture the acoustic data for validating the performance of the sparse array processing algorithms. The acoustic section of the array consists of 36 pressure sensors, which were uniformly distributed along the array hose to support linear array spatial processing. The towed array sensor signal was fed to a chain of analog signal acquisition hardware and then digitized using 16 bit analog to digital converter (ADC). The front end channel gain of the towed array receiver was configured to utilize the full dynamic range of the ADC. The digitized data was then transmitted to the onboard receiver installed in the tow ship. Power PC processor based PCB was used to receive the acoustic data and the same was stored in a server based data storage unit.

A towing trial was conducted in the deep ocean where water depth is more than 1 km to capture the acoustic signals radiated from the tow ship in the presence of ambient and flow noise. During this experiment, the sound speed profile was in situ measured and confirmed that the profile was very close to the typical sound speed profile for Indian Ocean based on climatological world ocean data base of Conkright⁹, *et al* and is characterized by a deep sonic duct with thickness slightly more than 50 m followed by a steep negative gradient of sound speed below this layer. The towed receiver array was kept below the layer (at 100 m depth) to ensure that the received acoustic energy was primarily due to the direct acoustic path. It is noteworthy that, under this operating scenario the complex multi-modal propagation of acoustic signal is negligible compared to the direct acoustic path.

The sensor data was passed through a spectrum analyzer to identify the dominant component emanating from the tow ship. The array data vector was then passed through a narrowband linear phase finite impulse response band pass filter centered at the dominant component. 18750 time samples (250 Monte Carlo simulations each uses 75 snapshots for conventional and 25 snapshots for CS array processing) were collected at the filtered output for all the 36 sensors to compare the compressed sensing localization performance against the conventional Capons spatial filter angular response.

Figure 6 shows the experimental data acquisition block diagram. It is noteworthy that, in a real compressively sensed towed array system, the spatial compression must be performed at the pre-amplifier output rather than at the ADC output to reduce the hardware complexity of the front end signal acquisition circuitry. However in this experiment, a conventional towed acoustic receiver array was used to capture the tow ship noise and the spatial compression was performed at the ADC output so that both conventional and compressively sampled measurements were made available at the input of the spatial array processor.

6.2 Experimental Results and Discussion

The towed receiver array data obtained from the experiment was subjected to spatial compression, and then the compressed measurements were fed to the CS-MVDR algorithm. Compressed measurements were obtained by projecting the received array data vector to a lower dimensional vector space spanned by a set of random basis vectors which constitutes the measurement matrix. The pseudo angular spectral response of conventional APS array using MVDR filter and SAPS array using CS-MVDR algorithm is shown in Fig. 7. It is seen that, the peak response of the CS-MVDR were slightly inferior compared to the MVDR estimator. The reduction in the signal power may be due to the reduced signal entry into the lower dimensional space. However the side lobe level performance of the CS-MVDR algorithm is much superior in comparison with the MVDR power pattern response. It clearly shows the superior strategic performance of the compressive sensing algorithm with the real towed array data.



Figure 6. Experimental data acquisition-block diagram.



Figure 7. Pseudospectrum of MVDR and CS-MVDR algorithms on the conventional APS array (N=10) and SAPS array (N=36, M=10) using the real towed array data.

7. CONCLUSION

In this paper, the inherent capabilities of the sparse angular reconstruction techniques were explored to design a high performance, hardware efficient towed array sonar for multiple acoustic source localization. The SAPS array architecture significantly reduced the analog signal acquisition hardware, transmission data rate, number of snapshots and software complexity for a specified acoustic performance. The details are presented in Table 1. Simulation results confirmed that, the localization performance of the proposed SAPS array is significantly superior in comparison with the conventional APS array utilizing the same analog signal acquisition hardware complexity. Theoretical formulation and Monte Carlo simulations were provided for sparse angular spectrum construction using the SAPS towed array. Using the recorded towing trial data, the tow ship localization performance with the conventional and the compressively sensed array were compared and the results were presented. It was observed that, the undesirable side lobe levels are significantly less with compressively sensed array in comparison with the conventional array.

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