

Viscoelastic Modelling of Solid Rocket Propellants using Maxwell Fluid Model

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ABSTRACT

Maxwell fluid model consisting of a spring and a dashpot in series is applied for viscoelastic characterisation of solid rocket propellants. Suitable values of spring constant and damping coefficient were employed by least square variation of errors for generation of complete stress-strain curve in uniaxial tensile mode for case-bonded solid propellant formulations. Propellants from the same lot were tested at different strain rates. It was observed that change in spring constant, representing elastic part was very small with strain rate but damping constant varies significantly with variation in strain rate. For a typical propellant formulation, when strain rate was raised from 0.00037/s to 0.185/s, spring constant K changed from 5.5 MPa to 7.9 MPa, but damping coefficient D was reduced from 1400 MPa-s to 4 MPa-s. For all strain rates, stress-strain curve was generated using Maxwell model and close matching with actual test curve was observed. This indicates validity of Maxwell fluid model for uniaxial tensile testing curves of case-bonded solid propellant formulations. It was established that at higher strain rate, damping coefficient becomes negligible as compared to spring constant. It was also observed that variation of spring constant is logarithmic with strain rate and that of damping coefficient follows power law. The correlation coefficients were introduced to ascertain spring constants and damping coefficients at any strain rate from that at a reference strain rate. Correlation for spring constant needs a coefficient H , which is function of propellant formulation alone and not of test conditions and the equation developed is $K_2 = K_1 + H \times \ln\{(d\varepsilon_2/dt)/(d\varepsilon_1/dt)\}$. Similarly for damping coefficient D also another constant S is introduced and prediction formula is given by $D_2 = D_1 \times \{(d\varepsilon_2/dt)/(d\varepsilon_1/dt)\}^S$. Evaluating constants H and S at different strain rates validate this mathematical formulation for different propellant formulations. Stress-strain curves for solid propellants can be generated at those strain rates at which actual testing is not possible. Close matching of test and predicted stress-strain curve indicates propellant behavior as visco-elastic Maxwell fluid.

Keywords: Solid rocket propellants, mechanical properties, viscoelasticity, Maxwell fluid, spring constant, damping coefficient

NOMENCLATURE

E	Elastic modulus or initial modulus
K	Spring constant for spring
D	Damping coefficient for dashpot
H	Correlation coefficient of spring constant for strain rate
S	Correlation coefficient of damping coefficient for strain rate (index or dimensionless)
σ	Stress
ε	Strain
t	Time
d/dt	Derivative wrt time
1, 2	Subscripts to differentiate test cases

1. INTRODUCTION

Solid rocket propellants in operational missiles and rockets are case-bonded composite propellants. These are formulated by dispersing powdered solid oxidiser (65-85 per cent) and metallic fuel (0-18 per cent) in polymeric

binder (10-18 per cent) matrix. The thick viscous slurry thus obtained after thorough mixing is cured at elevated temperature using isocyanate-curatives, as cross-linking agent. Upon curing propellant slurry becomes solid for evaluation of physical, chemical, mechanical, and ballistic properties. Mechanical properties of solid rocket propellants are important from structural integrity considerations and propellants must possess sufficient strength and elongation to absorb various handling, processing, transportation, storage, and operational loads^{2,3}. To ascertain mechanical properties of propellants, specimen are generally prepared⁴ as per ASTM D638 Type IV and tested with constants strain rate of loading machine at nominal strain rate of 0.0185/s in uniaxial tensile mode. Stress-strain curve was generated, which is nonlinear in nature and salient mechanical parameters like initial modulus, tensile strength, elongation at peak stress and elongation at break were ascertained from generated curves. However, these parameters alone are not able to represent nature of solid propellants completely.

Several assumptions have been put forth by researchers about nature of solid propellant as mechanical load-bearing material. The mechanical characterisation of solid propellants started with development of classic theory of physics. Sometimes, it is referred to possess rubber-like elasticity⁵, sometimes visco-elasticity^{6,7} sometimes time-temperature superposition^{8,9}. Many researchers in the recent past have simulated mechanical properties of HTPB-based composite solid propellant by empirical relations¹⁰⁻¹². While characterising mechanical properties for solid rocket propellants, tensile strength, modulus and percentage elongation are frequently mentioned, but nature of complete stress-strain curve is seldom presented and explained^{11,12}.

Invariably propellant shows stress-relaxation behaviour and at constant strain, stress generated in propellant specimen reduces with time. As stress relaxation behaviour can be best represented by Maxwell fluid model, an attempt has been made by the authors to simulate complete stress-strain curve of propellant during uniaxial tensile testing as a Maxwell fluid. Maxwell fluid is represented by a spring and a dashpot arranged in series and is the best way to represent stress-relaxation shown by propellants.

2. MATHEMATICAL FORMULATION

Stress-strain curve under uniaxial tensile test for solid rocket propellant is represented by tandem combination of single spring and single dashpot model, as shown in Fig. 1. Spring behaves as linear elastic component with spring constant of K and stress varies directly as strain in spring [Eqn (1)]. This component of the system represents truly elastic behaviour and is also referred to as Hookean solid model.

$$\text{Stress, } \sigma = K \times \varepsilon_1 \tag{1}$$

where, K = spring constant, ε_1 = Strain in spring due to stress σ .

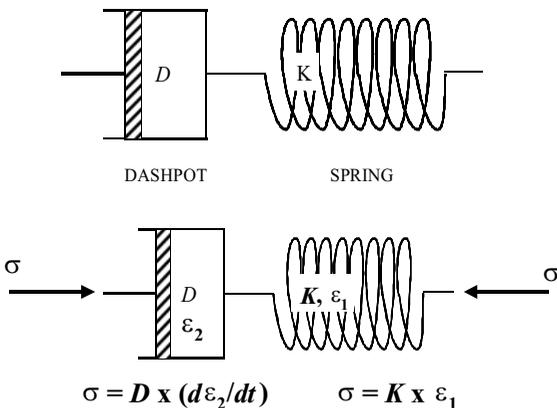


Figure 1. Maxwell model with single spring and single dashpot in series.

On the contrary, dashpot is characterised by damping coefficient D, and stress in this case is proportional to strain rate [Eqn (2)]. This is similar to Newton’s law of viscosity and represents viscous nature of linear Newtonian fluid.

$$\text{Stress, } \sigma = D \times (d\varepsilon_2/dt) \tag{2}$$

where, D = Damping coefficient of dashpot, $d\varepsilon_2/dt$ = Rate of straining.

Since both the components-spring and dashpot are connected in series in Maxwell model, stress at any time in both the components of the system is the same. However, the value of total strain is equal to sum of individual strains in both the components separately. This can be restated, as strain rate of system is sum of individual strain rates of both the components of the model. The governing equation is represented is Eqn (3) as

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \quad \text{or}$$

$$d\varepsilon/dt = (1/K) \times (d\sigma/dt) + \sigma/D \tag{3}$$

During tensile testing, if this model represents propellant specimen, then two independent constants K and D can characterise mechanical behaviour of propellants completely. Tensile testing is generally carried out at constant strain rate making left hand side of Eqn (4) constant. Initial conditions are specified by zero values of both stress and strain. With this boundary condition, correlation between stress and strain in propellant specimen is given by Eqn (4).

$$\sigma = D \times (d\varepsilon/dt) \times (1 - e^{-K \times \varepsilon / \{D \times (d\varepsilon/dt)\}}) \tag{4}$$

Elastic response of any specimen is given by slope of stress-strain curve at zero strain, which is defined as initial modulus or elastic modulus. As per Eqn (4), slope of stress-strain curve is obtained by differentiation of stress wrt strain. Actual differentiation of Eqn (4) results in Eqn (5) as

$$d\sigma/d\varepsilon = K \times e^{-K \times \varepsilon / \{D \times (d\varepsilon/dt)\}} \tag{5}$$

This indicates that slope of stress-strain curve for Maxwell fluid has exponential decay. However, value of slope at no stress known as initial modulus is given by K. It gives initial slope of stress-strain curve as K, which is representative of elastic response of the Maxwell fluid model. It indicates that initial stress is taken by spring alone, and with passage of time, dashpot shares stresses. The mathematical formulation depicted as Eqn (4), is used to generate complete stress-strain curve from bare minimum data and also to predict complete stress-strain curve at different strain rates.

3. EXPERIMENTAL RESULTS

Case-bonded composite propellant formulation containing 15 per cent hydroxy-terminated polybutadiene (HTPB)-based binder, 67 per cent ammonium perchlorate (AP) oxidiser and 18 per cent aluminum (Al) powder as main ingredients was processed. All ingredients were mixed in a vertical planetary mixer in proper sequence. Mixing process passes through binder mixing, incorporation of solid ingredients, and homogenisation. Curing agent, toluene di-isocyanate (TDI) was added in the mix at 38 °C – 40 °C and NCO:OH ratio of 0.8 was maintained. Final mixing was carried out for 30 min and propellant slurry was cast in control motors

around circular mandrel. Motors along with propellants were kept at 50 °C for 5 days in air ovens for curing of the propellants. After curing and decoring, propellant specimens were prepared for evaluation of mechanical properties as per ASTM D638 Type IV. Testing was carried out at different strain rates, varying from 0.00037/s - 0.185/s and stress-strain curves for each case were reproduced as Fig. 2. At each strain rate minimum 5 specimens were tested and reproducibility of data was ensured. Average values have been reported at each test condition.

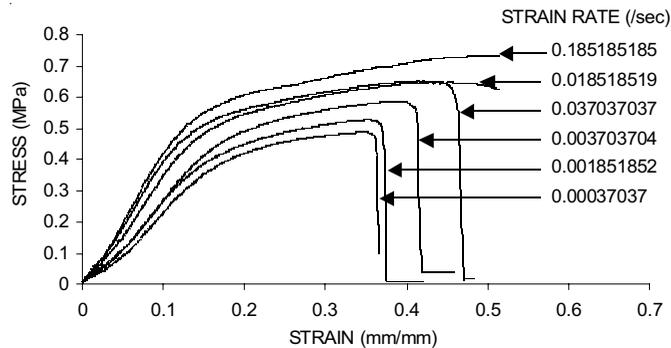


Figure 2. Stress-strain curves for the same propellant at different strain rates.

It is clear that as strain rate increases, value of initial slope representing modulus increases. Same is the trend for tensile strength represented by peak stress attained on stress-strain curve for any given strain rate. Percentage elongation also increases with increasing strain rate. It is clear that curves are showing an initial toe section where variation in slope is observed. However, as per ASTM D638, this part is to be deleted from tensile testing curve. For the nominal strain rate of 0.01852/s, suitable value of spring constant and damping coefficient was obtained by regression analysis for least square fit for errors. The stress-strain curve obtained from Maxwell model using Eqn (4) is reproduced in Fig. 3 along with actual test curve. As per Eqn (5), slope of stress-strain curve has exponential decay and constant slope part of actual stress-strain curve cannot be represented by Maxwell fluid model. Except for initial toe region, Maxwell model curve replicates actual stress-strain curve.

The value of initial modulus from actual test curve

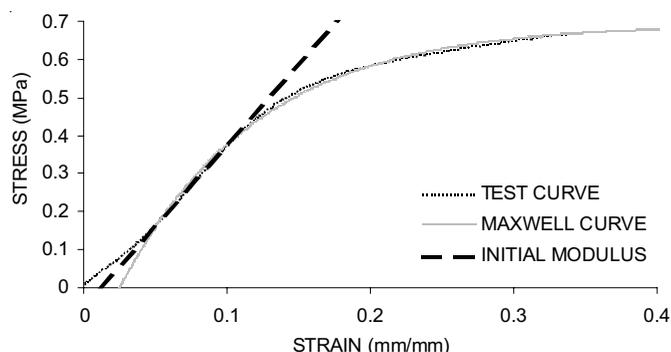


Figure 3. Stress-strain curves from testing a Maxwell fluid model.

is 4.2 MPa. The values of spring constant (K) and damping coefficient (D) are 7.3 MPa and 37.8 MPa-s respectively. Similarly for other strain rates, values of K and D can be enumerated.

4. ANALYSIS AND DISCUSSION

For each strain rate, elastic modulus, spring constant and damping coefficient were generated by least square fit and these values have been tabulated in Table 1. It is clear that value of elastic modulus and spring constants are not identical. However, their ratios (E/K) at different strain rates are varying between 0.54 and 0.60. It is observed that this ratio is more or less constant and is independent of strain rate. For an average estimate of ratio (E/K) as 0.575, spring constant for the propellant at different strain rates can be obtained from the elastic modulus.

Table 1. Value of constants of Maxwell fluid model at different strain rates.

Strain rate (/s)	Elastic modulus(E) (MPa)	Spring constant(K) (MPa)	Damping coefficient(D) (MPa-s)
0.185185	4.80	7.9	4.0
0.037037	4.70	7.5	18.0
0.018518	4.20	7.3	37.8
0.003704	3.55	6.1	158.0
0.001852	3.30	5.6	310.0
0.000370	3.00	5.5	1400.0

Variations of spring constant with strain rate have been plotted and best-fit curve by minimisation of least square of errors was obtained. It is observed that spring constant shows a logarithmic variation. Spring constant at reference strain rate can be used to obtain spring constant at any other strain rate using Eqn (6).

$$K_2 = K_1 + H \times \ln \left\{ \frac{(d\varepsilon_2/dt)}{(d\varepsilon_1/dt)} \right\} \quad (6)$$

where K_2 = spring constant at strain rate $d\varepsilon_2/dt$, K_1 = spring constant at strain rate $d\varepsilon_1/dt$, H = Spring constant strain rate correlation constant fixed for given propellant formulation. For the given propellant formulation, value of H is found as 0.449 MPa.

The value of damping coefficient D has a higher numerical value at lower strain rates. Since damping coefficient represents viscous nature of propellant, it is clear that at low strain rates, viscous part is dominant. However, as strain rate increases, value of damping coefficient reduces drastically, indicating lower contribution of viscous component at higher strain rates. It is also observed that numerically, damping coefficient is more sensitive to strain rate and can represent small variation in strain rate effectively. Compared to elastic modulus or spring constant, damping coefficient is showing better sensitivity to strain rates. Variation of damping coefficient with strain rate is found to follow the power law. Damping coefficient at any strain rate can be obtained from damping coefficient at reference strain rate using Eqn (7).

$$D_2 = D_1 \times \{(d_2/dt)/(d\varepsilon_1/dt)\}^S \quad (7)$$

where D_2 = damping coefficient at strain rate $d\varepsilon_2/dt$, K_1 = damping coefficient at strain rate $d\varepsilon_1/dt$, S = damping coefficient strain rate correlation constant fixed for given propellant formulation.

For the given propellant formulation, value of S is obtained as -0.941 by regression analysis using least square fit. To validate formulation proposed above, especially Eqns (6) and (7), different case-bonded composite propellant formulations were considered.

One of the propellant formulations was tested at three strain rates, i.e., 0.00185/s, 0.01852/s and 0.1111/s. 0.01852/s was taken as reference test condition and value of spring constant and damping coefficient were obtained as 7.6MPa and 38 MPa-s, respectively at this strain rate from stress-strain curves obtained by uniaxial tensile testing. Similarly, at low strain rate of 0.00185/s, value of spring constant and damping coefficient were obtained from test curve as 6.5 MPa and 320 MPa-s, respectively. Using Eqn (6), value of spring constant strain rate correlation constant (H) was obtained as 0.477 MPa. From this value of H spring constant at strain rate of 0.1111/s was found to be 8.45 MPa. Using Eqn (7), value of damping coefficient strain rate correlation coefficient (S) was obtained as -0.925 . Value of damping coefficient at strain rate of 0.1111/s was obtained as 7.24 MPa-s. The test curve is superimposed over Maxwell model curve and is reproduced in Fig. 4.

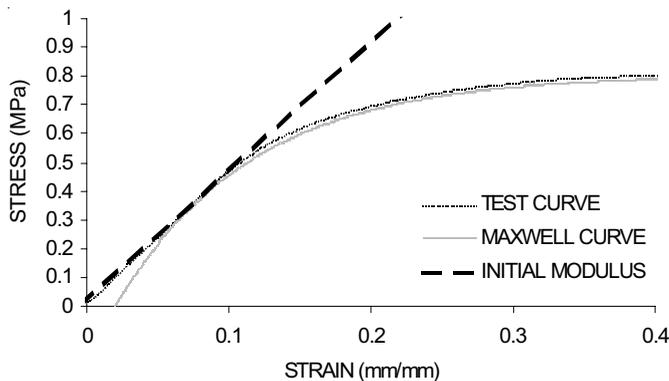


Figure 4. Validation of Maxwell fluid model for generation of stress-strain curve.

It is clear that to a fair degree of accuracy, the formulations are able to reproduce the stress-strain curve for the given propellant formulation at any strain rate from test curve at reference strain rates.

Another case-bonded composite propellant formulation was taken for validation of developed formulations. Uniaxial tensile testing was conducted at strain rates of 0.00185/s and 0.1111/s and results are considered for prediction of stress-strain curves at strain rates of 0.296/s and 0.370/s. Spring constants were obtained as 6.0 MPa and 7.9 MPa, respectively at two reference test conditions. Using Eqn (6), value of H was calculated as 0.464 MPa. Damping coefficients (D) was found to be 325 MPa-s and 7.4 MPa-s, respectively for the tested strain rates. Using Eqn (7), value of S was

calculated as -0.9238 . With data at strain rate of 0.00185/s as reference, at strain rate of 0.296/s, spring constant (K) and damping coefficient (D) were predicted to be 8.35 MPa and 2.99 MPa-s, respectively using Eqns (6) and (7). With these constants, stress-strain curve was generated using Maxwell fluid model Eqn (4) and actual uniaxial tensile test curve is superimposed in Fig. 5. Close matching was again observed. Similarly, Eqn (4) was utilised for prediction of stress-strain curve for strain rate of 0.370/s also for the given propellant formulation. Superimposed test curve and predicted curve is also reproduced in Figs. 5(a) and 5(b).

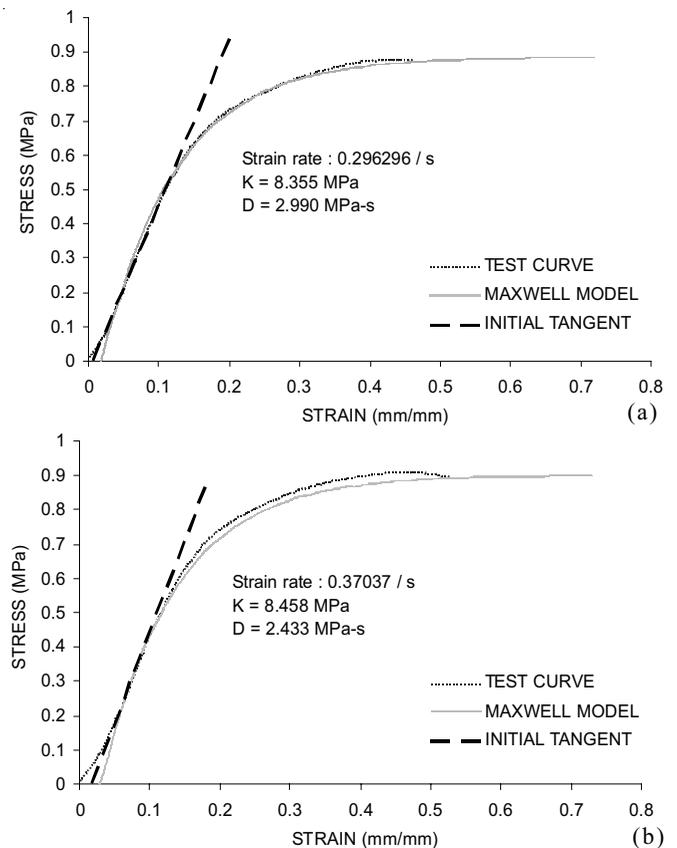


Figure 5. Validation of Maxwell model for another propellant formulation.

Using stress-strain curve for strain rate of 0.00185/s as reference stress-strain curve for 200 times higher strain rate, i.e., 0.370/s can be generated. In fact, in actual motor operation, during initiation, rocket propellants are subjected to very high rates of pressurisation or loadings or strain rates. It is very difficult to simulate those high strain rates in universal testing machine. The developed formulation is a handy and ready-to-use tool for such situations for prediction of stress-strain curves at any strain rate, once stress-strain curves at any two reference strain rates are generated.

In the initial phases of propellant development, propellants are tested at different strain rates for their complete characterisation. After complete characterisation and for already developed formulations, tensile testing is routinely carried out at nominal strain rate of 0.0185/s (equivalent

to test speed of 50 mm/min) for quality control checks, only. It is worth consideration that for propellants, mechanical properties vary significantly with time lapsed and is also very sensitive to change in raw materials, environmental conditions, and processing parameters. Even for developed and productionised propellant formulations, tensile testing at different strain rates is carried out to ascertain reproducibility of properties for any probable change in various control parameters. Any ballistic abnormality always demands generation of mechanical properties at different strain rates.

In place of testing propellant at one strain rate for quality control, better characterisation of solid propellant formulation is possible with generation of stress-strain curve at minimum two reference strain rates. The mathematical formulation developed in this study can be implemented for subsequent data generation. Application of this formulation can reduce testing of propellant specimen at different strain rates and full stress-strain curve can be generated from Maxwell fluid model directly at any strain rate.

5. CONCLUSIONS

Case-bonded solid propellant is modelled as Maxwell fluid. Propellant specimens tested at different strain rates are simulated with two material property variables namely, spring constant K and damping coefficient D . Complete stress-strain curve is generated with proposed formulation and close matching to actual stress-strain curve during uniaxial tensile testing is observed. A method to predict stress-strain curve of uniaxial tensile testing at any strain rate is developed using another two material constants H and S . Spring constant and damping coefficient are found to vary with strain rate in logarithmic and power law fit, respectively. The developed formulation is applied to different case bonded composite propellant formulations and close matching of prediction to test-curve validates the developed formulation. The approach presented in this paper can be applied to any case-bonded composite propellant formulation and with suitable coefficients complete stress-strain curve in uniaxial tensile testing can be generated. So far, material constant is given prime importance by various researchers and first time complete stress-strain curve is considered and reproduced by modelling. With developed formulations, complete stress-strain curve is generated even for those strain rates at which actual testing is not possible. This reduces propellant testing at different operating strain rates and prediction is possible using the formulations described in the paper without actual tensile testing of propellant specimens at different strain rates.

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designed propellant

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