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Natural Convection in Unsteady Couette Motion

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ABSTRACT

Unsteady free convective flow of an incompressible viscous fluid between two vertical parallel plates is considered for impulsive start of one of the plates. Expressions for velocity and temperature fields and their related quantities are obtained by the Laplace transform technique. The effect of Grashof number is to increase the velocity of both air and water and to decrease the skin-friction of these fluids.

1. INTRODUCTION

An analysis of flow formation in Couette motion, as predicted by classical fluid mechanics, was presented by Schlichting[1]. This problem is of fundamental importance as it provides the exact solution of the Navier-Stokes equation and reveals how the velocity profiles varies with time, approaching a linear distribution asymptotically, and how the boundary layer spreads throughout the flow field. The Couette flow of a viscous, incompressible and electrically conducting fluid between two infinite parallel plates in the presence of a uniform transverse magnetic field was studied by Katagiri[2]. The motion of the fluid is induced by the impulsive motion of one of the plates. By assuming the magnetic Reynolds number to be small and applying the Laplace transform technique he obtained the fluid velocity and the skin-friction on the surface of the lower plate. Muhuri[3] extended the study on the Couette flow between two porous walls when one of the walls moves with uniform acceleration and there is uniform suction and injection. Further, Singh and Kumar[4] studied the same problem when the magnetic lines of force are fixed relative to the moving plate.

In the recent years, the study of free convection phenomenon has been the object of extensive research. The intensity of research in this field is due to enhanced concerns in science and technology about buoyancy – induced motions in the atmosphere, in bodies of water and in quasi-solid bodies such as earth. Heat transfer effects under

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the conditions of free convection are now dominant in many engineering applications such as rocket nozzels, cooling of nuclear reactors, high sinks in turbine blades, high speed aircrafts and their atmospheric re-entry, chemical devices and process equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural field and groves of fruit trees, damage of crops due to freezing and pollution of the environment and so on.

The present work concerns with the effect of free convection on the unsteady Couette motion. In Section 2, we have given the formulation of the problem. The resulting system of coupled linear partial differential equations have been solved by the Laplace transform method. In Section 3, we have presented the results graphically for velocity and temperature fields. Numerical values for skin-friction and rate of heat transfer parameters have been tabulated.

2. MATHEMATICAL ANALYSIS

Consider the unsteady flow of a viscous and incompressible fluid between two infinite, vertical-parallel plates separated by a distance h. At time $t' \leqslant 0$, both the fluid and plates are assumed to be at rest. One of the plate starts moving at $t' > 0$ in its own plane with an impulsive velocity U, while the other plate at distance h apart, is kept fixed. Choose a cartesian co-ordinate system with x'-axis along the moving plate in the upward direction and y'-axis perpendicular to it. As the plates are infinite in length the velocity and temperature fields are functions of y' and t' only. The equation of motion in x'-direction and energy equation are given by

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_h) + \nu \frac{\partial^2 u'}{\partial y'^2}, \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2}, \tag{2}$$

and the equation of continuity being identically satisfied, where g is acceleration due to gravity, β is the volumetric co-efficient of expansion, ρ is the density, ν is the kinematic viscosity, K is the thermal conductivity and C_p is the specific heat under constant pressure.

The initial and boundary conditions are

$$u' = 0, \quad T' = T'_h \quad \text{at} \quad t' \leqslant 0 \quad \text{and} \quad 0 \leqslant y' \leqslant h$$

$$\left.\begin{array}{lll} u' = U, & T' = T'_w & \text{at } y' = 0 \\ u' = 0, & T' = T'_h & \text{at } y' = h \end{array}\right\} \text{for } t' > 0 \tag{3}$$

By use of the following dimensionless quantities :

$$y = y'/h,\ t = t'\nu/h^2,\ u = u'/U,\ P = \mu\, Cp/K,$$
$$\theta = (T' - T'_h)/(T'_w - T'_h),\ G = \beta g\,(T'_w - T'_h)\, h^2/\nu U, \tag{4}$$

Eqns. (1) and (2) become

$$\frac{\partial u}{\partial t} = G\theta + \frac{\partial^2 u}{\partial y^2}. \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P}\frac{\partial^2 \theta}{\partial y^2}, \tag{6}$$

with the initial and boundary conditions

$$\begin{aligned} &u = 0, \quad \theta = 0 \quad \text{at} \quad t \leqslant 0 \quad \text{and} \quad 0 \leqslant y \leqslant 1 \\ &\left.\begin{aligned} &u = 1, \quad \theta = 1 \quad \text{at} \quad y = 0 \\ &u = 0, \quad \theta = 0 \quad \text{at} \quad y = 1 \end{aligned}\right\} \quad \text{for} \quad t > 0 \end{aligned} \tag{7}$$

Applying the Laplace transformation in Eqns. (5) and (6), we have

$$\frac{d^2\bar{u}}{dy^2} - s\bar{u} = -G\bar{\theta}, \tag{8}$$

$$\frac{d^2\bar{\theta}}{dy^2} - Ps\bar{\theta} = 0, \tag{9}$$

under the boundary conditions

$$\begin{aligned} &\bar{u} = \frac{1}{s}, \quad \bar{\theta} = \frac{1}{s} \quad \text{at} \quad y = 0; \\ &\bar{u} = 0, \quad \bar{\theta} = 0 \quad \text{at} \quad y = 1. \end{aligned} \tag{10}$$

The solution of Eqns. (8) and (9) under the boundary conditions (10) is given by

$$\begin{aligned} \bar{u} = {} & \frac{1}{s}\sum_{k=0}^{\infty}\{e^{-a\sqrt{s}} - e^{-b\sqrt{s}}\} + \frac{G}{(P-1)s^2}\sum_{k=0}^{\infty}[\{e^{-a\sqrt{s}} \\ & - e^{-a\sqrt{Ps}}\} - \{e^{-b\sqrt{s}} - e^{-b\sqrt{Ps}}\}], \end{aligned} \tag{11}$$

$$\bar{\theta} = \frac{1}{s}\sum_{k=0}^{\infty}\{e^{-a\sqrt{Ps}} - e^{-b\sqrt{Ps}},\} \tag{12}$$

where $a = 2k + y$ and $b = 2 + 2k - y$.

After the inverse transformation, the velocity and temperature fields are given by the expressions

$$\begin{aligned} u = {} & \sum_{k=0}^{\infty}\left[erfc\left(\frac{a}{2\sqrt{t}}\right) - erfc\left(\frac{b}{2\sqrt{t}}\right)\right] \\ & + \frac{G}{(P-1)}\sum_{k=0}^{\infty}\left[\left\{\left(t + \frac{a^2}{2}\right) erfc\left(\frac{a}{2\sqrt{t}}\right) - a\sqrt{\frac{t}{\pi}}\, e^{-a^2/4t}\right.\right. \\ & \left. - \left(t + \frac{a^2P}{2}\right) erfc\left(\frac{a\sqrt{P}}{2\sqrt{t}}\right) + a\sqrt{\frac{Pt}{\pi}}\, e^{-Pa^2/4t}\right\} \end{aligned}$$

$$-\left\{\left(t+\frac{b^2}{2}\right) erfc\left(\frac{b}{2\sqrt{t}}\right) - b\sqrt{\frac{t}{\pi}}\, e^{-b^2/4t}\right.$$

$$-\left(t+\frac{Pb^2}{2}\right) erfc\left(\frac{b\sqrt{P}}{2\sqrt{t}}\right)$$

$$\left.\left. + b\sqrt{\frac{Pt}{\pi}} e^{-Pb^2/4t}\right\}\right], \tag{13}$$

$$\theta = \sum_{k=0}^{\infty}\left[erfc\left(\frac{a\sqrt{P}}{2\sqrt{t}}\right) - erfc\left(\frac{b\sqrt{P}}{2\sqrt{t}}\right)\right] \tag{14}$$

Using the expressions (13) and (14), the skin-friction at the moving plate and the rate of heat transfer from the moving plate to the fluid are given by

$$\tau = \frac{h\tau'}{\mu U} = -\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= \frac{1}{\sqrt{\pi t}} \sum_{k=0}^{\infty} \{e^{-k^2/t} + e^{-(1+k)^2/t}\} + \frac{G}{P-1}\sum_{k=0}^{\infty}\left[2\sqrt{\frac{t}{\pi}}\{e^{-k^2/t}\right.$$

$$+ e^{-(1+k)^2/t}\} - 2\sqrt{\frac{Pt}{\pi}}\{e^{-Pk^2/t} + e^{-P(1+k)^2/t}\}$$

$$+2k\left\{P erfc\left(\frac{k\sqrt{P}}{\sqrt{t}}\right) - erfc\left(\frac{k}{\sqrt{t}}\right)\right\} + 2(1+k)\left\{P erfc\left[\frac{(k+1)\sqrt{P}}{\sqrt{t}}\right]\right.$$

$$\left.\left. - erfc\left(\frac{1+k}{\sqrt{t}}\right)\right\}\right], \tag{15}$$

and

$$Nu = \frac{q'h}{K(T'_w - T'_h)} = -\left.\frac{\partial \theta}{\partial y}\right|_{y=0}$$

$$= \sqrt{\frac{P}{\pi t}} \sum_{k=0}^{\infty}\left[e^{-Pk^2/t} + e^{-P(1+k)^2/t}\right], \tag{16}$$

respectively, where τ is the non-dimensional skin-friction and Nu stands for the Nusselt number which represents here the dimensionless co-efficient of heat transfer. For $G = 0$, Eqn. (13) gives the result of Schlichting[1].

3. DISCUSSION

In order to study the effects of the various parameter on the velocity field and temperature field, numerical calculations are carried out for different values of G, P and t. The values of P are taken as 0.71 and 7.0 which physically corresponds to air and water, respectively. The value of G indicates the state of plates. Since G depends on the plates, it can take positive, zero or negative values depending on the temperature of the plates. If $T'_W - T'_h < 0$, then the moving plate temperature is less than the temperature of the other plate and hence the convection currents flow towards the moving plate and in this case the moving plate is heated from the externally supplied heat energy of the fluid. Hence $G < 0$ corresponds to an external heating of the moving plate. Similarly $G > 0$ corresponds to external cooling of the moving plate and $G = 0$ corresponds to the absence of the convection currents.

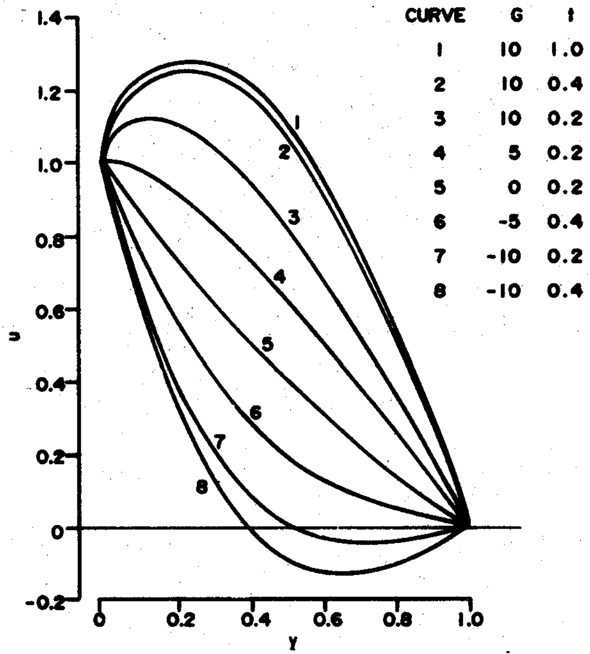

Figure 1. Velocity profiles of air ($P = 0.71$).

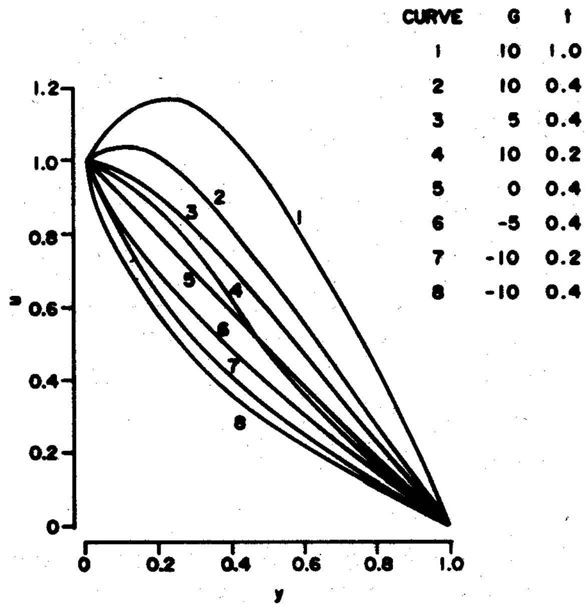

Figure 2. Velocity profiles of water ($P = 7.0$)

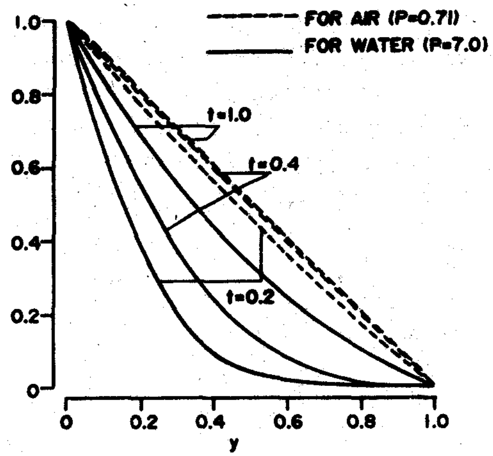

Figure 3. Temperature profiles.

Table 1. Numerical values of skin-friction

G	t	Values of τ	
		$P=0.71$	$P=7.0$
−10.0	0.2	3.9313	2.6448
−10.0	0.4	4.1096	2.8232
− 5.0	0.2	2.5964	1.9531
− 5.0	0.4	2.5008	1.8576
5.0	0.2	−0.0733	0.5699
5.0	0.4	−0.7167	−0.0735
10.0	0.2	−1.4082	−1.2168
10.0	0.4	−2.3255	−1.0391

Table 2. Values of rate of heat transfer

t	Values of Nu	
	$P=0.71$	$P=7.0$
0.2	1.0794	3.3377
0.4	0.8414	2.3601

Figs. 1 and 2 show the velocity profiles of air ($P = 0.71$) and water ($P = 7.0$) respectively. It is seen from these figures that the effect of Grashof number is to increase the velocity of air and water. We also observe that, in case $G < 0$ (Fig. 1) the velocity of air takes negative values and this means that there is reverse type of air motion. For the case $G > 0$ the velocity of air and water increases with time whereas decreases for $G < 0$. The temperature profiles of air and water are shown in Fig. 3. We conclude that the temperature of air and water increases with time. The numerical values of the skin-friction and those of rate of heat transfer are entered in Tables 1 and 2 respectively. We observed that skin-friction of both air and water decreases as the values of G increases. The values of Nusselt number decreases with time and the rate of heat transfer in water is greater than air.

REFERENCES

1. Schlichting, H., Boundary Layer Theory, (McGraw-Hill, New York), 1979.
2. Katagiri, M., *J. Phys. Soc. Japan,* **17** (1962), 393.
3. Muhuri, P.K., *J. Phys. Soc. Japan,* **18** (1963), 1671.
4. Singh, A.K. & Kumar, N., *Wear 89* (1983), 125.