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A Note on the Constant Pressure Phase in an Orthodox Gun with Moderated Charges using Hydro-Dynamical Model

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ABSTRACT

The existing work on the constant pressure guns has been improved upon by introducing the concept of Lagrange mean density function for the propellant gases behind the shot. Lagrange hydrodynamical model has been considered and the expressions have been established to work out the shape factor of the second component of the moderated charge, which will maintain a constant pressure during the burning of this component. This is followed by the numerical computations of a problem to establish the physical interpretation of the results.

1. INTRODUCTION

A moderated charge consists of identical grains, each grain consists of layers of different compositions, the first layer to burn being a cool, slow burning type and the succeeding layers being hotter and faster burning surface.

Kapur[1,2] developed the general theory for the moderated charge and Ray[3,4] investigated the possibility of getting a constant driving pressure by using a moderated charge, consisting of two layers. Recently, using a Lagrange density approximation viz,

$$\rho = C_z \left[K_0 + A_x - \frac{C(1-Z)}{\delta} \right]$$

for the propellant gases, Narvilkar[5,6] has solved the internal ballistics equations. Dr. Chugh[7-14] had used this density function to show that the theoretical results thus obtained give closer results to the experimental observation made by ARDE, Canada

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and others as compared to Hunt Hind system. This prompted many workers Kothari, Verma, Prasad etc. to use different density functions. In this paper, it is established that it is always possible, using Lagrange density approximation for the combustion gas, to find a moderate charge with two components which produces constant driving pressure during the burning of the second component. It is assumed that the first component is known and the second component is also known except for the size and shape. Relation from which the size and the shape of the second propellant component are calculated have been determined. Generally, the maximum pressure occurs before the all burnt point of the first component of the moderated charge. If the design is such that the first component has monotonically increasing pressure space curve till the all burnt point, the constant pressure of the second phase then equals the maximum pressure in the entire pressure space curve. To utilise a specified area in the pressure space curve for obtaining the muzzle velocity, the constant pressure value will be higher in the later case against the former. Then the design of the gun will be complex and heavy, till the muzzle end.

The practical situation is the former where the first component reaches a maximum pressure before its all burnt point. The same situation is explained by Table 1. The numerical computation of the motion is shown in Fig. 1. A theoretical approach to calculate maximum pressure in tabular form is appended at A. The shot-travel and the velocity are also calculated during the second constant pressure phase.

Table 1. Experimental data

Vol. of the chamber	120 cu. in.	Shot travel	60 in.
Bore area	7.0543 sq. in.	Shot start pressure	3.5 tsi
Shell wt.	13.32 lbs	K_H	0.1
		γ	1.25

Particulars of the moderated charge

Component charge	Form coefficient	Rate of burning (in./sec.)	Web size (in.)	Force constant (in - tons/lb)	Weight (lb.)	Covolume (cu.in./lb.)	Density (lb./cu.in.)
No. 1	1	0.75	0.018	1900	1.165	27	0.06061
No. 2				1900	1.165	27	0.06061

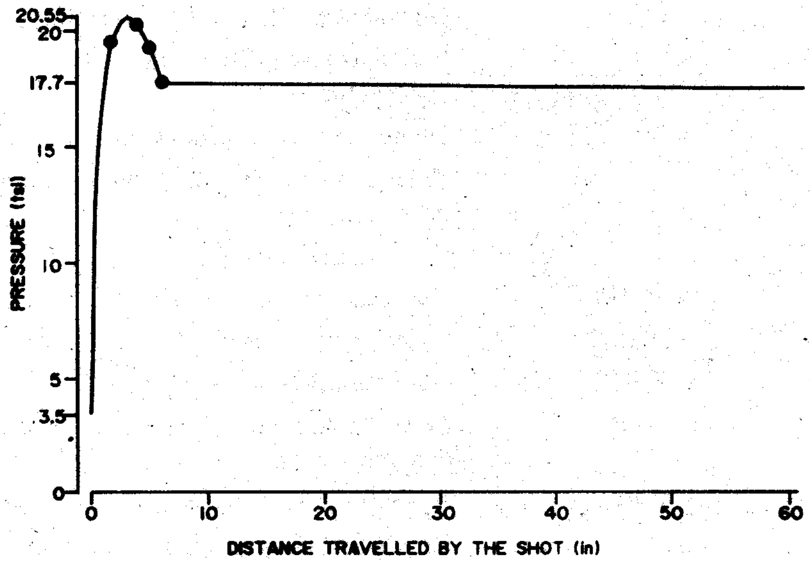

Figure 1. Pressure-space curve

NOTATIONS

Symbols used		Description of the symbols
Dimensional form	Non-dimensional form	
A		Cross-sectional area of the barrel
β		Burning rate constant of the propellant
C		Propellant weight
D		Web size of the propellant
δ		Density of the propellant
F		Force constant of the propellant
f		Fraction of the web size remained at time t
K_0		Chamber capacity
l		Effective length of the free space in the chamber
m		Weight of the shot
t	$\tau = \frac{V_o t}{l}$ where $V = \beta FC/AD$	Time parameter
V_C		Covolume of the propellant

x	$\xi=1+x/l$	Distance travelled by the shot at time t
z		Fraction of weight of the propellant burnt at time t
θ		Shape factor of the propellant
E_P		Kinetic energy of the propellant gas and unburnt propellant
E_h		Heat energy lost to the gun
P_b	$\zeta_b = P_bAl/FC$	Gas pressure at the breech
P_m	$\zeta_m = P_mAl/FC$	Space – mean pressure
P_s	$\zeta_s = P_sAl/FC$	Gas pressure at the shot base
u	$U=u/V_0$	The propellant gas velocity at a distance y from the breech at time t
V	$\eta=V/V_0$	Velocity of the shot at time t
W		External work done on the projectile
y	$Y=y/l$	Distance of the cross-section of gas flow, under consideration from the breech at time t
y_b	$Y_b=y_b/l$	Distance of the burning surface of the propellant from the breech at time t
y_s	$Y_s=y_s/l$	Distance of the shot base from the breech at time t
ρ		Lagrange of density function
	$\gamma=C_p/C$	Ratio of the specific heats of the propellant

Suffixes 1, 2, *AB*1 and *AB*2 denotes the value of the parameter under consideration for the first component charge, second component charge, at the all burnt point of the first component and at the all burnt point of the second component charge respectively.

2. BALLISTIC EQUATIONS WHEN THE FIRST COMPONENT BURNS

During the burning of the first component the Lagrange gas density approximation

viz. $\rho_1 = \dfrac{C_1 z_1}{K_0 + Ax - \dfrac{C_2}{\delta_2} - \dfrac{C_1(1-z_1)}{\delta_1}}$, has been adopted for the density of

the propellant gas. The basic equation as given by Narvilkar[5] are :

$$\frac{df_1}{d\eta} = -\frac{1}{M_1}\cdot(\zeta_b/\zeta_s) \tag{1}$$

$$\frac{d\xi}{d\eta} = \eta/M_1\zeta_s \tag{2}$$

$$\zeta_b = \zeta_s\left(1 + \frac{\epsilon_1 z_1}{2}\right) - \frac{\epsilon_1 z_1 \alpha_1}{2M_1}\cdot\frac{d^2 z_1}{d\tau^2} \tag{3}$$

$$\zeta_m = \zeta_s\cdot\left(1 + \frac{\epsilon_1 z_1}{3}\right) - \frac{\epsilon_1\cdot\alpha_1 z_1}{6M_1}\cdot\frac{d^2 z_1}{d\tau^2} \tag{4}$$

$$z_1 = \zeta_m(\xi - B_1 z_1) + \eta^2\left[\frac{\hat{\gamma} - 1)}{2M_1} + \delta' z_1\right]$$

$$+\delta'\alpha_1 z_1 \frac{dz_1}{d\tau}\left[\alpha_1 \frac{dz_1}{d\tau} - \eta\right] \tag{5}$$

Where

$$\alpha_1 = \frac{C_1}{Al\delta_1}, \quad \epsilon_1 = \frac{C_1}{1.05m}$$

$$B_1 = \left(V_{c1} - \frac{1}{\delta_1}\right)\frac{C_1}{Al}$$

$$Al = K_0 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}$$

$$M_1 = F_1 C_1/(1.05mV_0^2)$$

$$\delta' = \frac{\epsilon_1(\hat{\gamma} - 1)}{6M_1(1 + K_H)}$$

and

$$(\hat{\gamma} - 1) = (\gamma - 1)(1 + K_H)$$

The term $\frac{\epsilon_1\alpha_1 z_1}{2M_1}\cdot\frac{d^2 z_1}{d\tau^2}$ of the Eqn. (3) can be neglected since its maximum value is of the order 3×10^{-4} and is very small as compared to the maximum value of the first term $\zeta_s\left(1 + \frac{\epsilon_1 z_1}{2}\right)$ viz. of the order 0.14. Similarly the term $\frac{\epsilon_1\cdot\alpha_1 z_1}{6M_1}\,\frac{d^2 z_1}{d\tau^2}$ of the Eqn. (4) can be eliminated since its maximum value is of the order 1×10^{-4} which is negligible as compared to the maximum value of the first term $\zeta_s\cdot\left(1 + \frac{\epsilon_1 z_1}{3}\right)$ viz. of order 0.135. Also the maximum value of the term $\delta'\alpha_1 z_1 \frac{dz_1}{d\tau}\left[\alpha_1 \frac{dz_1}{d\tau} - \eta\right]$ is of the order -3×10^{-4} which is very small as compared to the value of 0.987 of the left out R.H.S. terms of the Eqn. (5) and so it can be neglected. So the Eqns. (1) and (2) along with the following equations :

$$\zeta_b = \zeta_s\left(1 + \frac{\epsilon_1 z_1}{2}\right) \tag{6}$$

$$\zeta_m = \zeta_s\left(1 + \frac{\epsilon_1 z_1}{3}\right) \tag{7}$$

and

$$z_1 = \zeta_m(\xi - B_1 z_1) + \eta^2\left[\frac{\gamma - 1)}{2M_1} + \delta' z_1\right] \tag{8}$$

forms the modified system of internal ballistic equations during the burning of the first component.

For solving these equations, we suppose that the first component is known and C_2 and δ_2 (or C_2/δ_2) as known, for Al involves these quantities. Fourth order Runge-Kutta method is used to solve the differential Eqns. (1) and (2), with the initial conditions, viz. at

$$\tau = 0, \quad \eta = 0, \quad \xi = 1, \ \& \ \zeta_s = \zeta_b = \zeta_m = \zeta_{ss}$$

where, ζ_{ss} is non-dimensional value of the shot start pressure. Initial value of Z_1 is found from Eqn. (8). For integrations of equations during the second stage of burning, we will require the values of $(\zeta)_{AB1}$ $(\eta)_{AB1}$ and $(\tau_b)_{AB1}$

3. BALLISTIC EQUATIONS WHEN THE SECOND COMPONENT BURNS

The Lagrange mean density during the second stage of burning becomes $\rho_2 = \dfrac{C_2 z_2 + C_1}{\left[K_0 + Ax - \dfrac{C_2(1 - z_2)}{\delta_2}\right]}$ consider motion of the gas produced during the second stage of burning of the propellant. The equation of continuity is

$$\frac{\partial \rho_2}{\partial t} + \rho_2 \frac{\partial u}{\partial y} = \frac{C_2 \dfrac{dz_2}{dt}}{\left[K_0 + Ax - \dfrac{C_2(1 - z_2)}{\delta_2}\right]} \tag{9}$$

Also equation of conservation of momentum is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial y} = -\frac{1}{\rho_2}\frac{\partial P}{\partial y} \tag{10}$$

where $v\ (y,t)$ is the gas velocity at a distance y from the breech at any time t after the movement of the shot. Making the equations dimensionless and solving continuity Eqn. (9) and momentum Eqn. (10), we get

$$U = \eta + \frac{\left[\eta + \alpha_2 \dfrac{dz_2}{d\tau}\right](Y - Y_s)}{[\xi + \alpha_1 + \alpha_2 z_2]} \tag{11}$$

and

$$\zeta = \zeta_s - \epsilon_2 \frac{d\eta}{d\tau}\frac{[\delta_2\alpha_2 z_2 + \alpha_1\delta_1](Y - Y_s)}{M_1\alpha_1\delta_1[\xi + \alpha_1 + \alpha_2 z_2]}\left[1 + \frac{(Y - Y_s)}{2(\xi + \alpha_1 + \alpha_2 z_2)}\right] \tag{12}$$

where $\epsilon_2 = \dfrac{C_1 + C_2 z_2}{1.05m}$ and $\alpha_2 = C_2/Al\delta_2$

Also while marking equations dimensionless the known parameters of the first component are used.

At $Y = Y_b$, $\zeta = \zeta_b$ so, pressure Eqn. (12) gives

$$\zeta_b = \zeta_s\left[1 + \frac{\epsilon_2\left(z_2\frac{\alpha_2\delta_2}{\alpha_1\delta_1} + 1\right)}{2}\right] \tag{13}$$

Using value of ζ from pressure Eqn. (12) and integrating, we get

$$\zeta_m = \frac{1}{(Y_s - Y_b)}\cdot\int_{Y_b}^{Y_s}\zeta dY = \zeta_s\left[1 + \frac{\epsilon_2\left(z_2\frac{\alpha_2\delta_2}{\alpha_1\delta_1} + 1\right)}{3}\right] \tag{14}$$

Similar to the earlier phase, terms containing $\frac{az_2^2}{d\tau^2}$ are neglected from the Eqns. (13) and (14) since their maximum values are negligible as compared to the values of the remaining terms.

Kinetic energy of the propellant gas during second stage of burning is given by

$$E_P = \frac{A}{2}\int_{Y_b}^{Y_s}\rho_2 u^2 dy = \frac{Al}{2}\rho_2 V_0^2\int_{Y_b}^{Y_s}U^2 dY$$

This gives

$$E_P = \frac{AlV_0^2\eta^2\left[1 - \frac{\alpha_2}{\eta}\frac{dz_2}{d\tau}\right][\alpha_1\delta_1 + \alpha_2\delta_2 z_2]}{6}$$

Kinetic energy of the shot is $mv^2/2$ and friction losses due to bore resistance can be assumed equivalent to 0.05 *x* Kinetic energy of the shot. Let W accounts for the work done by the reaction products in providing Kinetic energy to the shot and propellant gases as well as the dissipation in overcoming bore resistance and heat transfer to the gun barrel. So we have

$$W = \frac{1.05mV^2}{2} + \frac{AlV_0^2\eta^2\left[1 - \frac{\alpha_2}{\eta}\frac{dz_2}{d\tau}\right](\alpha_1\delta_1 + \alpha_2\delta_2 z_2)}{6} + \frac{1.05mV_0^2\eta^2K_H}{2}$$

where energy losses due to heat transfer is assumed as

$$E_h = \frac{1.05mV_0^2\eta^2K_H}{2}$$

Now energy equation for the second phase of burning,

$$F_1C_1 + F_2C_2z_2 = P_m\left[K_0 + A_x - \frac{C_2}{\delta_2} - C_2z_2\left(V_{c_2} - \frac{1}{\delta_2}\right) - C_1V_{c_1}\right] + (\gamma - 1)W$$

becomes

$$1 + \frac{F_2C_2}{F_1C_1} z_2 = \zeta_m[\xi - B1 - B2z_2] + \frac{(\bar{\gamma} - 1)\eta^2}{2M_1} + \delta'\eta^2\left(1 + \frac{\alpha_2\delta_2}{\alpha_1\delta_1} z_2\right) \tag{15}$$

where

$$B_2 = \left(V_{c_s} - \frac{1}{\delta_2}\right) \frac{C_2}{Al}$$

The terms containing $\left(\frac{dz_2}{d\tau}\right)^2$ and $\eta\left(\frac{dz_2}{d\tau}\right)$ are neglected, since their maximum values are very small as compared to the maximum value of the remaining terms of the equation. We have rate of burning equation and form function equation

$$\frac{df_2}{d\eta} = -\frac{\beta_2 D_1}{\beta_1 D_2 M_1}\left(\frac{\zeta_b}{\zeta_s}\right) \tag{16}$$

$$z_2 = (1 - f_2)(1 + \theta f_2) \tag{17}$$

respectively

Further we have equation of shot motion given by

$$d\xi/d\eta = \eta/M_1\zeta_s \tag{18}$$

we have to obtain a solution of Eqns. (15) to (18) with initial conditions.

$$\xi = (\xi)_{AB}, \qquad \zeta_b = (\zeta_b)_{AB1}, \qquad \zeta_m = (\zeta_m)_{AB1}$$
$$\zeta_s = (\xi)_{ABl.} \qquad \eta = (\eta)_{AB1}, \quad \text{at } f_2 = 1$$

Let us assume that a solution of the above equations is possible with

$$\zeta_b = (\zeta_b)_{AB1}, \qquad \zeta_s = (\zeta_s)_{AB1}, \qquad \zeta_m = (\xi_m)_{AB1} \tag{19}$$

and we will find the conditions that this solution may give

$$\xi = (\xi)_{AB1}, \qquad \eta = (\eta)_{AB1} \qquad \text{at } f_2 = 1$$

and that the system of Eqns. (15) to (18) may remain consistent for the solution (19).

The rate of burning Eqn. (16) and equation of motion of the shot (18) with relation (19) becomes

$$\frac{df_2}{d\eta} = -\frac{\beta_2 D_1}{\beta_1 D_2 M_1}\left[(\zeta_b)_{AB1}/(\zeta_s)_{AB1}\right] \tag{20}$$

and

$$\frac{d\xi}{d\eta} = \eta/M_1(\zeta_s)_{AB1} \tag{21}$$

From Eqn. (20) taking f_2 as independent variable, on integraion, we get

$$\eta = (\eta)_{AB1} + \frac{\beta_1 D_2 M_1}{\beta_2 D_1} [(\zeta_s)_{AB1}/(\zeta_b)_{AB1}](1 - f_2) \tag{22}$$

Using relations (22) and (19) in energy Eqn. (15) we get

$$1 + \frac{F_2 c_2 z_2}{F_1 c_1} = (\zeta_m)_{AB1} [\xi - B1 - B2 z_2)$$

$$+[(\eta)_{AB1} + \frac{\beta_1 D_2 M_1}{\beta_2 D_1} \{(\zeta_s)_{AB1}/(\zeta)_{AB1}\}(1 - f_1)]^2 \left[\frac{(\bar{\gamma}-1)}{2M_1} + \delta'\left(1 + \frac{\alpha_2 \delta_2 z_2}{\alpha_1 \delta_1}\right)\right] \tag{23}$$

Now above energy Eqn. (23) and form function Eqn. (17) will give ξ as a function f_2, during the second stage of burning.

We have from Eqn. (20)

$$\frac{d\eta}{df_2} = - \frac{\beta_1 D_2 M_1}{\beta_2 D_1} [(\zeta_s)_{AB1}/(\zeta_b)_{AB1}]$$

or

$$\frac{d}{df_2}\left[\frac{d\xi}{df_2} \cdot \frac{df_2}{d\tau}\right] = - \frac{\beta_1 D_2 M_1}{\beta_2 D_1} [(\zeta_s)_{AB1}/(\zeta_b)_{AB1})] \tag{24}$$

Also we have

$$\frac{df_2}{d\tau} = - \frac{\beta_2 D_1}{\beta_1 D_2} (\zeta_b)_{AB1} \tag{25}$$

Using Eqn. (25) in Eqn. (24), we get

$$\frac{d^2\xi}{df_2^2} = \frac{\beta_1^2 D_2^2 M_1}{\beta_2^2 D_1^2} [(\zeta_s)_{AB1}/(\zeta_b)^2_{AB1}] \tag{26}$$

Now impose condition that energy Eqn. (23) should give $\xi = (\xi)_{AB1}$, $\eta = (\eta)_{AB1}$ at $f_2 = 1$ and further that Eqn (23) be consistent with Eqn. (26).

Now $\xi = (\xi)_{AB1}$, $f_2 = 1$ and $z_2 = 0$ will satisfy Eqn. (23), if

$$1 = (\zeta_m)_{AB1} [(\xi)_{AB1} - B1] + (\eta)^2_{AB1} \left[\frac{(\bar{\gamma} - 1)}{2M_1} + \delta'\right]$$

which is true, for

$$\xi = (\xi)_{AB1}, \quad \eta = (\eta)_{AB1}, \quad \zeta_m = (\zeta_m)_{AB1}, \quad f_1 = 0, \quad z_1 = 1$$

satisfy energy Eqn. (8) for first stage of burning and this equation for those values of the variable is identical with the above equation. Now differentiating energy Eqn. (23) and form function Eqn. (17) w.r.t. f_2, we get

$$\frac{F_2C_2}{F_1C_1}\frac{dz_2}{df_2} = (\zeta_m)_{AB1}\left[\frac{d\xi}{df_2} - B_2\frac{dz_2}{df_2}\right]$$
$$+\left[(\eta)_{AB1} + \frac{\beta_1 D_2 M_1}{\beta^2 D_1}\{(\zeta_s)_{AB1}/(\zeta_b)_{AB1}\}(1-f_2)\right]^2 \frac{\delta'\alpha_2\delta_2}{\alpha_1\delta_1}\frac{dz_2}{df_2}$$
$$-\frac{2\beta_1 D_2 M_1}{\beta_2 D_2}[(\zeta_s)_{AB1}/(\zeta_b)_{AB1}]\left[\frac{(\bar{\gamma}-1)}{2M_1} + \delta'\left(1+\frac{\alpha_2\delta_2 z_2}{\alpha_1\delta_1}\right)\right]$$
$$\times\left[(\eta)_{AB1} + \frac{\beta_1 D_2 M_1}{\beta_2 D_1}\{(\zeta_s)_{AB1}/(\zeta_b)_{AB1}\}(1-f_2)\right] \quad (27)$$

and

$$\frac{dz_2}{df_2} = (1-f_2)\,\theta_2 - (1+\theta_2 f_2) \quad (28)$$

Also we have using Eqn. (19)

$$\frac{d\xi}{df_2} = -\frac{\beta_1 D_2}{\beta_2 D_1}[(\eta)_{AB1}/(\zeta_b)_{AB1}] \quad (29)$$

Using above Eqn. (29) in Eqn. (27) and then applying condition that at $f_2 = 1$, $\eta = (\eta)_{AB1}$

$$\frac{dz_2}{df_2} = -(1+\theta_2) \quad \text{and} \quad z_2 = 0 \quad \text{we get}$$

$$1+\theta_2 = \frac{\alpha_0(\eta)_{AB1}\left[(\zeta_m)_{AB1} + M_1(\zeta_s)_{AB1}\left\{\frac{(\bar{\gamma}-1)}{M_1} + 2\delta'\right\}\right]}{(\zeta_b)_{AB1}\left[\beta_0 + (\zeta_m)_{AB1}\,B2 - (\eta)^2_{AB1}\,\delta'\frac{C_2}{C_1}\right]} \quad (30)$$

where

$$\frac{F_2C_2}{F_1C_1} = \beta_0\,,\quad \frac{\beta_1 D_2}{\beta_2 D_1} = \alpha_0$$

and we have

$$\frac{\alpha_2\delta_2}{\alpha_1\delta_1} = \frac{C_2}{C_1}$$

Now in order to satisfy the condition that Eqns. (26) and (23) should be consistent we differentiate Eqn. (23) twice w.r.t. f_2 so we have

$$\frac{F_2C_2}{F_1C_1}\frac{d^2z_2}{df_2^2} = (\zeta_m)_{AB1}\left[\frac{d^2\xi}{df_2^2} - B2\frac{d^2z_2}{df_2^2}\right]$$
$$+\frac{\delta'\alpha_2\delta_2}{\alpha_1\delta_1}\frac{d^2z_2}{df_2^2}\left[(\eta)_{AB1} + \frac{\beta_1 D_2 M_1}{\beta_2 D_1}\frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}}(1-f_2)\right]^2$$
$$-2\delta'\frac{\alpha_2\delta_2}{\alpha_1\delta_1}\frac{dz_2}{df_2}\left[(\eta)_{AB1} + \frac{\beta_1 D_2 M_1}{\beta_2 D_1}\frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}}(1-f_2)\right]\frac{\beta_1 D_2 M_1}{\beta_1 D_1}$$
$$\times\left[\frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}}\right] - \frac{2\beta_1 D_2 M_1}{\beta_2 D_1}[(\zeta_s)_{AB1}/(\zeta_b)_{AB1}]$$

$$\times \left\{ \delta' \frac{\alpha_2 \delta_2}{\alpha_1 \delta_1} \frac{dz_2}{df_2} \left[(\eta)_{AB1} + \frac{\beta_1 D_2 M_1}{\beta_2 D_1} \frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}} \cdot (1 - f_2) \right] - \frac{\beta_1 D_2 M_1}{\beta_2 D_1} \frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}} \left[\frac{(\bar{\gamma} - 1)}{2M_1} + \delta' \left(1 + \frac{\alpha_2 \delta_2}{\alpha_1 \delta_1} z_2 \right) \right] \right\} \tag{31}$$

Also we have

$$\frac{d^2 z_2}{df_1} = -2\theta_2$$

and at

$$f_2 = 1, \qquad z_2 = 0, \qquad \eta = (\eta)_{AB1}$$

$$\frac{dz_2}{df_2} = -(1 + \theta_2)$$

and

$$\frac{d^2 \xi}{df_2^2} = \frac{\beta_1^2 D_2^2 M_1 (\zeta_s)_{AB1}}{\beta_2^2 D_1^2 (\zeta_b)_{AB1}^2}$$

Using above relations in Eqns. (31) and (30), we get expression for θ_2.

Since it is assumed that properties of the first component of propellant are known and second component is also known except for the size and shape, we will get B_2 and β_0. Eqn. (30) and the above expression of θ_2 gives two relations connecting four parameters, a_0, β_0, θ_2 and B_2, out of these B_2 and β_0 assumed to be known. Hence, we may look upon Eqn. (30) and above expression in θ_2 as two relations for θ_2 and a_0. Now it is to be shown that the values should be positive and θ_2 satisfies $-1 < \theta_2 \leqslant 1$ for practical values of constants involved in Eqn. (30) and expression for θ_2.

The Eqn. (30) and the expression for θ_2 gives expression for a_0.

The expression for θ_2, using this expression for a_0, becomes

$$\theta_2 = \frac{\alpha_0 (\eta)_{AB1} \left[(\zeta_m)_{AB1} + M_1 (\zeta_s)_{AB1} \left\{ \frac{(\bar{\gamma} - 1)}{M_1} + 2\delta' \right\} \right]}{(\zeta_b)_{AB1} \left[\beta_0 + (\zeta_m)_{AB1} B2 - (\eta)_{AB1}^2 \delta' \frac{C_2}{C_1} \right]} - 1 \tag{32}$$

4. SHOT TRAVEL AND VELOCITY DURING THE BURNING OF THE SECOND COMPONENT

We have from velocity Eqn. (22)

$$\eta = (\eta)_{AB1} + \alpha_0 M_1 \frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}} \cdot (1 - f_2)$$

$$\frac{d\xi}{d\tau} = (\eta)_{AB1} + \alpha_0 M_1 \frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}} \cdot (1 - f_2)$$

Using the Eqn. (25) and above equation, we get

$$\frac{d\xi}{df_2} = -\frac{\alpha_0}{(\zeta_b)_{AB2}}\cdot\left[(\eta)_{AB1} + \alpha_0 M_1 \frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}}(1-f_2)\right]$$

Integrating this, we will get

$$(\xi - (\xi)_{AB1}) = \frac{\alpha_0}{(\zeta_b)_{AB1}}\cdot\left[(\eta)_{AB1}(1-f_2) + \frac{\alpha_0 M_1 (\zeta_s)_{AB1}}{2(\zeta_b)_{AB1}}(1-f_2)^2\right] \tag{33}$$

Let $\xi = (\xi)_{AB2}$, when second component burns, at this point we have $f_2 = 0$ so Eqn. (33) becomes

$$[(\xi)_{AB2} - (\xi)_{AB1}] = \frac{\alpha_0}{(\zeta_b)_{AB1}}\cdot\left[(\eta)_{AB1} + \frac{\alpha_0 M_1 (\zeta_s)_{AB1}}{2(\zeta_b)_{AB1}}\right]$$

or

$$\frac{[(\xi)_{AB2} - (\xi)_{AB1}]}{(\xi)_{AB1}} = \frac{\alpha_0\left[(\eta)_{AB1} + \dfrac{\alpha_0 M_1 (\zeta_s)_{AB1}}{2(\zeta_b)_{AB1}}\right]}{(\zeta_b)_{AB1}\cdot(\xi)_{AB1}}$$

Therefore,

$$\text{travel ratio} = \frac{\text{shot travel during the second stage of burning}}{\text{shot travel during the first stage of burning}}$$

$$= \frac{\alpha_0\left[(\eta)_{AB1} + \dfrac{\alpha_0 M_1 (\zeta_s)_{AB1}}{2(\zeta_b)_{AB1}}\right]}{(\zeta_b)_{AB1}\,(\xi)_{AB1}} \tag{34}$$

Also using velocity Eqn. (22) at $f_2 = 0$, $\eta = (\eta)_{AB2}$

we have

$$(\eta)_{AB2} = (\eta)_{AB1} + \alpha_0 M_1 \frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}}$$

Therefore using above equation, we have velocity ratio

$$\frac{(\eta)_{AB2}}{(\eta)_{AB1}} = 1 + \alpha_0 \frac{M_1}{(\eta)_{AB1}}\frac{(\zeta_s)_{AB1}}{(\zeta_b)_{AB1}} \tag{35}$$

5. AFTER ALL BURNT OF SECOND COMPONENT

On integrating Eqn. (21) of motion of the shot, we get

$$\xi = (\xi)_{AB2} + \frac{[\eta^2 - (\eta)^2_{AB2}]}{2M_1(\zeta_s)_{AB1}}$$

or

$$\eta = \sqrt{(\eta)^2_{AB2} + 2\{\xi - (\xi)_{AB2}\}\, M_1(\zeta_s)_{AB1}} \tag{36}$$

Eqn. (36) gives velocity of the shot after all burnt.

6. DISCUSSION OF THE RESULTS AND CONCLUSIONS

Using experimental data given in Table 1, the present technique is studied quantitatively and the results are presented in Table 2. It is found that value of a_0 is positive and θ_2 satisfies. $-1 < \theta_2 \leq 1$. Table 3 contains the particulars of the composite charge data and the results obtained from the Lagrange mean-density method, taken from Narvilkar's[6] research paper regarding composite charge. The weight of the moderated charge is assumed same as the weight of the composite charge. Since the constant pressure has been maintained during the burning of the second component of the moderated charge, the muzzle velocity obtained must be more than the muzzle velocity obtained by considering composite charge. This can be varified from Table 2 and Table 3.

Table 2. Results obtained from the present method by using moderated charge data

$a_0 = 0.1017$	$\theta_2 = -0.0598$
Muzzle velocity = 2582, ft./sec.	Constant mean pressure = 17.7 tsi
Maximum pressure = 20.55 tsi	

Table 3. Particulars of the composite charge of three component charges

Component charge	Form coefficient	Rate of burning (in./sec.)	Web size (in.)	Force constant (in - tons/lb.)	Weight lb	oc	dr	Covolume (cu.in./lb.)	Density (lb./cu.in.)
No. 1	1	0.75	0.018	1900		4	$11\frac{1}{2}$	27	0.06061
No. 2	-0.172	0.75	0.0322	1900		4	12	27	0.06061
No. 3	-0.172	0.75	0.0414	1900	1	11	13	27	0.06061

Results obtained from Lagrange mean density method by using composite charge data

Muzzle velocity	2023 ft./sec.
Maximum pressure	16.5 tsi

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APPENDIX 'A'

Tabular Form of the Maximum Pressure

The Eqn. (1), using the Eqns. (6) and (7) along with the form function

$$z_1 = (1 - f_1)/1 + \theta_1 f_1)$$

simplifies to

$$\frac{df_1}{d\eta} = -\frac{1}{M_1}\cdot\left[1 + \frac{\epsilon_1 z_1}{2}\right]$$

This equation is integrable and the following relation is obtained :

$$f_1 = \frac{[\psi\, e^{-\epsilon_1 a\theta\eta/M_1} + \phi]}{\left[\frac{\psi}{a_2} e^{-\epsilon a\theta\eta/M_1}\ \frac{\phi}{a_1}\right]}$$

where

$$\phi = f_{10} - 1 \qquad a_1 = a + \frac{(1-\theta)}{2\theta}$$

$$\psi = 1 + f_{10}/a_1 \qquad a_2 = a - \frac{(1-\theta)}{2\theta}$$

$$a = \left(\sqrt{4\theta\left(1 + \frac{2}{\epsilon_1}\right) + (1-\theta)^2}\right)\Big/2\theta$$

and f_{10} is the value of f_1 at shotstart.

Using the form function equation, the expression for z_1 becomes

$$z_1 = -\theta\frac{[\psi\, e^{-\epsilon_1 a\theta\eta/M_1} + \phi]^2}{\left[\frac{\psi}{a_2} e^{-\epsilon_1 a\theta\eta/M_1} + \frac{\phi}{a_1}\right]^2} + (\theta - 1)\frac{[\psi\, e^{-\epsilon_1 a\theta\eta/M_1} + \phi]}{\left[\frac{\psi}{a_2} e^{-\epsilon_1 a\theta\eta/M_1} + \frac{\phi}{a_1}\right]} + Z$$

where

$$Z = z_{10} - f_{10}(\theta - 1 - \theta f_{10})$$

and z_{10} is the value of z_1 at shotstart.

Further from the equation of shot motion (2), pressure relations (6) and (7), energy Eqn. (8) and the above relation for z_1, we obtain the following equation similar to Hunt-Hind (18).

$$\int\frac{M_1 d\xi}{(\xi - B_1 z_1)} = \log H$$

where

$$\log H = \int_0^{\eta} \frac{(\bar{a}_1\eta\, e^{-2\epsilon_1 a\theta\eta/M_1} + \bar{a}_2\eta\, e^{-\epsilon_1 a\theta\eta/M_1} + \eta\bar{a}_3)\, d\eta}{[\psi^2 e^{-\epsilon_1 a\theta\eta/M_1}(\theta + \eta^2\bar{a}_4) + \psi e^{-\epsilon_1 a\theta\eta/M_1}(\bar{a}_5 + \eta^2\bar{a}_6) - \phi(\bar{a}_7 + \eta^2\bar{a}_8)]}$$

$$\bar{a}_1 = \psi^2\left(\frac{1}{a_2^2} - e_0\right)$$

$$\bar{a}_2 = \psi\left(\frac{2\phi}{a_1 a_2} - 2e_0 + \frac{b_0}{a_2}\right)$$

$$\bar{a}_3 = \phi\left(\frac{\phi}{a_1^2} - e_0\phi + \frac{b_0}{a_1}\right)$$

$$\bar{a}_4 = \frac{c_0 + d_0}{a_2^2} - \theta\delta'$$

$$\bar{a}_5 = 2\theta\phi + \frac{(\theta - 1)}{a_2}$$

$$\bar{a}_6 = \frac{2[c_0 + d_0].}{a_2} - \frac{(\theta - 1)\delta'}{a_2} - 2\phi\theta\delta'$$

$$\bar{a}_7 = \phi\theta - \frac{(\theta - 1)}{a_1}$$

$$\bar{a}_8 = \frac{\phi}{a_1^2}(c_0 + d_0) + \frac{(\theta - 1)}{a_1}\delta' - \phi\theta\delta'$$

$$a_0 = \frac{1 + \epsilon_1 z}{3}, \qquad b_0 = \frac{\epsilon_1}{3}(\theta - 1), \qquad c_0 = \frac{(\gamma - 1)}{2M_1}$$

$$d_0 = \delta' z, \qquad e_0 = \epsilon_1\theta/3$$

The analysis subsequently remains the same as in the above reference and therefore is not repeated. One can easily verify if the first order solutions of f_1 in ε_1 are approximated, the effect of modified gas density function disappears.

Secondly the integral of log H contains exponentials and the form is non-integrable. The solution of log H in closed form can be developed in the tabular form only.

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