Def Sci J, Vol 40, No. 3, July 1990, pp 275-282

Transient Heat Transfer in Composite Solids with Non-Linear Boundary Condition

M.G. Chopra and I.J. Marwah

Defence Science Centre, Delhi–110 054

ABSTRACT

Transient heat transfer in composite media with non-linear radiation boundary condition has been studied by implicit finite difference scheme. It is observed that as the diffusivity ratio decreases there is more transfer of heat from radiating surface towards the non-radiating surface.

NOMENCLATURE

C	specific heat
$K = K_1/K_2$	ratio of thermal conductivities
$k = k_1/k_2$	ratio of thermal diffusivities
l	thickness of slab
R	number of intervals
s	source parameter
t	time
$T_{i,n}$	non-dimensional temperature at ith nodal point and nth time step
u	temperature
X	distance along thickness of slab
$x = X/l$	non-dimensional distance
Δx	distance step
ρ	density
τ	non-dimensional time
$\Delta\tau$	time step

Received 19 August 1988

1. INTRODUCTION

Radiative heat transfer studies are a common phenomenon when we are working in the domain of high temperature. They become more involved because of the non-linearity resulting from the Stephan-Boltzman radiation term. Radiation heat transfer studies are of immense importance in heat exchangers for space studies and a number of scientists have worked on different issues of this specialisation. Kreith[1] has discussed the detailed problem regarding the radiation heat exchanger design. Gray and Miller[2] have also studied the thermal radiation for reflection and refraction in the same way as visible electromagnetic radiation. The numerical method used for the coupled differential equations was reported by Von Rosenberg[3] in which he has solved the differential equations by balancing enthalpy and reactant materials of a packed bed reactor and the equations were coupled through the term describing the chemical reactions. In a particular case Harris and Weaver[4] obtained the solution of equations which described the transmission of potential impulse along a nerve. Douglas and Jones[5] explained the Crank-Nikolson technique for solving non-linear partial differential equations.

In the present investigation, the composite problem with the non-linear boundary conditions at one of its faces has been solved numerically using Crank-Nikolson Method. The non-linear term arising in the boundary conditions leads to the problem of solving a non-linear algebraic equation in the finite difference algorithm.

2. FORMULATION AND SOLUTION

Let a composite solid extending from $x = 0$ to $x = 1$ consist of two infinitely extending plates of thickness 0.5 each. Let there be a continuous flux across the junction and the sides $x = 0$ and $x = 1$ be maintained at temperatures 0 and 1 respectively. Initially both the solids are at 0°C. Suddenly the face $x = 1$ is brought into contact with a medium which receives heat by radiation.

The energy equations and the accompanying initial and boundary conditions using the non-dimensional numbers

$$x = \frac{X}{l};\ \tau = \frac{k_1 t}{l^2};\ T_1 = \frac{u_1}{u_0};\ T_2 = \frac{u_2}{u_0},\ \text{and}\ k = \frac{k_1}{k_2} \tag{1}$$

become

$$\begin{aligned} \frac{\partial^2 T_1}{\partial x^2} &= \frac{\partial T_1}{\partial \tau} \quad \text{for} \quad 0 < x < 0.5 \\ \frac{\partial^2 T_2}{\partial x^2} &= k\frac{\partial T_2}{\partial \tau} \quad \text{for} \quad 0.5 < x < 1.0 \end{aligned} \tag{2}$$

$$\begin{aligned} &T_1 = T_2 = 0 \quad \text{for all} \quad x \text{ and } \tau = 0 \\ &T_1 = 0 \quad \text{for} \quad x = 0 \text{ and } \tau > 0 \end{aligned} \tag{3}$$

$$s(1 - T_2^4) = \frac{\partial T_2}{\partial x} \quad \text{at } x = \quad \text{for } \tau > 0$$

where s is a non dimensional parameter depending upon source. The flux condition at the interface may be written as :

$$\partial T_1 \; - \; \frac{1}{k}\,\frac{\partial T_2}{\partial x} \qquad \text{at } x = 0.5 \tag{4}$$

Dividing the region $x = 0$ to $x = 1$, in R equal intervals and denoting the temperature at the ith nodal point and nth time step by $T_{i,n}$ the second order correct implicit finite difference analogue of Eqn. (2) may be expressed as follows :

For $2 \leqslant i \leqslant \frac{R}{2} - 1$

$$\begin{aligned} &T_{i-1,n+1} + (-2 - a)T_{i,n+1} + T_{i+1,n+1} \\ &\quad = -T_{i-1,n} + (2 - a)T_{i,n} - T_{i+1,n} \end{aligned} \tag{5}$$

where

$$a \qquad 2\frac{(\Delta x)^2}{\Delta \tau}$$

For $\quad i + \frac{R}{2} \leqslant i \leqslant R \quad 2$

$$\begin{aligned} &T_{i-1,n+1} + (-2 - ka)T_{i,n+1} + T_{i+1,n+1} \\ &\quad = -T_{i-1,n} + (2 - ka)T_{i,n} - T_{i+1,n} \end{aligned} \tag{6}$$

For $i = 1$

$$a)T_{1,n+1} + 2T_{2,n+1} = (2 \quad a)T_{1,n} - T_{2,n} \tag{7}$$

For $i = R/2$

$$\begin{aligned} &kT_{(R/2)-1,n+1} - (k + 1)T_{(R/2),n+1} + T_{(R/2)+1,n+1} \\ &\quad = -kT_{(R/2)-1,n} + (k + 1)T_{(R/2),n} - T_{(R/2)+1,n} \end{aligned} \tag{8}$$

where

$$k \quad \frac{k_1}{k_2}$$

Boundary condition at $i = R$ is obtained by using backward difference method :

$$T_{i-1,n+1} + \left(-2 - \frac{ka}{2}\right)T_{R,n+1} + T_{R+1,n+1} = -kaT_{R,n} \tag{9}$$

For $i = R$

$$T_{R-1,n+1} + \left(\quad 2 \quad \frac{ka}{2}\right)T_{R,n+1} + T_{R+1,n+1} \qquad -kaT_{R,n} \tag{10}$$

Using radiation boundary condition, at $i = R$

$$s(1 - T^4_{R,n+1}) - \frac{T_{R+1,n+1} - T_{R-1,n+1}}{2\Delta x} = 0 \tag{11}$$

substituting the value of $T_{R+1,\,n+1}$ in Eqn. (10)

$$2T_{R-1,n+1} + \left(-2 - \frac{ka}{2}\right)T_{R,n+1}$$
$$= (-kaT_{R,n} - 2s\Delta x) + 2s\Delta x T^4_{R,n+1}$$

Writing

$$\beta_i = b_i - \frac{a_i c_{i-1}}{\beta_{i-1}}, \qquad \gamma_i = d_i - \frac{a_i \gamma_{i-1}}{\beta_i}$$

$$\beta_R = \left(-2 - \frac{ka}{2}\right) - \left(\frac{2}{\beta_{R-1}}\right)$$

$$T_{R,n+1} = \gamma_R = \frac{d_R - a_R\gamma_{R-1}}{\beta_R}$$

$$T_{R,n+1} = (-ka - 2s\Delta x - a_R\gamma_{R-1})/\beta_R$$
$$+ (2s\Delta x/\beta_R)T^4_{R,n+1}$$

This Eqn. can be written as

$$f(T) = 0$$

with $f(T) = p + qT^4_{R,n+1} - T_{R,n+1}$

where

$$p = -(ka + 2s\Delta x + a_R\gamma_{R-1})/\beta_R$$
$$q = 2s(\Delta x)/\beta_R$$

This fourth order equation has been solved by using *regula falsi* method. The two values initially have been supposed to be 0 and 1 and the root of the Eqn. (14) has been estimated using

$$T^{(3)} = \frac{f^{(1)}T^{(2)} - f^{(2)}T^{(1)}}{f^{(1)} - f^{(2)}}$$

where the superscripts denote the iterative values and subscripts are omitted from $T_{R,\,n+1}$. After $T^{(3)}$ is found, $f^{(3)}$ is obtained from Eqn. (14). $T^{(4)}$ is computed from Eqn. (15) with $T^{(3)}$ and $f^{(3)}$ replacing $T^{(1)}$ and $f^{(1)}$ on the right hand side of Eqn. (15). This process carries on till the convergence is reached with tolerance 10^{-4}.

3. RESULTS AND DISCUSSION

Figure 1 shows the temperature distribution for both the regions. It is clear from the figure that as the time increases, the transfer of heat to the non-radiating region also increases. Figure 2 depicts the steady state temperature distribution for various values of source parameter *s*. As the source strength increases the temperature in both the regions also increases.

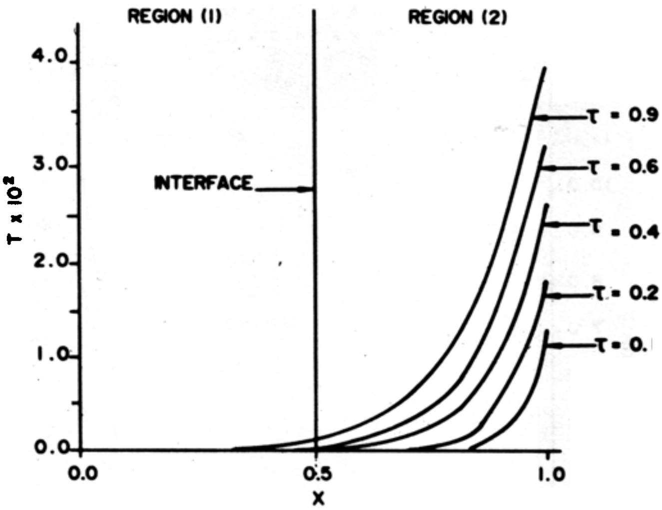

Figure 1. **Temperature history of the composite solid at $s = 0.1$, $K = 4.0$ and $k = 7.2$.**

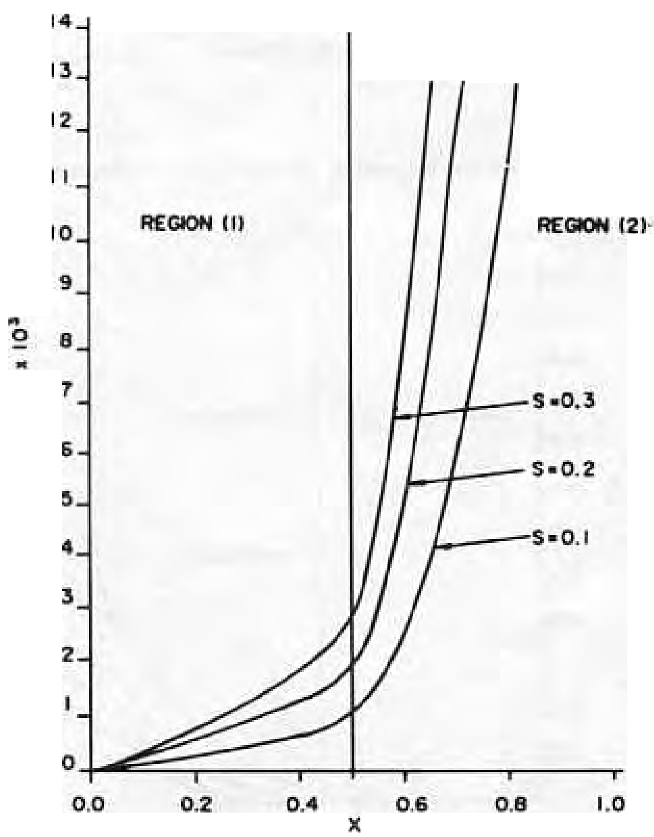

Figure 2. **Steady state temperature distribution for various values of source parameter s at $K = 4.0$ and $k = 7.2$.**

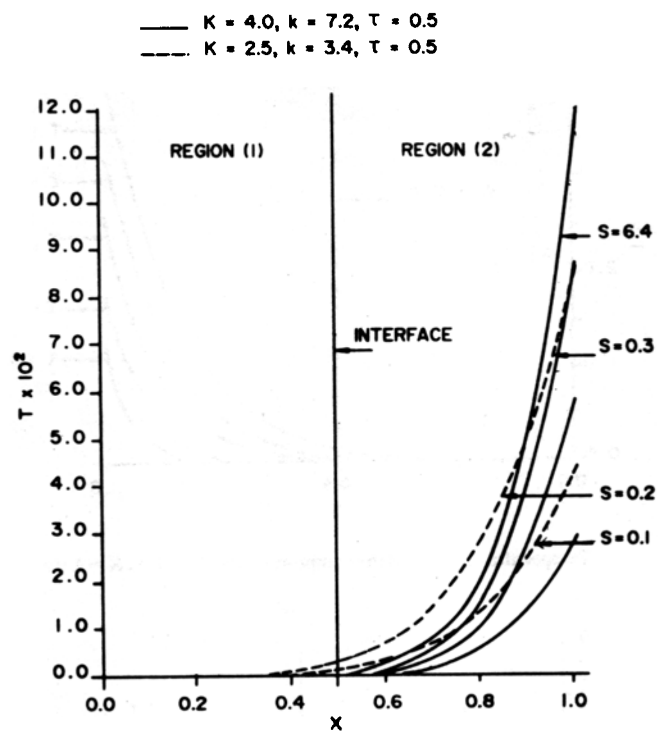

Figure 3. Transient heat temperature for various values of source parameter s.

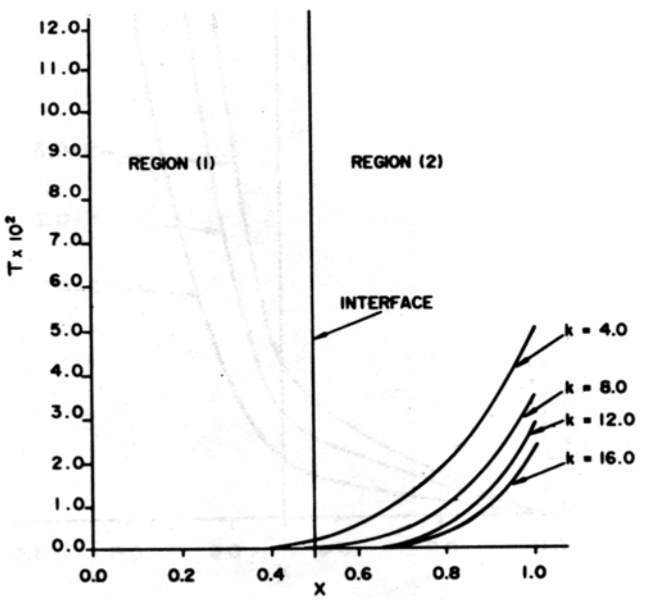

Figure 4. Temperature distribution for various diffusivity ratios at $s = 0.1$, $k = 4.0$ and $\tau = 0.8251$.

The transient temperature distribution for various source parameters is shown in Fig. 3. The graph has two types of curves, full and dotted depending upon the values of the physical quantities. In this figure too it is obvious that as source strength increases the temperature also increases in both the regions.

The temperature distribution for various values of diffusivity ratio of the two regions has been shown in Fig. 4. It is observed that as diffusivity ratio decreases there is more transfer of heat from the radiating surface to the non-radiating surface.

ACKNOWLEDGEMENTS

Thanks are due to Dr. R.S. Srivastava for many stimulating discussions and Shri V.K. Arora, Director, Defence Science Centre, for permission to publish this paper.

REFERENCES

1. Kreith, C.F., Radiation Heat Transfer for Spacecraft and Solar Power Plant Design, (International Text Book Company, Scranton), 1962.
2. Gray, W.A. & Muller, R., Engineering Calculation in Radiative Heat Transfer, (Pergamon Press, Oxford), 1974.

   Rosenberg, D.U.V., Methods of the Numerical Solutions of Partial Differential Equations, (American Elsevier Publishing Company Inc., New York), 1969.
4. Harris, T.R. & Weaver-Robert, E.C., *Chem. Eng. Progr. Symp. Ser.* **62** (1966), pp. 109-122.
5. Douglas, Jim Jr. & Schmidt, H.E., *Ind. Appl. Maths.*, **11** (1963), 195.