Def Sci J, Vol 40, No. 2, April 1990, pp 133–138

Area Search for a Moving Target

J.N. Nanda

Science Consultant, 8052/C-8, Vasant Kunj, New Delhi-110 037

ABSTRACT

When a target has an *apriori* existence in an area A and a fraction ϕ of the area is searched, there are well-known expressions for the detection probability when the target is stationary. In this paper the detection probability is worked out for a more important case when the target is in motion. It must be assumed, however, that the target manages to remain in the area in which it has *apriori* existence by permitting suitable changes in its direction of motion. The detection probability will depend on the ratio of the speeds of the target and the searcher in a complex way. The computation should involve a computer programme but analytical expressions can be approximately derived for $\phi << 1$. The calculated probability is less than ϕ which is the detection probability for continuous search for a stationary target and more than the value for a random search.

1. INTRODUCTION

The searches considered here are of a submerged submarine by means of helicopters carrying dunking sonar, or of a surfaced submarine by patrolling planes carrying radar. For a stationary target simple coverage of the area allegedly containing the submarine is involved till detection. Any practical search has to be by means of repeated searching looks in adjacent or overlapping area elements progressively covering the entire area or the part of the area when detection is successful. The detection probability is given by simple expressions dealt in various text books when continuous or a random search is resorted to for a stationary target. When the target is in motion or taking evasive action, the detection probability is given by approximate expressions that will be derived in this paper, but the method as outlined can be the basis of computer-aided results. The approximate analytical expressions however are fairly close to the expected computation for $\phi << 1$. A mathematical theory of search

Received on 30 June 1987, revised 22 July 1989

of a moving submarine has earlier been worked out by Balasubramanian' for evaluation of the search tactics of surface vessels. A more complex problem has been tackled here rather in a simple and practical way, and incidentally a new approach has been evolved to search prob'ems.

Let each area element howsoever small, have a uniform *apriori* occurrence probability of the target, associated with the element. The calculation will proceed over elements of area 2 WI where 2 W is the effective range of the detection equipment perpendicular to the small length l in the direction of motion of the detecting aircraft. For easy visualisation, the target in any element of **area is** supposed to have constant step in length and speed for simplicity. The direction of the steps will be random for random walk or evasive action and constant for purposeful motion of the submarine. For a homogeneous *apriori* distribution of targets, it is implied that as many targets leave an area element as enter the same in any time element howsoever small. The searcher takes time to move at speed Vfrom an area element of length l to the next. It is also assumed that the detection equipment is cent per cent effective and accurate. Each target step may also be considered as taking time τ, the snapshot detection time is zero unless otherwise stated.

2. PROBABILITY OF FAILURE TO DETECT IN THE *r*th AREA ELEMENT

In Fig. 1 area elements a, b, c, d, etc., each of width 2Wand length l are shown. The target area A is of n such elements. The search starts with the element a. The probability of not detecting any target in element a is $(1-2Wl/A)$. The probability of not detecting any target in next element b is $(1-2Wl/(A-2Wl-\gamma_1)$ where γ_1 is **zero,** if the target is stationary and has not been detected in element a; it is $2Wl$ if the detection in element b is independent of prior effort in a or if the targets are moving with such high speed that the initial probability is maintained in element a immediately after the failure to detect in it. For a finite speed of the target, γ_1 will be less than $2Wl$

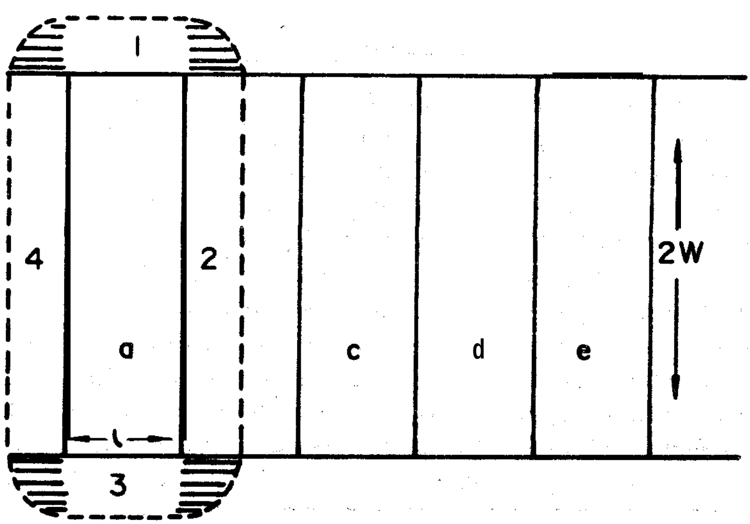

Figure 1. Area elements *a*, *b*, *c*, *d*, *e*... and zones of approach 1, 2, 3, 4 into element *a*.

depending on the number of targets stepping into element a from its surroundings since the departure of the detector from that element. The limiting distance from which this element can get a target in the single step is $v\tau$ where v is the speed of the target and τ is the fixed time for a single step. Thus the area surrounding the element a upto the distance vl/V is important for such entry. We can neglect the shaded portions in Fig. 1 since the interconnection is only a point. From the rectangular portions (1) and (4), half the targets moving randomly will move away from element a; of the remaining targets, the approach will be with a speed component v sin θ where θ is the angle of the trajectory with the boundary of the element a. The average of sin θ over $180°$ is $2/\pi$. Thus the area equivalent of targets entering the element will be $2l^2v/(\pi V) = a$ (let). Similarly, the contribution from the neighbouring zones (2) or (4) will be $2Wlv/(\pi V) = \beta$ (let). Thus for the second area element b (i.e., $r+1=2$), $\gamma_1 = a + 2\beta$. Failure to detect in element a or element b together is

$$F_2 = (1-2Wl/A)\left(1-\frac{2Wl}{A-(2Wl-\gamma_1)}\right) \tag{1}$$

In case the starting element a is such that no targets could enter from the left, γ_1 will be only $a+\beta$. For Table 1 where the area equivalent to targets for each element are indicated for various steps, this simplification has been used. For the step involving detection in b, the element c is not involved which continues to have the apriori probability. At the end of each step the exchange is spelled out only for the adjoining elements since targets more than a step away may not enter any element. Some of the elements that moved into a from b will move back into b (fraction $1/\pi$). Similarly, a fraction $1/\pi$ of a will move back to the zone of origin. The remaining targets will spread there. The rth term denominator in the expression for failure is $[A-(r-1).2Wl - \gamma_{r-1}]$.

In Table 2, the gammas being the sum of all targets (area equivalents) present in all elements upto the 7th element are given. By inference, for the $(r+1)$th element we have

$$\gamma_r = [3.14r-6.7 + 6.7 \times (0.68)^r]a + r\beta \tag{2}$$

The Eqn. (2) can be used for $r < r_0$ where r_0 is determined from the equation

$$\gamma_{r_0} = 2Wl \tag{3}$$

Since by observation alone the presence of possible targets cannot go beyond $2Wl$ in any element and for ($r > r_0$, the appropriate γ_r will be 2 Wl. The probability for failure in n' steps (where $n' = n.\phi$) is given by

$$F_{n'} = (1-2Wl/A)\left(1-\frac{2Wl}{A-2Wl-\gamma_1}\right)\left(1-\frac{2Wl}{A-2\times 2Wl-\gamma_2}\right)\ldots$$
$$\left(1-\frac{2Wl}{A-\gamma_0 2Wl-2Wl}\right)\left(1\quad\frac{2Wl}{-\gamma_0 2Wl}\right)\ldots$$
$$\left(1-\frac{2Wl}{A-(n'-1)2Wl}\right) \tag{4}$$

i.e., a double product of r_0 terms and $n'-r_0$ terms respectively. When $n' << n$ (actually $n' < n/3$) each product can be approximately written as the appropriate power of the

Table 1. Targets in term of area present at each snapshot

Time	Element No.	Description	Area elements						
			a	b	c	d	e	f	g
0	1	Present	0						
		Moved in	$a+\beta$						
		Moved out	0						
1	2	Present	$a+\beta$	0					
		In	a	$a+1.32\beta$					
		out	$-0.32(a+\beta)$	0					
2	3	Present	$1.68a+0.68\beta$	$a+1.32\beta$	0				
		In	a	$a+0.32\times0.68\beta$	$a+1.42\beta$				
		out	$-0.32(1.68a+0.68\beta)$	$-0.32(a+1.32\beta)$	0				
3	4	Present	$2.14a+0.46\beta$	$1.68a+1.12\beta$	$a+1.42\beta$	0			
		In	a	$a+0.15\beta$	$a+0.36\beta$	$a+1.45\beta$			
		Out	$-(0.68a+0.15\beta)$	$-(0.54a+0.36\beta$	$-(0.32a+0.45\beta)$	0			
4	5	Present	$2.46a+0.31\beta$	$2.14a+0.91\beta$	$1.68a+1.33\beta$	$a+1.45\beta$	0		
		In	a	$a+0.1\beta$	$a+0.27\beta$	$a+0.43\beta$	$a+1.46\beta$		
		Out	$-(0.79a+0.1\beta)$	$-(0.68a+0.27\beta)$	$-(0.54a+0.43\beta)$	$-(0.32a+0.46\beta)$	0		
5	6	Present	$2.67a+0.21\beta$	$2.46a+0.72\beta$	$2.14a+1.17\beta$	$1.68a+1.43\beta$	$a+1.46\beta$	0	
		In	a	$a+0.07\beta$	$a+0.23\beta$	$a+0.37\beta$	$a+0.38\beta$	$a+1.47\beta$	
		out	$-(0.85a+0.07\beta)$	$-(0.79a+0.23\beta)$	$-(0.68a+0.37\beta)$	$-(0.54a+0.45\beta)$	$-(0.32a+0.47\beta)$	0	
6	7	Present	$2.86a+0.14\beta$	$2.67a+0.56\beta$	$2.46a+1.03\beta$	$2.14a+1.34\beta$	$1.48a+1.37\beta$	$a+1.47\beta$	0

Table 2. Notionally occupied area for each element (γ)

Element $(r+1)$	γ_r	
1	$\gamma_0 = 0$	
2	$\gamma_1 = a + \beta$	i.e., targets present in a and b at time (1) or $P_a + P_b$ at t_1
3	$\gamma_2 = 2.7a + 2\beta$	i.e., $\sum_{r=3}^{r=1} P_{r+1}$ at time t_2
4	$\gamma_3 = 4.8a + 3\beta$	
5	$\gamma_4 = 7.4a + 4\beta$	
6	$\gamma_5 = 10.0a + 5\beta$	
7	$\gamma_6 = 12.8a + 6\beta$	
The general term :		
$r+1$	$\gamma_r = \left[3.14r - 6.7\times(0.68)^r\right] a + r\beta$	

middle term. More properly a computer programme should be used to numerically calculate the failure probability. The detection probability is given by $1 - F_{n'}$. A couple of examples are solved in the following paragraphs.

2.1 Search Patrols

Let $A = 1000 \times 1000$ km^2, $2W = 20$ km, being the surface range of a down looking radar, from the low height of the searching aircraft. Let $l = 5$ km and $V/v = 10$. the area searched is given by $\phi = 0.25$,

Now $n = 10{,}000$ and $n.\phi = 2500$ $(=n')$

a = 1.6 and $\beta = 3.2 = 2a$

Using Eqns. (2) and (3) we get $(r_0) = 14$. Using the aforesaid approximation for the products

$$F_{n'} = \left(1 - \frac{2Wl}{A - 7\times 2Wl - \gamma_7}\right)^{14}\left(1 - \frac{2Wl}{A - 1257\times 2Wl}\right)^{2486} \tag{5}$$

Now $\gamma_7 = 47.6$ and $F_{n'} = 0.752$ and the detection probability $= 1 - 0.752 = 0.248$. For the same random search effort the detection probability is $1 - \exp(-0.25) = 0.221$.

2.2 Search of a Submarine Under Water Taking Evasive Action

The area A requiring a thorough search is given by the strategic traffic cone as well as the possible range of the torpedoes estimated to be carried by the submarine. Let this area be A = 20×50 km^2 and let $V/v = 25$. The dunking sonar usually will have small range in the disturbed seas. Let $2W = 0.5$ km. Let the time of flight from one step to the next be τ and the time for observation be δ; and for this example let us assume $\delta = 2\tau$ so that a and β are three times the values from the expressions already given for them. Let l = 0.5 km. (The size l of each step will also depend on the turning radius of the submarine).

We have $a = 6\, l^2 v/(\pi V) = 1.91 \times 10^{-2}$ and $\beta = 0.5\, a$. Now $r_0 = 6$ and $n' = 1000$, since $\phi = 0.25$ and $n = 4000$.

For this case

$$F_{n'} = \left(1 - \frac{0.25}{1000 - 4 \times 2Wl - \gamma_4}\right)^6 \left(1 - \frac{0.25}{1000 - 503 \times 2Wl}\right)^{994}$$

$$= 0.751 \tag{6}$$

and the detection probability = 1 - 0.751 = 0.249.

3. DISCUSSION

The steps of the target and the snapshots have been **synchronised.** Usually this will not happen, the effective step length then **will** be less or V/v will be effectively larger improving the detection probability.

For a barrier patrol the search can be made definite by adjusting the frequency of traverses so that the time taken by air traverse is less than the time taken-by the submarine to cross the barrier. The detection is also beset with errors on account of **operator** fatigue and lower than ideal efficiency of the equipment. The detection effort should therefore be more than estimated from the calculation.

REFERENCES

1. Balasubramanian, N., *Def. Sci. J.*, **9(2), (1959),** 1-9.
2. Washburn, A.R., Surface Detection, **(ORS,** Virginia), 1981.