Def Sci J, Vol 40, No. 3, July 1990, pp 231-242

Interaction of Explosive Shocks with Airborne Cylindrical Targets of Elliptical Cross-Section

V.P. Singh and S.K. Singh

Centre for Aeronautical Systems Studies and Analyses, Bangalore–560 093

ABSTRACT

A theoretical model to find pressure-space history around a cylindrical target of elliptical cross-section has been presented for assessing the damage by warheads exploding in the vicinity of an airborne vehicle. The energy hypothesis has been used to find the attenuation law from a point explosion. The Whitham-Bryson and Gross theory for shock interaction has been modified to take into account the finite strength of incident shock. The theoretical results have also been compared with Heilig's experimental results.

1. INTRODUCTION

The determination of dynamic pressures exerted on the target becomes necessary in order to assess the damage caused by airblasts of anti-aircraft shells or any other warhead exploding in the vicinity of the airborne vehicles. This problem has gained importance due to its relation with the investigation of large blast fields around aerospace vehicles in an attempt to accumulate a database for survivability and vulnerability studies.

It is well-known that the blast wave is the predominant mechanism for damage, especially for larger ammunitions. Depending on the angles and distances, a structure might be impulsively loaded by incident-reflected shock pair or by a single stronger Mach stem. The interaction of a shock wave with various targets has been of interest for the last three decades and literature both on theoretical and experimental aspects has appeared since World War-II. In this pioneering work, Von-Neumann[1] gave the two- and three-shock theories. Whitham[2-4] gave approximate theories for interaction with two and three dimensional objects assuming the incident shock to be plane.

Received 12 December 1987

Based on Whitham's methods, Bryson and Gross[5] studied the diffraction of strong shocks by cones, cylinders and spheres both theoretically and experimentally using high-speed Schliren-systems. Heilig[6,7] presented the experimental results for shock wave diffraction by a cylinder using Cranz-Schard in camera and Mach-Zehunder interferograms. More recent workers, such as Kutler and Shanker[8] and Shanker *et al.*[9] and Yang *et al.*[10] have used refined computational methods and have also presented the pressure and density plots for shock diffraction. Ben-Dor and Glass[11,12] carried out experiments and compared the results with Kutler and Shanker[8] and other theoretical results. Griffith[13] has given an excellent review on virtually all aspects of shock waves.

In the present work, a theoretical model has been proposed for obtaining pressure-space history around any airborne target, which has been assumed to be of cylindrical shape for simplicity. Both circular and elliptical cross-sections have been taken into consideration. The theory used by Bryson and Gross[5] for shocks of finite strength has been modified. The former had the limitation that it had assumed a constant value for Chester's function $K(M)$, which in turn assumes an infinitely large value for the incident shock strength M. Another limitation was that it assumed the outset of the Mach stem right from the point of normal impact of the shock, which does not appear to be physically realisable. On the other hand, Heilig[6,7] had shown that the beginning point of Mach stem lies somewhere between the stagnation point and the zenith of the cylinder; depending upon the strength of the incident shock, and it can be found as limiting case for either two or three shock theories.

The attenuation of shock wave in air using the 'energy hypothesis' of Singh and Bola[14], has also been taken into consideration. Using this hypothesis, the pressure at any point near explosion can be expressed as a function of distance from the point of blast.

In the absence of any well-accepted theory to give the net pressure on the cylindrical surface for Mach reflection, an empirical correction has been used which indicates that nearly 20-30 per cent of the pressure behind the incident shock contributes towards the total pressure on the surface. This is based on the assumption (Kinney and Graham[15]) that the Mach stem results in limit, as a fusion of the incident and reflected shocks. However, being the first attempt of its own type this needs experimental verification.

The results have also been compared with experimental results of Heilig[7], obtained from shock tube experiments. It shows good agreement at low shock strength. In the present analysis, the usual two-shock theory to obtain the critical angle (α_{crit}) has been used. In contrast to Bryson-Gross assumption of infinite incident shock strength, the present model takes into account finite shock strength, thus two first order simultaneous differential equations for length and strength of Mach stem are obtained.

A striking feature of this model is that it indicates the existence of a threshold Mach stem length for a physically realisable solution. This implies that outset of Mach stem takes place somewhere in the region of regular reflection itself. Further, as it has been noted by several authors (for example Itoh and Itaya[16]), the Mach stem formation and its growth strongly depends on the curvature of the target's boundary, an attempt has been made for a more generalised surface by taking the cross-section as an ellipse. In fact, the present method can be generalised to analyse the shock-interaction with targets whose cross-section can be approximated by any arbitrary conic or a combination of conic-sections.

The results obtained for elliptical cross-section also exhibit varying trends for lengths and strength of Mach stem. For increasing values of eccentricity from zero (i.e., circular section) to 0.5, the Mach stem length and pressure distribution exhibit decreasing trend, while for eccentricity values exceeding 0.5 the trend is reversed, presumably because of the model is applicable only for low eccentricities.

2. PROPAGATION AND ATTENUATION OF SHOCK WAVES

Due to the geometry as well as a number of dissipative mechanisms, the shock front undergoes attenuation while propagating in any material medium. For underwater explosion, attenuation laws based on Whitham's method and energy hypothesis were studied by Singh and Bola[14] and Singh[17]. It was also shown that in case of underwater shocks, energy hypothesis gives better agreement with experimental data[17].

Following Singh and Bola[14] and Bhutani[18], the energy hypothesis for spherical blast wave is given by

$$[E] - \frac{3\alpha Q}{4\pi R^3 p_0}\left\{\frac{\gamma - 1}{\gamma + 1} + \frac{2a^2}{(\gamma + 1)U^2}\right\} \tag{1}$$

Where Q is the energy of explosion released at $R=0$, R is the radius of expanding blast wave at any instant, a is the velocity of sound and U is the velocity of blast wave.

The constant α is given by

$$\alpha = \underset{R\to 0}{\mathrm{Lt}} \frac{\rho_1}{\rho_0} \tag{2}$$

Also from the jump-condition for energy

$$\frac{1}{2}u_1^2 + \frac{1}{\gamma - 1}\left(\frac{p_1}{\rho_1} - \frac{p_0}{\rho_0}\right) \tag{3}$$

which gives

$$E] = \frac{2(U^2 - a^2)\{2\gamma U^2 - (\gamma - 1)a^2\}}{\gamma(\gamma + 1)U^2} \tag{4}$$

Combining Eqns. (1) and (4), we get the biquadratic equation for U as

$$AU^4 \quad BU^2 \quad C = 0$$

where

$$\frac{4}{\gamma + 1}$$

$$B \quad \frac{2(3\gamma - 1)a^2}{\gamma(\gamma + 1)} \quad \frac{3(\gamma - 1)\alpha Q}{4\pi R^3 p_0}$$

$$C = \frac{2(\gamma - 1)a^4}{\gamma(\gamma + 1)} - \frac{6\alpha Q a^2}{4\pi R^3 p_0}$$

Solving Eqn. (5) the strength of the spherical blast wave at any point in the vicinity of explosion can be obtained. In the present problem, the incident spherical waves have been approximated as plane. Strictly speaking this assumption holds only when $R \rightarrow \infty$.

3. REGULAR REFLECTION OF SHOCK WAVES FROM A CURVED BOUNDARY

The regular reflection of plane shock waves from a plane boundary has been studied by Von Neumann[1], Polachek and Seeger[19], Kinney and Graham[15]. This theory has been applied to find the reflected shock strength and the angle of reflection until the angle of incidence reaches a critical value, beyond which there is no regular reflection and the reflection phenomena has to be studied using three-shock theory. An out line of the theory is explained in the following paragraphs.

Let IO be the incident plane shock, making an angle α with the rigid target boundary (assumed to be plane over a small region around the point of incidence O). After reflection, this is shown as OR, making an angle β with the wall; u_1, u_2, u_3 being the fluid velocities in regions I, II, and III respectively, u_1 and u_3 being parallel to boundary (Fig. 1). Now the two physical conditions imposed on the flow are: (a) Velocity of OR cannot be more than that of OI, and (b) net normal component of fluid flow is zero. These conditions when compounded with well-known Rankine-Hugoniot jump conditions yield the following set of equations[15].

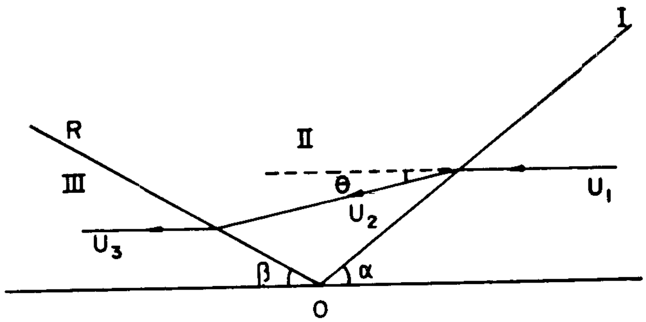

Figure 1. Oblique reflection from a plane surface.

$$\tan\theta = \frac{5(M_x^2 - 1)\tan\alpha}{(5 + M_x^2)\tan^2\alpha + 6M_x^2} \tag{6}$$

$$[M_2 \sin(\alpha - \theta)]^2 = (5 + M_x^2)/(7M_x^2 - 1) \tag{7}$$

$$\frac{\tan\beta}{\tan(\beta + \theta)} = \frac{6 + \{M_2 \sin(\beta + \theta)\}^2}{6\{M_2 \sin(\beta + \theta)\}^2} \tag{8}$$

$$M_3 \sin\beta)^2 = \frac{5 + \{M_2 \sin(\beta + \theta)\}^2}{7\{M_2 \sin(\beta + \theta)\}^2 -} \tag{9}$$

where

$$M_x = M_1 \sin \alpha, \; M_1 = u_1/c_1, \; M_2 = u_2/c_2 \text{ and } M_3 = u_3/c_3$$

are the Mach numbers, and c_1, c_2, and c_3 are local sound velocities in the respective regions.

The above system is numerically solved for any given values of α and M, giving both the strength (M_3) and the angle of reflection β for the reflected shock. This also yields a specific critical angle (α_{crit}) for every known value of the strength of incident shock (M_x).

4. MACH REFLECTION FROM ELLIPTICAL CYLINDER SURFACE

As the angle of incidence (α) increases above α_{crit}, the phenomenon enters the domain of Mach reflection (or three-shock reflection). Bryson and Gross[5] have given the diffraction theory of shocks from circular cylinder surface, assuming the Chester's function $K(M)$ to be constant. But this assumption, proposed by Whitham[2] holds only when the incident shock strength $M \rightarrow \infty$. Singh and Murthy[20], presented a modification for finite (and also unknown) strength of the Mach stem in the case of underwater shocks. The same approach was followed here for air blast and the theory was generalised by taking the cross-section of the cylinder as elliptical. Assuming the most general case, the explosion may be assumed to occur anywhere in the vicinity of the target.

Suppose the explosion takes place at the point E in the vicinity of the target, so that the shock front (assumed plane for simplicity) is normally incident at the point L of the target whose cross-section is taken as the standard ellipse (Fig. 2) given by, Then,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \tag{10}$$

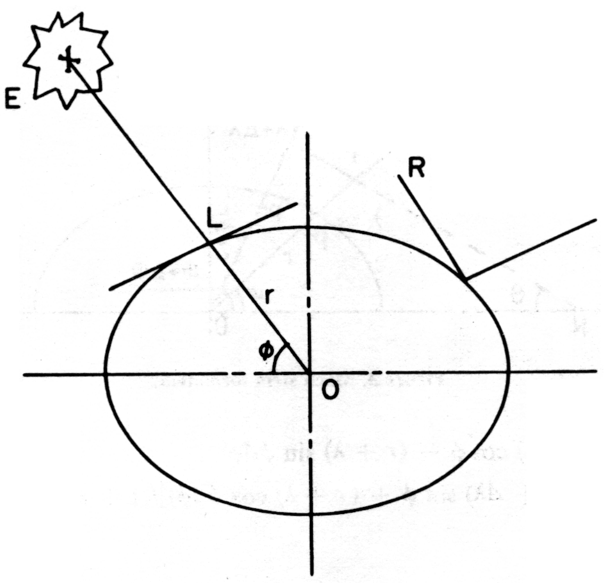

Figure 2. Normal and regular oblique reflection.

Let ϕ be the eccentric angle of the ellipse at any point on it. For small values of eccentricities, it is reasonable to assume $\phi \simeq \alpha$ (in the case of a circle, ϕ exactly equals α).

At L, the reflection is governed by simple rules of normal incidence and as we move away from the point L (with increasing ϕ), the reflection phenomenon remains within two shock theory. The Mach stem formation begins when α exceeds α_{crit} ($\simeq$ 40 degrees). By experimental studies, the Mach stem has been found to be nearly normal to the target's surface. We may assume it to be directed towards the centre of the ellipse (Bryson and Gross[5]).

The undisturbed rays would have occupied a length TN, which is now constrained to a length TP ($= \lambda$). Therefore $A = \lambda$ per unit length of cylinder and $A_0 = TN$ per unit length of cylinder.

Using the elementary properties of conic, it can be shown that

$$TN = (r \quad \lambda) \sin\phi \left(\frac{1}{m_0^2} + \quad \right)^{1/2} \tag{11}$$

where $m_0 \quad -\frac{b}{a}\cot\phi$ and $r = (a^2\cos^2\phi + b^2\sin^2\phi)^{1/2}$

Hence $A_0 = k(r + \lambda)\sin\phi$ and $A = \lambda$ (12)

where $k = \left(\frac{1}{m_0^2} + \quad \right)^{1/2}$

Let P' be a neighbouring point to P. Let L be the distance between the shock fronts at $P(\phi)$ and P' $(\phi + \Delta\phi)$, then it can be shown that (Fig. 3)

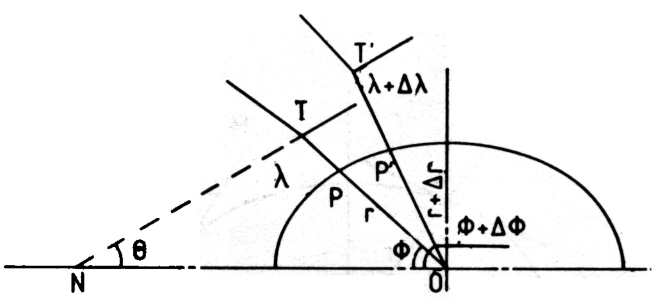

Figure 3. Mach stem formation.

$$L = [(\Delta r + \Delta\lambda)\cos\phi - (r + \lambda)\sin\phi\Delta\phi$$
$$- m\{(\Delta r + \Delta\lambda)\sin\phi + (r + \lambda)\cos\phi\Delta\phi\}]/(1 + m^2)^{1/2}$$

where

$m = \tan\theta$. Also $L = -M_0\Delta\alpha$ and $\Delta\alpha = a_0\Delta t$ (Whitham[1])

Taking the limit as $\Delta\phi \to 0$ we get

$$\frac{d\lambda}{d\phi} = (r + \lambda)\tan(\theta + \phi) - \left(r + \frac{\lambda}{2}\right)\frac{M_0}{M}\sec(\theta + \phi)$$
$$+ \frac{(a^2 - b^2)}{2r}\sin 2\phi$$

where we have used the results

$$\frac{d\alpha}{d\phi} = \left(r + \frac{\lambda}{2}\right)\Big/M$$

and $\quad \dfrac{dr}{d\phi} = -\dfrac{1}{2r}(a^2 - b^2)\sin 2\phi.$

Also from Whitham[2], we have

$$\frac{A}{A_0} \quad \frac{f(M)}{f(M_0)} \tag{14}$$

where $f(M) = \exp\left\{-\int \dfrac{2M\,dM}{(M^2 - 1)K(M)}\right\}$

$$K(M) = 2\left[\left(1 + \frac{2}{r+1}\cdot\frac{1-\mu^2}{\mu}\right)(2\mu + \quad + M^{-2})\right]^{-1}$$

and $\quad \mu^2 = \dfrac{(\gamma - 1)M^2 + 2}{2rM^2 - (r - 1)}$

Using Eqns. (12) and (13), Eqn. (14) gives finally

$$\frac{dM}{d\phi} = \frac{(M^2 - 1)K(M)}{2M}\left[\cot\phi - \frac{r}{\lambda}\left\{\tan(\theta + \phi) - \frac{2r + \lambda}{2(r + \lambda)}\cdot\frac{M_0}{M}\right.\right.$$
$$\left.\left.\sec(\theta \quad \phi)\right\} \quad \frac{1}{2\lambda r}(a^2 - b^2)\sin 2\phi\right]$$

The system of Eqns. (13) and (15) determine the length and strength of the Mach stem, originating due to scattering of a plane shock front of strength M_0 at the point on the ellipse. The outset of Mach stem is governed by the condition $\alpha > \alpha_{crit}$

$$\text{where } \alpha = \tan \left\{\frac{\frac{b}{a}(\cot\phi - \cot\phi_0)}{+\frac{b^2}{a^2}\cot\phi\cot\phi_0}\right\} \tag{16}$$

4.1 Particular Cases

4.1.1 Normal Incidence at the End of Major Axis

In this case the Eqns. (13) and (15) simplify to the following pair

$$\frac{d\lambda}{d\phi} = (r + \lambda)\tan\phi \quad \frac{\left(r + \frac{\lambda}{2}\right)}{\cos\phi}\cdot\frac{M_0}{M} + \frac{1}{2r}(a^2 \quad b^2)\sin 2\phi$$

$$\frac{dM}{d\phi} = \frac{(M^2 - 1)K(M)}{2M}\left[\cot\phi - \frac{r}{\lambda}\left\{\tan\phi \quad \frac{2r + \lambda}{2(r + \lambda)}\sec\phi\cdot\frac{M_0}{M}\right.\right.$$
$$\left.\frac{1}{2\lambda r}(a^2 - b^2)\sin 2\phi\right]$$

4.1.2 Incidence on a Circular Cylinder of Radius Unity (Non-Dimensional)

The Eqns. (13) and (15) are simplified to the following :

$$\frac{d\lambda}{d\phi} = (1 + \lambda)\tan\phi - \frac{\left(1 + \frac{\lambda}{2}\right)}{\cos\phi}\cdot\frac{M_0}{M}$$
$$\frac{dM}{d\phi} = \frac{(M^2 - 1)K(M)}{2M}\left[\cot\phi - \frac{1}{\lambda}\left\{\tan\phi - \frac{2 + \lambda}{2(1 + \lambda)}\sec\phi\cdot\frac{M_0}{M}\right]\right. \tag{20}$$

Eqns. (19) and (20) are similar to those derived by Singh & Murthy[20]. In this case

$$f(M) = \exp\left\{\int\frac{dM}{MK(M)}\right\}$$

where $K(M) \quad \{1 + (b - 1)M\}\Big/\left[1 + \frac{\{1 + (b - 1)M\}\sqrt{(2M - 1)}}{(M - 1)}\right] \times \left[1 + \frac{M}{\sqrt{(2M + 1)}}\right]$

and b being a parameter in the equation of state for the case of underwater shocks around a circular cylinder.

5. RESULTS AND DISCUSSIONS

Eqns. (9) and (10) have been integrated using Runge-Kutta method of fourth order and results are shown in Figs. 4 and 5. Four values of eccentricity, viz., e = 0.0, 0.25, 0.50 and 0.75 have been taken. The first value, i.e., $e = 0.0$ corresponds to the case of circular cylinder, whereas $e = 0.75$ gives us highly elliptical cylinder. In Fig.4 variation of Mach stem length has been plotted against the angle of incidence for the typical case of $M_0 = 3.75$. As mentioned in section 1, this curve indicates a threshold value of λ at the critical angle. The Mach stem grows exponentially for higher values of ϕ. The nature of curve is almost similar to the one obtained by Singh and Murthy[20] for underwater shocks. It was also observed that the nature of curves for other Mach numbers is also exactly similar, exhibiting the same trends for varying eccentricities. The critical angles for different shock strengths, its corresponding threshold value of Mach stem length and the distance from the point of explosion (the lengths being non-dimensionalised with respect to the cylinder's average radius) are given in Table 1.

Figure 5 shows the variation of the pressure on the cylindrical surface against ϕ for M_0 = 2.15, 2.95 and 3.75 along with the experimental results obtained by Heilig from shock tube experiments. A remarkable agreement for low strength shocks with

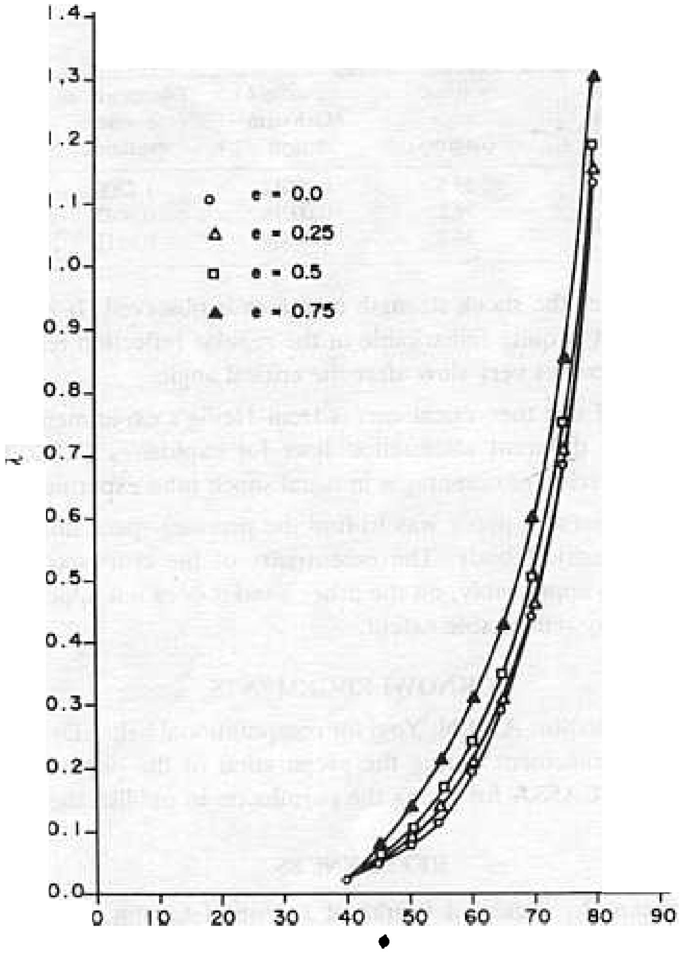

Figure.4. Variation of λ with ϕ.

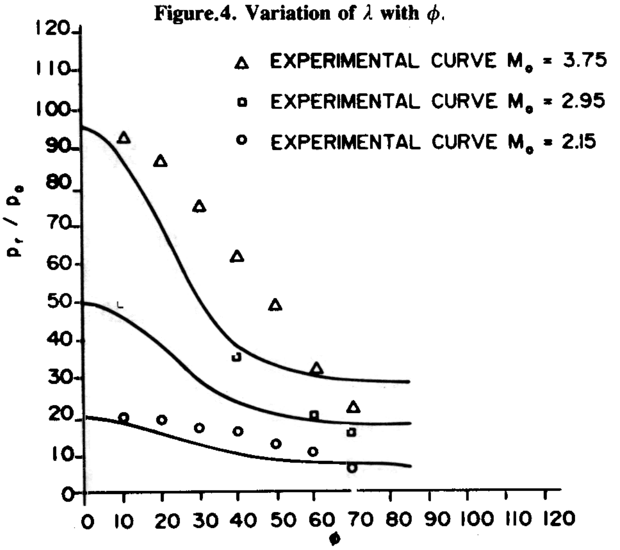

Figure 5. Variation of pressure with ϕ.

Table 1

Mach number of shock (strength)	Critical angle (degrees)	Threshold Mach stem length	Distance from the point of explosion
2.15		0.00012	1.2308
2.95		0.00018	0.8683
3.75		0.00046	0.6917

increasing deviation as the shock strength increases is observed. It is also seen that the attenuation effect is quite remarkable in the regular reflection region, while the decay in pressure becomes very slow after the critical angle.

The deviation of our theoretical curves from Heilig's experimental ones might be attributed to the different attenuation laws for explosives (RDX/TNT in the present case) and piercing of diaphragm in usual shock tube experiments.

The aim of the present paper was to find the pressure-space history around an arbitrary-shaped cylindrical body. The eccentricity of the cross-section affects the growth of Mach stem appreciably, on the other hand it does not appear to affect the pressure profile to any remarkable extent.

ACKNOWLEDGEMENTS

We are thankful to Shri A.M.N. Yogi for computational help, Dr. S.M. Sharma Sc 'E' for the encouragement during the preparation of the paper and to Dr. R Natarajan, Director, CASSA for giving the permission to publish the paper.

REFERENCES

1 Von Neumann, J., Collected Works of J. Von Neumann, 6 V., A.W. Taub, (Ed), (Pergamon, New York), 1963, pp. 238-299.

2. Whitham, G.B., *J. Fluid Mech.*, **2** (1957), 145-171.

3. Whitham, G.B., *J. Fluid Mech.*, **4** (1958), 337-360.

4. Whitham, G.B., *J. Fluid Mech.*, **5** (1959), 369-386.

Bryson, A.E. & Gross, R.W.F., *J. Fluid Mech.*, **10** (1961), 1-16.

6 Heilig, W.H., *Physics of Fluids Suppl.*, **1-2** (1969), 153-157.

7 Heilig, W.H., *In* Proceedings of Eleventh International Symposium on Shock Tubes and Waves, (University of Washington Press, Seattle), 1977, pp. 288-295.

8. Kutler, P. & Shanker, V., *AIAA J.*, **15** (1977), 197-203.

9. Shanker, V. *et al.*, *AIAA J.*, **16** (1978), 4-5.

10. Yang, J.Y. *et al.*, *AIAA J.*, **25** (1987), 683-689.

11. Ben-Dor, G. & Glass, I.I., *J. Fluid Mech.*, **92** (1979), 459-496.

12. Ben-Dor, G. & Glass, I.I., *AIAA J.*, **16** (1978), 1146-1153.

13 Griffith, W.C., *J. Fluid Mech.*, **106** (1981), 81-101.

14 Singh, V.P. & Bola, M.S., *Ind. J. Pure and Applied Math.*, **7** (1976), 1405-1410.

15. Kinney, G.P. & Graham, K.J., Explosive Shocks in Air, (Springer-Verlag, New York), 1985.
16. Itoh, S. & Matsuki, Itaya, *In* Proceedings of the Twelfth International Symposium Shock Tubes and Waves (Hebrew University, Jerusalem), 1979, pp. 314-323.
17. Singh, V.P., *Def. Sci. J.*, **32** (4), (1982), 327-332.
18. Bhutani, O.P., *J. Math. Analysis Applic.*, **7** (1966), 565-575.
19. Polachek, H. & Seeger, R.J., Fundamentals of Gas-Dynamics, (Princeton University Press, Princeton), 1958.
20. Singh, V.P. & Murthy, D.S., *In* Proceedings of the Seminar on Anti-Submarine Warfare, IAT, Pune, 1985, pp. 81-88.