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An Efficient ΣΔ-STAP Detector for Radar Seeker using RPCA Post-processing

Zhao Lei, Luo Xiling*, Sun Jinping, and Lu Songtao#

School of Electronic and Information Engineering, Beihang University, Beijing-100 191, China
#Department of Electrical and Computer Engineering at Iowa State University, IA 50011, USA
**E-mail: luoxiling@buaa.edu.cn*

ABSTRACT

Adaptive detection of moving targets in sea clutter environment is considered as one of the crucial tasks for radar seekers. Due to the severe spreading of the sea clutter spectrum, the ability of space-time adaptive processing with sum and difference beams (ΣΔ-STAP) algorithms to suppress the sea clutter is very limited. This paper, investigated the low-rank property of the range-Doppler data matrix according to the eigenvalue distribution from the eigen spectrum, and proposed an efficient ΣΔ-STAP detector based on the robust principle component analysis (RPCA) algorithm to detect moving targets, which meets the low-rank matrix recovery conditions. The proposed algorithm first adopts ΣΔ-STAP algorithm to preprocess the sea clutter, then separates the sparse matrix of target component from the range-Doppler data matrix through the RPCA algorithm, and finally, effectively detects moving targets in the range-Doppler plane. Simulation results demonstrate the effectiveness and robustness of the proposed algorithm in the low signal-to-noise ratio scenarios.

Keywords: Space-time adaptive processing, low-rank matrix recovery, principal component analysis, accelerated proximal gradient

1. INTRODUCTION

In recent years, great development has been achieved in adaptive radar detection under Gaussian and compound-Gaussian clutter environment[1-5]. Many detectors have been used in radar signal processing as powerful tools of moving target detection, and have a constant false alarm rate (CFAR) property under homogenous data assumption, such as the generalized likelihood ratio test (GLRT)[1-2], the adaptive matched filter (AMF)[1], and the normalized adaptive matched filter (NAMF)[5], and so on. However, the heterogeneous data will result in performance degradation of these detectors in a real environment, since they cannot estimate the clutter covariance matrix accordingly such that greatly mitigates their CFAR properties[4]. To overcome these problems, robust algorithm design in improving the detection performance is practical for space-time adaptive processing (STAP) techniques[6-9].

Classical principle component analysis (PCA) has been widely used in data analysis and compression as one of the most popular tools[10-11]. PCA mainly studies the exact recovery problem from a corrupted low-rank data owing to small errors and noise, and provides the optimal estimation of the lower-dimensional subspace from the observed data. However, PCA cannot effectively deal with incomplete or missing real-world data under large corruption. Recently, a new theoretical framework, called the robust principle component analysis (RPCA), has been proposed for corrupted low-rank data recovery[12-13], which can be applied in many engineering domains, such as background modeling, image processing, and face recognition, and so on. The variant of the Douglas-Rachford splitting method (VDRSM) is used to solve the recovery problem for object detection by exploiting the separable structure in both objective function and the constraint[14]. Principal component pursuit (PCP) is successfully applied to separate ground clutter and moving target in heterogeneous environments[15]. However, the velocity of moving target cannot be estimated due to the inexact extraction.

In this paper, authors find that the range-Doppler data matrix has low-rank property by analyzing its eigenvalue distribution, and consider to incorporate this property into STAP framework to further improve the performance of moving target detection. Therefore, authors extend the idea of the RPCA algorithm to space-time adaptive processing with sum and difference beams (ΣΔ-STAP)[16], and propose an efficient ΣΔ-STAP detector based on the RPCA algorithm to detect moving targets, which meets the low-rank matrix recovery conditions. The proposed algorithm can accurately separate the sparse matrix of moving target from the range-Doppler data matrix after ΣΔ-STAP processing with the NAMF detector, which has the advantages in preserving the target signal during the process of clutter suppression. Simulation results show that the proposed algorithm greatly improves the performance of moving target detection in radar seeker, and also performs robustly in the case of low signal-to-noise ratio (SNR).

2. PROBLEM STATEMENT AND SIGNAL MODEL

Authors consider a radar seeker with the sum and difference beams that transmits a sequence of M coherent pulses during the coherent processing interval (CPI) and

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samples the radar returns on sum-channel and delta-channel. For each pulse, it collects K temporal samples from each channel, where each temporal sample corresponds to a range cell. The entire received data can therefore be organized in a three-dimensional data cube denoted as $\mathrm{X}=[x, x_1, L, x_K]$, where $\mathbf{x} \in \mathbb{C}^{2M\times 1}$ represents the test data in the cell under test (CUT), $\mathrm{x}_k \in \mathbb{C}^{2M\times 1}$, k =1, 2, ..., K, denotes the secondary samples, and $\mathbb{C}$ stands for the complex number field.

Sum-channel data x_Σ and delta-channel data x_Δ can be expressed as follows

$$\mathrm{x}_\Sigma = [x_{\Sigma 1}, \cdots, x_{\Sigma m}, \cdots, x_{\Sigma M}]^T \tag{1}$$

$$\mathrm{x}_\Delta = [x_{\Delta 1}, \cdots, x_{\Delta m}, \cdots, x_{\Delta M}]^T \tag{2}$$

where $x_{\Sigma m}$, $x_{\Delta m}$ are the m^{th} elements of the sum-channel data and delta-channel data, respectively, m = 1, 2,...,M, and $(\bullet)^T$ denotes the transpose.

The test data at a range cell can be rearranged as

$$\mathrm{x}=\begin{bmatrix}\mathrm{x}_\Sigma \\ \mathrm{x}_\Delta\end{bmatrix} = [x_{\Sigma 1}, \cdots, x_{\Sigma m}, \cdots, x_{\Sigma M}, x_{\Delta 1}, \cdots, x_{\Delta m}, \cdots, x_{\Delta M}]^T \tag{3}$$

In the line-of-sight (LOS) direction, the steering vector of the sum-channel and delta-channel can be written as

$$\begin{aligned}\mathrm{s}=\begin{bmatrix}\mathrm{s}_\Sigma \\ \mathrm{s}_\Delta\end{bmatrix} &= [s_{\Sigma 1}, \cdots, s_{\Sigma m}, \cdots, s_{\Sigma M}, s_{\Delta 1}, \cdots, s_{\Delta m}, \cdots, s_{\Delta M}]^T \\ =\begin{bmatrix}\mathrm{s}_\Sigma \\ 0\end{bmatrix} &= [s_{\Sigma 1}, \cdots, s_{\Sigma m}, \cdots, s_{\Sigma M}, 0, \cdots, 0, \cdots, 0]^T\end{aligned} \tag{4}$$

where s_Σ is the steering vector of the sum-channel, and s_Δ is the steering vector of the delta-channel. Note that the response of the delta-channel in a certain direction is usually null, so the steering vector s_Δ can be assumed to be zero.

Now we address the detection problem, which can be formulated in terms of the following binary hypothesis test

$$\begin{cases}\mathrm{H}_0\text{:x=c+n} \\ \mathrm{H}_1\text{:x=d+c+n}\end{cases} \tag{5}$$

where x, c, n, and d are the test data, clutter, noise and signal, respectively. The signal d can be modeled as $\mathrm{d}=\alpha\mathrm{s}$, where s is the steering vector and α is the unknown complex amplitude.

Assuming that the clutter covariance matrix R is known, the NAMF detector is given by

$$\Lambda=\frac{\left|\mathrm{s}^H \mathrm{R}^{-1}\mathrm{x}\right|^2}{\left(\mathrm{s}^H \mathrm{R}^{-1}\mathrm{s}\right)\left(\mathrm{x}^H \mathrm{R}^{-1}\mathrm{x}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{6}$$

where $(\bullet)^H$ denotes the conjugate transpose, and η is the detection threshold[9]. Then, the test statistic of the NAMF detector is compared with a corresponding threshold to determine whether a target is present or not.

The NAMF detector needs to estimate the clutter covariance matrix R in Eqn. (6). However, the NAMF detector has a great loss in performance due to limited sample support, which results in inaccurate estimation of the clutter covariance matrix, especially in heterogeneous environment. In the conventional STAP algorithms, the clutter covariance matrix R can be obtained by the maximum likelihood (ML) estimator which makes use of secondary samples from adjacent range cells to estimate the unknown clutter covariance matrix[6-7], and the sample covariance matrix $\hat{\mathrm{R}}$ is estimated by

$$\hat{\mathrm{R}} = \frac{1}{K}\sum_{k=1}^{K} xx^H \tag{7}$$

The NAMF detector has excellent capability of sidelobe clutter suppression but at the cost of low target sensitivity. Therefore, it is not sensitive to the influence of any signal mismatch where the actual signal is not aligned with the presumed steering vector. Since the sidelobe clutter spreads severely in range-Doppler plane, the NAMF detector is exploited to preprocess the sidelobe clutter.

3. TARGET DETECTION BASED ON RPCA POST-PROCESSING

Recently, the RPCA algorithm is proposed that can accurately recover the low-rank component and the sparse component of observed data corrupted by large errors and noise, and has obvious advantage over classical PCA algorithm in the exact recovery problem. After STAP processing with the NAMF detector, we can get a range-Doppler spectrum image as a data matrix, where moving target has the sparsity in the range-Doppler domain while sea clutter forms a relatively low-rank property. Hence, the RPCA algorithm can be applied to detect moving targets in the range-Doppler plane by solving the convex optimization problem. Meanwhile, the recovery problem can be seen as a semi-definite programming (SDP) problem and solved by the accelerated proximal gradient (APG) algorithm[13].

3.1 RPCA algorithm for STAP

After estimating the covariance matrix $\hat{\mathrm{R}}$ in equation (7), we can obtain the range-Doppler data matrix D using the test statistic Λ from Eqn. (6). For example, the (k,i) entry of matrix D is calculated by employing the steering vector **s** at the k-th frequency to the data **x** of the i-th range cell. The detailed process about how to produce range-Doppler data in the ΣΔ-STAP can be found[16,18].

The new matrix $\mathrm{D} \in \mathbb{C}^{M\times K}$ formed by the range-Doppler data matrix can be decomposed into two matrices, named the low-rank matrix and the sparse matrix, which respectively correspond to the sea clutter component and the moving target one in our problem. Then the new matrix D has the form

$$\mathrm{D=L+S} \tag{8}$$

where $\mathrm{L} \in \mathbb{C}^{M\times K}$ is a low-rank matrix, and $\mathrm{S} \in \mathbb{C}^{M\times K}$ is a sparse matrix.

Taking the singular value decomposition (SVD) of D, we have

$$\mathrm{D=U\Sigma V}^H \tag{9}$$

where $\mathrm{U} \in \mathbb{C}^{M\times M}$ and $\mathrm{V} \in \mathbb{C}^{K\times K}$ are the orthogonal matrices, and $\Sigma \in \mathbb{C}^{M\times K}$ is the diagonal matrix.

Assume that land S can be represented as follows

$$L = \sum_{i=1}^{r} \sigma_i u_i v_i^H \tag{10}$$

$$S = \sum_{i>r} \sigma_i u_i v_i^H \tag{11}$$

where $r \le \min\{M, K\}$ denotes the rank of L as $r = rank(L)$, $u_1,\cdots,u_r \in \mathbb{C}^{M\times 1}$ and $v_1,\cdots,v_r \in \mathbb{C}^{M\times 1}$ are two sets of the singular vectors of U and V, and $\sigma_1,\cdots,\sigma_r \ge 0$ are the singular values of Σ, respectively.

Then, to separate the low-rank matrix and the sparse matrix, we can solve the following convex optimization problem

$$\min_{L,S} \|L\|_* + \delta\|S\|_1 \quad \text{s.t.} \quad D = L + S \tag{12}$$

where $\|\bullet\|_*$ and $\|\bullet\|_1$ denote the nuclear norm and ℓ_1 norm, respectively. $\delta > 0$ is the weighted parameter for balancing that scales as $1/\sqrt{N}$ [12], and selection of the appropriate parameter is discussed in the later Section. Actually, the convex optimization algorithm addressed in Eqn. (12) is usually intractable in theory and practice. Instead of directly solving the Eqn. (12), we can solve the following dual problem equivalently:

$$\min_{L,S} \|L\|_* + \delta\|S\|_1 + \frac{1}{2\mu}\|M - L - S\|_F^2 \tag{13}$$

where $\|\bullet\|_F$ denotes the Frobenius norm, and $\mu > 0$ is the penalty parameter for the violation of the linear constraint.

The APG algorithm is a fast algorithm to solve this dual problem, and its MATLAB codes are available online[17].

3.2 Algorithm description

The procedure of the proposed algorithm is described in detail as follows. We first divide the complex matrix into two matrices according to its real and imaginary parts, and then use the APG algorithm to solve these two dual problems formed by real and imaginary matrices, respectively. After the separation of the real and imaginary parts of the sparse matrix, we unite them into the complex sparse matrix of moving target. Then, we can detect moving targets in the range-Doppler plane.

Now the procedures of the proposed algorithm are given as :

Step (1) Use the ML estimator in Eqn (7) to estimate the clutter covariance matrix $\hat{\mathrm{R}}$ from the secondary samples;

Step (2) Generate a range-Doppler data matrix D by the NAMF detector in Eqn (6);

Step (3) Divide the complex matrix D into real and imaginary matrices, D_R, D_I;

Step (4) Use the APG algorithm to solve the two dual problems in Eqn (13), and obtain two sparse real matrices after low-rank and sparse matrices separation, S_R, S_I;

Step (5) Unite the real and imaginary matrices into the sparse complex matrix, $\mathrm{S} = \mathrm{S}_R + i * \mathrm{S}_I$, and get the final moving target detection result.

Since the iterative process is involved in the APG algorithm, more computation is required in the proposed algorithm than that in the original ΣΔ-STAP detector. The details of computational performance regarding the APG algorithm are additionally discussed and compared[17].

4. EXPERIMENTAL RESULTS

Authors used simulated sea clutter data to validate the performance of target detection in radar seekers. Simulation parameters are set as follows: PRF = 1 kHz, the platform velocity $v = 2000$ m/s, the number of pulses in a CPI $M = 32$, the number of range cells $K = 512$, the antenna scanning angle is 2°, the shape parameter of the K-distributed sea clutter is set as $\omega = 2.5$, the clutter-to-noise ratio (CNR) is 50dB, and the root mean square (RMS) of sea clutter velocity dispersion $\sigma_{sea} = 0.5$ m/s. Meanwhile, there are two moving targets inserted into the simulated sea clutter data, and their parameters are listed in Table 1.

Table 1. Target parameters for numerical simulation

	Range cell	Radial velocity	Line-of-sight angle	SNR
Target I	100	-2.5 m/s	0°	30dB
Target II	300	3.0 m/s	0°	30dB

Firstly, we illustrate the low-rank matrix property of the range-Doppler data matrix. Fig. 1 shows the eigenvalue distribution of the observed range-Doppler data matrix. There are exactly 32 large eigenvalues corresponding to the inflexion of the eigenvalue distribution curve, and others are relatively small compared with 32 large eigenvalues. Therefore, it is reasonable that the proposed algorithm with sparse recovery can suppress the clutter properly in this scenario.

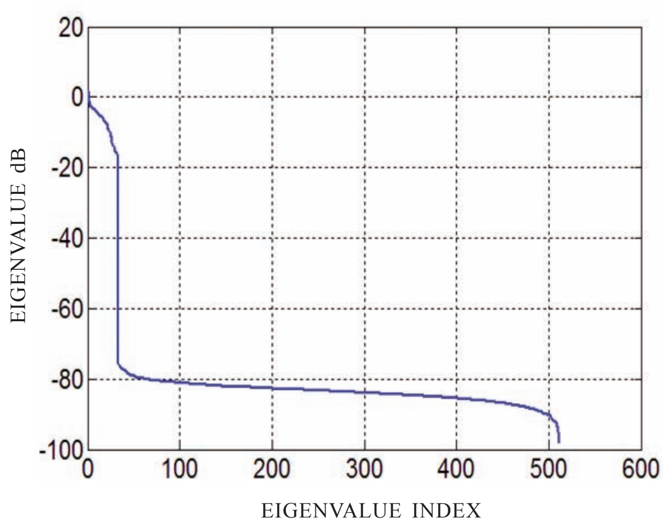

Figure 1. Eigenvalue distribution for the range-Doppler data matrix.

Then, we compare the performance of moving target detection in terms of different SNRs and weighted parameters according to the Monte Carlo simulations[13]. Figure 2 displays the test statistic of two moving targets with the parameter N for different SNRs, where $N = 1/\delta^2$ ranges from 32 to 512. As the value of N becomes larger, the test statistic of moving target I keeps invariant at approximately the same rate for different SNRs in Fig. 2(a). While the case in Fig. 2(b) is different from that in Fig. 2(a), the test statistic of moving target II increases with N, and gradually to the extreme point 1. The reason of this phenomenon is that the strong clutter has influence on the result of targets detection.

Some moving targets may not be detected if N is too small (δ too large)[15]. Also, some clutter residue still exists, meaning that the setting of weighted parameter δ is very important for the performance of moving target detection. Hence, we

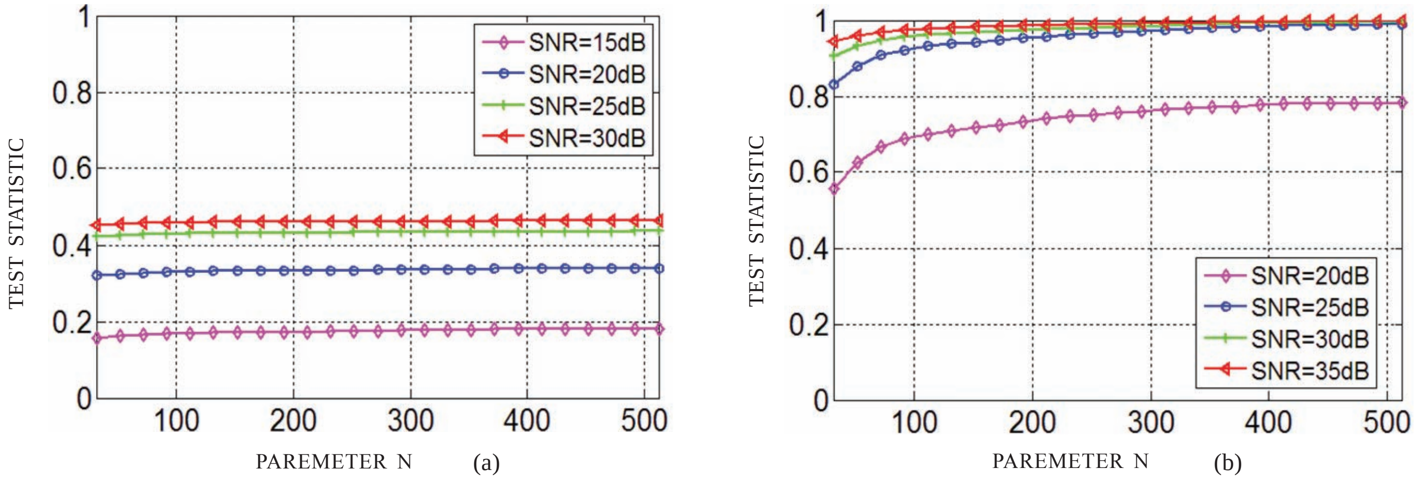

Figure 2. Performance of target detection with the proposed algorithm: (a) Simulation results of target I and (b) Simulation results of target II.

can make a trade off between clutter suppression and target detection. However, in our simulation, the parameter N has little influence on moving target with positive Doppler frequency in range-Doppler area, and a bit influence on moving target with negative Doppler frequency. That is to say, the parameter N selection has little effect on the detection performance, so it is almost negligible, where $\delta = 1/\sqrt{32}$ ($N = \min\{M, L\}$) is selected in the simulation.

Next, the sea clutter range-Doppler spectrum with the scanning angle 2° in radar seeker is shown in Fig. 3(a), and

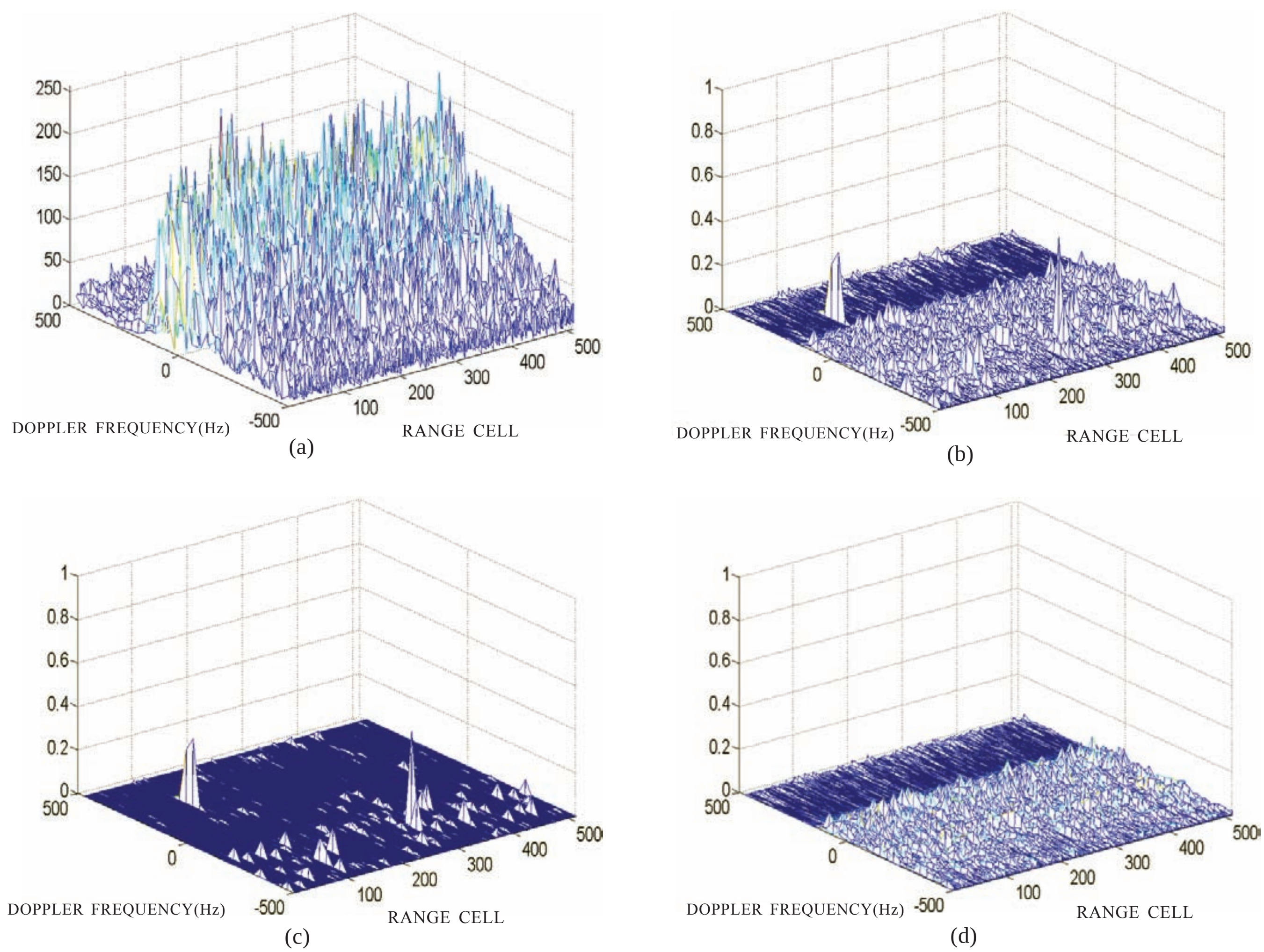

Figure 3. Simulation results with the NAMF detector and the proposed algorithm. (a) Range-Doppler spectrum for sea clutter data, (b) Test statistic of the NAMF detector, (c) Test statistic of target component with the proposed algorithm, and (d) Test statistic of clutter component with the proposed algorithm.

the phenomenon analysis can be referred in[18]. Because the sea clutter spectrum severely spreads in a large part of the range-Doppler plane, two moving targets easily fall into strong clutter. In this case, radar seeker can difficultly detect moving targets. Meanwhile, we plot the curves of test statistic using the NAMF detector and the proposed algorithm in Figs. 3(b), 3(c) and 3(d). Figure 3(b) shows that the NAMF detector can detect two moving targets, but cannot effectively suppress the strong sea clutter, which may lead to high probability of false alarm and the failure of detecting moving targets with a small RCS. The proposed algorithm can also effectively detect two moving targets accompanied with clutter suppression from Figs. 3(c) and 3(d), and have a better performance in suppressing clutter than the NAMF detector. Therefore, the simulation results in Fig. 3 demonstrate the effectiveness and advantages of the proposed algorithm.

Finally, we compare two algorithms in terms of detection performance using 100 snapshots as the secondary samples with $200/P_{fa}$ Monte Carlo trials. To reduce the computational burden, the probability of false alarm (P_{fa}) is set as $P_{fa} = 10^{-3}$ and the corresponding threshold is evaluated. For the sake of convenient simulation, the number of moving targets added in the simulated sea clutter is 20, where half of them are set as the parameters of target I listed in Table 1, and others are set as the parameters of target II. The probability of detection (P_d) is computed as the ratio between the number of detectable targets and the total number of targets. Figure 4 presents the performance of moving target detection. It can be clearly observed that the proposed algorithm outperforms the NAMF detector about 10 dB in improving the performance of moving target detection at $P_d = 0.5$.

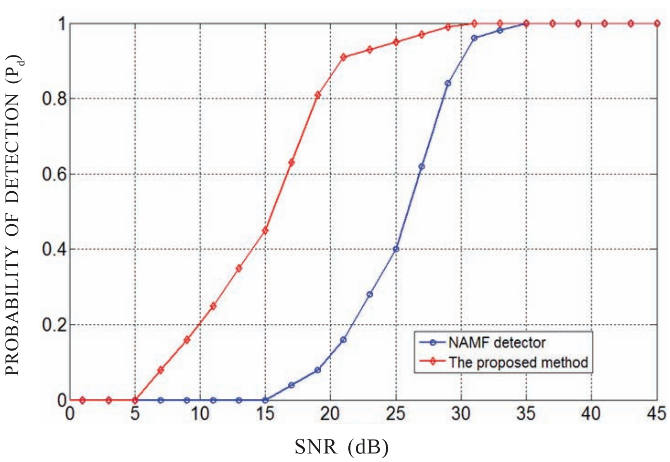

Figure 4. P_d versus SNR with $P_{fa} = 10^{-3}$.

5. CONCLUSIONS

In this paper, an efficient ΣΔ-STAP detector based on the low-rank matrix recovery for moving target detection was proposed in radar seeker. Compared with ΣΔ-STAP processing with NAMF detector, the proposed algorithm can compensate the deficiency of insufficient clutter suppression, and effectively detect moving targets in the range-Doppler plane. Meanwhile, the proposed algorithm can avoid the application of lots of homogeneous samples for STAP training, which is practical in application.

REFERENCES

1. Kelly, E.J. A CFAR adaptive matched filter detector. *IEEE Trans. Aerosp. Electron. Syst.*, 1992, **28**(1), 208-216.
2. Conte, E.; Lops, M. & Ricci, G. Asymptotically optimum radar detection in compound-Gaussian clutter. *IEEE Trans. Aerospace Electro. Sys.*, 1995, **31**(2), 617-625.
3. Sangston, K.J.; Gini, F.; Greco, M.V. & Farina, A. A structures for radar detection in compound-Gaussian clutter. *IEEE Trans. Aerospace Electro. Sys.*, 1999, **35**(1), 445-458.
4. Richmond, C.D. Performance of the adaptive sidelobe blanker detection algorithm in homogeneous environments. *IEEE Trans. Signal Proc.*, 2000, **48**(5), 1235-1247.
5. Rangaswamy, M. Statistical analysis of the non-homogeneity detector for non-Gaussian interference backgrounds. *IEEE Trans. Signal Proc.*, 2005, **53**(6), 2101-2111.
6. Brennan, L.E. & Reed, I.S. Theory of adaptive radar. *IEEE Trans. Aerospace Electro. Sys.*, 1973, **9**(2), 237-252.
7. Ward, J. Space-time adaptive processing for airborne radar. Technical Report TR-1015, MIT Lincoln laboratory, Lexington, MA, Dec. 1994.
8. Wang, H. & Cai, L. On adaptive spatial-temporal processing for airborne surveillance radar systems. *IEEE Trans. Aerospace Electro. Sys.*, 1994, **30**(3), 660-670.
9. Wang, Y.; Bao, Z. & Peng, Y. STAP with medium PRF mode for non-side-looking airborne radar. *IEEE Trans. Aerospace Electro. Sys.*, 2000, **36**(2), 609-620.
10. Eckart, C. & Young, G. The approximation of one matrix by another of lower rank. *Psychometrika*, 1936, **1**(3), 211-218.
11. Jolliffe, I.T. Principal component analysis. New York: Springer-Verlag New York, Inc., Ed. 2nd, 2002.
12. Wright, J.; Ganesh, A.; Rao, S. & Ma, Y. Robust principal component analysis: exact recovery of corrupted low-rank matrices by convex optimization. *In* Proceedings of NIPS, 2009.
13. Lin, Z.; Ganesh, A.; Wright, J.; Wu, L.; Chen, M. & Ma, Y. Fast convex optimization algorithms for exact recovery of a corrupted low-rank matrix. Technical Report UILU-ENG-09-2214, UIUC, Aug. 2009.
14. Zhang, H. & Liu, L. Recovering low-rank and sparse components of matrices for object detection. *Electron. Lett.*, 2013, **49**(2), 109-111.
15. Yan, H.; Wang, R.; Li, F.; Deng, Y. & Liu, Y. Ground moving target extraction in a multichannel wide-area surveillance SAR-GMTI system via the relaxed PCP. *IEEE Geosci. Remote Sens. Lett.*, 2013, **10**(3), 617-621.
16. Brown, R.D.; Schneible, R.A.; Wicks, M.C.; Wang, H. & Zhang, Y. STAP for clutter suppression with sum and difference beams. *IEEE Trans. Aerospace Electro. Sys.*, 2000, **36**(2), 634-646.
17. http://www.perception.csl.illinois.edu/matrix-rank/home.html.
18. Zhao, L.; Sun, J.; Xu, X. & Tian, J. Space-time adaptive processing for sea clutter and jamming suppression in radar

seekers. *In* Proceedings of 2nd International Conference on Signal Processing Systems (ICSPS), Dalian, China, Jul. 2010.

CONTRIBUTORS

Mr Zhao Lei received the MS (circuits and systems) from Xidian University, Xi'an, China, in 2009. He is currently pursuing his PhD (Electrical engineering) at Beihang University, Beijing, China. His current research interests include: Space-time adaptive processing, synthetic aperture radar image interpretation, and digital image processing.

Dr Luo Xiling received the BE and PhD from Beihang University, Beijing, China, in 1996 and 2003, respectively. He is presently working as an Associate Professor at the School of Electronics and Information Engineering, Beihang University, Beijing, China. He has published 20 research papers, co-edited one book. His research interests include air traffic management and radar data processing.

Dr Sun Jinping received the MSc and PhD from Beihang University (BUAA), Beijing, China, in 1998 and 2001, respectively. He is currently a Professor with the School of Electronic and Information Engineering, BUAA. His research interests include : High-resolution radar signal processing, image understanding, and robust beamforming.

Mr Lu Songtao received his MS (Communication and information engineering) from Beihang University, Beijing, China, in 2011. He is currently pursuing the PhD in the Department of Electrical and Computer Engineering at Iowa State University, USA. His primary research interests include : Statistical signal processing, interference mitigation and wireless communications.