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Plane and Cylindrical Strong Shocks in Magnetogas Dynamics

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ABSTRACT

Chisnell-Chester-Whitham method has been used to investigate the propagation of strong diverging plane and cylindrical shock waves in an infinitely electrically conducting ideal gas in the presence of a constant. axial magnetic field under two distinct situations; (a) when the ratio of densities on either side of the shock nearly equals $(\gamma + 1)/(\gamma - 1)$, where γ is the adiabatic index of the gas, or (b) when the applied magnetic field is large. It is found that the plane shock wave moves with a constant shock strength. An increase in the magnetic field leads to an increase in the shock velocity.

1. INTRODUCTION

Numerical results obtained by Chaturani and Ranga Rao[1] and Kumar and Kulshrestha[2] describing the propagation of strong plane and cylindrical hydromagnetic variable energy blast waves through an infinitely electrically conducting ideal gas have shown good agreement with experimental observations. In their study the magnetic field is assumed to be axial and initially of constant strength. In the present paper, Chisnell-Chester-Whitham (CCW) method[3-7] has been used to represent analytically the propagation of diverging strong plane and cylindrical shock waves in an infinitely electrically conducting ideal gas in the presence of an axial magnetic. field. From the shock conditions it is seen that a strong shock may be obtained by two distinct means: (a) as in the purely non-magnetic case, when the ratio of densities on either side of the shock nearly equals $(\gamma + 1) / (\gamma - 1)$, or (b) when the applied magnetic field is large, that is when the ambient magnetic pressure is large compared with the ambient gas pressure. Expressions for the shock velocity, pressure and particle velocity immediately behind the shock have been obtained.

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Finally, the results accomplished here have also been compared with those for weak **shocks**[6].

1.1 Basic Equations, Boundary Conditions and Analytical Relations for Shock Velocity and Shock Strength

The equations of motion for **the** gas enclosed by the shock front are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{2\rho}\frac{\partial H^2}{\partial r} = 0 \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\alpha\rho u}{r} = 0 \tag{2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{3}$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial u}{\partial r} + \frac{[illegible]}{r} = 0 \tag{4}$$

where r is the radial w-ordinate; u, p, ρ, H and μ are particle velocity, pressure, density,. axial magnetic field and magnetic permeability of the gas respectively; a = 0 and 1, for plane and cylindrical symmetry of the problem respectively and $a^2 = \gamma p / \rho$.

The magnetohydrodynamic shock wnditions **can** be written in terms of single parameter $\xi = \rho_1 / po$ as

$$\rho_1 = \rho_0\xi, \quad H_1 = H_0\xi, u_1 = U\frac{\xi - 1}{\xi}$$

$$U^2 = \frac{2\xi}{(\gamma + 1) - (\gamma - 1)\xi}\left[a_0^2 + \frac{1}{2}b_0^2\{(2 - \gamma)\xi + \gamma\}\right]$$

$$p_1 = p_0 + \frac{2\rho_0(\xi - 1)}{(\gamma + 1) - (\gamma - 1)\xi}\left\{a_0^2 + \frac{\gamma - 1}{4}b_0^2(\xi - 1)^2\right\} \tag{5}$$

where the subscripts 1 and 0 denote conditions behind and **infront** (equilibrium state) of. the shock respectively. a,, $(= \sqrt{\gamma\, p_0 / \rho_0})$ is the speed of sound and b_0 $(= \sqrt{\mu H_0^2 / \rho_0})$ the Alfven speed.

In the limiting case of a strong shock p_1 / p_0 is large. In the magnetic case this is achieved in two. ways : case (i), the purely non-magnetic way when $\xi - (\gamma + 1) / (\gamma - 1)$ is small, and case (ii), when $b_0^2 \gg a_0^2$, i.e., $\mu H_0^2 \gg \gamma p_0$, the magnetic pressure is very much greater than the gas pressure, in the equilibrium state. In terms of ξ, the boundary conditions from Eqn. **(5)**, now become

$$\rho_1/\rho_0 = \xi, \; H_1/H_0 = \xi, \quad u = U\frac{(\xi - 1)}{\xi}$$

$$\frac{p_1}{p_0} = \chi(\xi)\left(\frac{U}{a_0}\right)^2 + \frac{\xi(\gamma + 1) - (\gamma - 1)}{(\gamma + \gamma) - (\gamma - 1)\xi} \tag{6}$$

where $$\chi(\xi) = \frac{\gamma(\gamma - 1)(\xi - 1)^3}{2\xi\{(2 - \gamma)\xi + \gamma\}}$$

For diverging shocks the characteristic form of system of flow Eqns. **(1)–(4),** i.e., the form in which equation contains derivatives in only one direction in (r, t) plane, can be written as

$$dp + \mu H\,dH + \rho c\,du + \frac{\rho c^2 \alpha u}{u + c}\frac{dr}{r} = 0 \tag{7}$$

where $c^2 = a^2 + b^2 = (\gamma p + \mu H^2) / \rho$.

Final step is to substitute the shock conditions in Eqn. (6) into Eqn. (7) and using equilibrium condition $u = 0 =$ Wt., and $dp_0 = 0$ for a uniform density distribution, we get

$$\frac{dU^2}{dr}B + \frac{U^2}{r}\alpha C = 0 \tag{8}$$

where
$$B = \frac{\chi(\xi)}{\gamma} + \frac{1}{2}\left(\frac{\chi(\xi)}{\xi}\right)^{1/2}(\xi - 1)$$

and
$$C = \frac{\chi(\xi)(\xi - 1)}{(\xi - 1) + \{\xi\chi(\xi)\}^{1/2}}$$

i.e.,. a first order differential equation for U^2 which determines the shock.

On solving Eqn. **(8),** we get

$$U^2 = K' r^{-\alpha C/B} \tag{9}$$

where K' is a constant of integration.

Subsequently the non-dimensional expression for shock velocity can be written as

$$\frac{V}{a_0} = K r^{-\alpha C/2B} \tag{10}$$

where $$K = \sqrt{K'} / a_0$$

The pressure and the particle velocity immediately behind the shock can be written as

$$\frac{p}{p_0} = \chi(\xi)K^2 r^{-\alpha C/B} + \frac{\xi(\gamma + 1) - (\gamma - 1)}{(\gamma + 1) - (\gamma - 1)\xi} \tag{11}$$

and $$\frac{u}{a_0} = K r^{-\alpha C/2B}\frac{(\xi - 1)}{\xi}$$

when $$1 < \xi < \frac{\gamma + 1}{\gamma - 1}$$

It should be noted from Eqn. (5) that in order the shock may be strong (a) for ξ-values just greater than 1, β^2 ($= b_0^2 / a_0^2$) should be large and (b) for ξ values close to $(\gamma + 1) / (\gamma - 1)$, the shock will also be strong for smaller values of β^2.

2. RESULTS AND DISCUSSIONS

Taking the initial strength of the shocks, say, at $r = 25$, as given in the Table 1 for given values of density ratio and magnetic field, the variation of the shock velocity, the pressure and the particle velocity just behind the shock with propagation distance r are numerically calculated and displayed in Figs. 1-3 respectively. It may be noted from Eqn. (10) that for a given value of ξ and β^2, the plane shock moves with constant strength. An increase in the magnetic field leads to an increase in the shock velocity.

The expressions for nondimensional shock velocity, pressure, density and particle velocity immediately behind the shock for weak diverging plane and cylindrical shocks in magnetogas dynamics with weak and strong magnetic fields are respectively given as (Kumar et al.[6])

$$\frac{U}{a_0} = 1 + \frac{\gamma + 1}{4} K_1 r^{-\alpha/(2+\beta^2)}$$

$$\frac{p}{p_0} = 1 + \gamma K_1 r^{-\alpha/(2+\beta^2)}$$

$$\frac{\rho}{\rho_0} = 1 + K_1 r^{-\alpha/(2+\beta^2)}$$

$$\frac{u}{a_0} = K_1 r^{-\alpha/(2+\beta^2)} \tag{12}$$

where K_1 is a constant of integration, and

$$\frac{U}{a_0} = \beta\left\{1 + \left(\frac{3}{4} - \frac{2-\gamma}{4\beta^2}\right) K_2 r^{-2\alpha\beta^2/(4\beta^2+1)}\right\}$$

$$\frac{p}{p_0} = 1 + \gamma K_2 r^{-2\alpha\beta^2/(4\beta^2+1)}$$

$$\frac{\rho}{\rho_0} = 1 + K_2 r^{-2\alpha\beta^2/(4\beta^2+1)}$$

$$\frac{u}{a_0} = \beta K_2 r^{-2\alpha\beta^2/(4\beta^2+1)} \tag{13}$$

where K_2 is a constant of integration.

2.1 Cylindrical Flew

Taking $(U / a_0) = 1.2$ at $r = 200$ and $\beta^2 = 0.1$ and $(U / a_0) = 1.2$ at $r = 200$ and $\beta^2 = 1.1$ respectively for weak and strong magnetic fields, variation of flow variables with propagation distance r for $\beta^2 = 0.1, 0.25$ and 0.3 and $\beta^2 = 1.1, 1.2, 1.4, 2.0$ and 2.5 have been given in Tables 2(a) and 2(b) respectively. It can be noted from

Table 1. Variation of (U/a_0) with ξ and β^2 (initially $r = 25$ and $\gamma = 1.4$)

$\xi = 1.5$			$\xi = 3.0$			$\xi = 4.5$			$\xi = 5.8$			$\xi = 5.9$		
β^2	U/a_0	K	β^2	U/a_0	K	β^2	U/a_0	K	β^2	U/a_0	K	β^2	U/a_0	K
16	4.5642	7.3105	5	6.7082	12.1526	0.1	4.0360	7.3355	0.1	13.4299	24.9803	0.1	19.1699	39.5511
14	5.3385	8.5501	10	9.2195	16.7026	0.8	6.2901	11.5351	0.3	15.8399	29.4630	0.3	22.6519	46.7475
20	6.3245	10.1293	14	50.8166	19.5954	5.0	12.9910	23.8210	0.5	17.9396	33.3680	0.5	25.6643	52.9500
25	7.0415	11.2781	20	12.8452	23.2709	10	17.9512	32.9167	2.0	20.1932	54.2930	2.0	41.8563	86.3570
35	8.2915	13.2803	25	14.3178	25.9388	14	21.1002	38.6930	3.0	34.7287	64.5970	2.5	45.9960	94.8980
50	9.8742	15.8153	35	16.8819	30.5840	20	25.0998	46.1042	5.0	43.7387	81.3560	3.0	49.7951	102.7374

Table 2(a). Cylindrical flow : variation of flow variables with propagation distance for weak shock with weak magnetic field for $\gamma = 1.4$

$\beta^2 = 0.1$					$\beta^2 = 0.3$					$\beta^2 = 0.25$				
r	U/a_0	p/p_0	u/a_0	ρ/ρ_0	r	U/a_0	p/p_0	u/a_0	ρ/ρ_0	r	U/a_0	p/p_0	u/a_0	ρ/ρ_0
100	1.2782	1.6492	0.4637	1.4637	100	1.3367	1.7858	0.5613	1.5613	100	1.3220	1.7514	0.5367	1.5367
200	1.2000	1.4666	0.3333	1.3333	200	1.2492	1.5814	0.4152	1.4152	200	1.2366	1.5222	0.3944	1.3944
250	1.1798	1.41%	0.2997	1.2997	250	1.2261	1.5277	0.3769	1.3769	250	1.2143	1.5006	0.3572	1.3572
300	1.1692	1.3947	0.2819	1.2819	300	1.2089	1.9874	0.3482	1.3482	300	1.1976	1.4610	0.3294	1.3294
350	1.1573	1.3670	0.2622	1.2622	350	1.1953	1.4559	0.3256	1.3236	350	1.1845	1.43%	0.3076	1.3076
400	1.1477	1.3446	0.2462	1.2462	400	1.1843	1.4302	0.3072	1.3072	400	1.1759	1.4058	0.2898	1.2898
500	1.1329	1.3102	0.2216	1.2216	500	1.1673	1.3909	0.2788	1.2788	500	1.1575	1.3675	0.2625	1.2625
1000	1.0959	1.2237	0.1598	1.1598	1000	1.1238	1.2883	0.2063	1.2063	1000	1.1157	1.2700	0.1929	1.1929

Table 2(b). Cylindrical flow : variation of flow variables with propagation distance for weak shock with strong magnetic field for $\gamma = 1.4$

p-1.1					$\beta^2 = 1.2$				
r	U/a_0	p/p_0	u/a_0	ρ/ρ_0	r	U/a_0	p/p_0	u/a_0	ρ/ρ_0
100	1.2493	1.4062	0.3043	1.2902	100	1.3011	1.3944	0.3086	1.2819
200	1.2000	1.3063	0.2294	1.2187	200	1.2498	1.2962	0.2317	1.2116
250	1.1868	1.2797	0.2095	1.1998	250	1.2363	1.2700	0.2113	1.1926
300	1.1759	1.2596	0.1945	1.1854	300	1.2260	1.2504	0.1959	1.1788
350	1x91	1.2438	0.1827	1.1741	350	1.2179	1.2349	0.1838	1.1678
400	1.1628	1.2309	0.1729	1.1649	400	1.2114	1.2239	0.1739	1.15%
500	1.1528	1.2108	0.1579	1.1506	500	1.2012	1.2027	0.1586	1.1447
1000	1.1228	1.1589	0.1191	1.1132	1000	1.1747	1.1522	0.1190	1.1081

$\beta^2 = 1.4$					$\beta^2 = 2.0$				
r	U/a_0	p/p_0	u/a_0	ρ/ρ_0	r	U/a_0	p/p_0	u/a_0	ρ/ρ_0
100	i.3988	1.3759	0.3177	1.2683	100	1.6564	1.3425	0.3459	1.2446
200	1.3438	1.2801	‘0.2368	1.2001	200	1.5922	1.2517	0.2542	1.1797
250	1.3294	1.2548	0.2154	1.1820	250	1.5754	1.2279	0.2302	1.1628
300	1.3184	1.2359	0.1993	1.1685	300	1.5628	1.2102	0.2123	1.1502
350	1.3099	1.2209	0.1868	1.1577	350	1.5529	1.1962	0.1982	1.1402
400	1.3029	1.2088	0.1765	1.1491	400	1.5449	1.1849	0.1868	1.1321
500	1.2921	1.1899	0.1605	1.1356	500	1.5327	1.1675	0.1692	‘1.1197
1000	1.2643	1.1415	0.11%	1.1010	1000	1.5012	1.1230	0.1243	1.0880

p-2.5				
r	U/a_0	p/p_0	u/a_0	ρ/ρ_0
100	1.1835	1.3269	0.3691	1.2335
200	1.7671	1.2386	0.2695	1.1704
250	1.7515	1.2156	0.2435	1.1540
300	1.7357	1.1984	0.2242	1.1418
350	1.7252	1.1850	0.2090	1.1321
400	1.7168	1.1741	0.1967	1.1244
500	1.7040	1.1574	0.1777	1.1124
1000	1.6705	1.1148	0.1297	1.0820

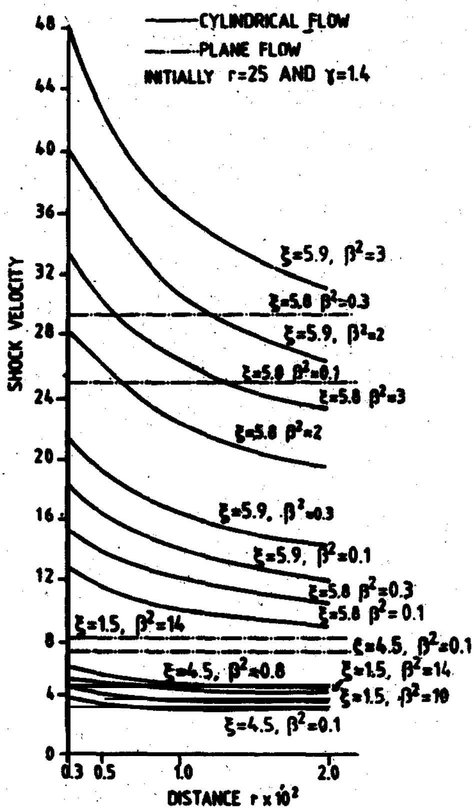

Figure 1. Variation of shock velocity with propagation distance for strong shocks.

Table 2(a) that an increase in the strength of the impressed weak axial magnetic field leads to the increase in the flow variable values. In the presence of strong axial magnetic field, shock velocity and particle velocity increase whereas the pressure and density decrease with increase in magnetic field (Table 2(b)).

2.2 Plane Flow

Plane shock wave in presence of magnetic field propagates with constant shock strength. In the presence of weak magnetic field, flow variables corresponding to constant shock strength level are independent of the magnetic field (Eqn. (12)). Their dependence upon γ is shown in the Table 3(a). It is important to note that an increase in γ leads to an increase in the shock velocity and the pressure, whereas the particle velocity and density remain constant. Dependence of flow variables upon y for the non-magnetic case' reveal that the effect of the presence of the weak magnetic field is indirectly to increase the shock velocity and pressure and to make the particle velocity and density constant (Table 3(a)).

The shock velocity and the particle velocity (Table 3(b)) corresponding to constant shock strength level increase with increase in the value of strong magnetic field, whereas the pressure and the density remain constant.

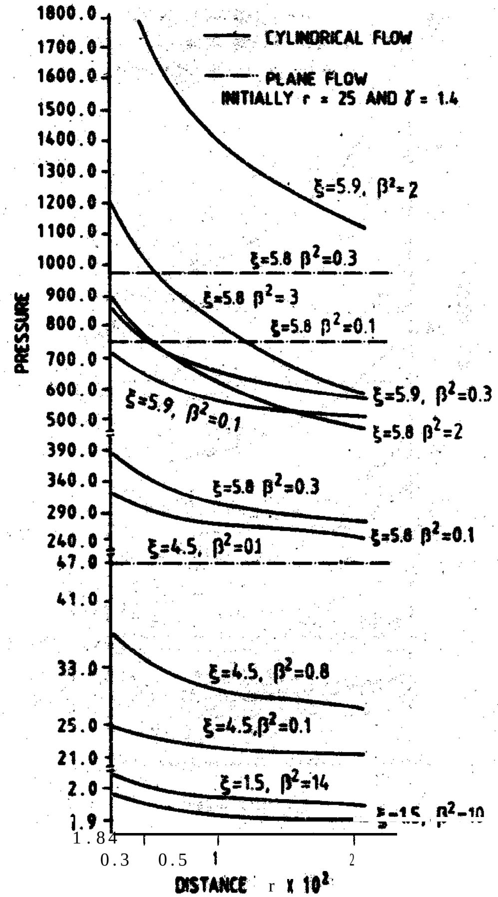

Figure 2. Variation of pressure immediately behind the shock front with propagation distance for strong shocks.

Table 3(a). Plane flow : variation of shock velocity, pressure, particle velocity and density immediately behind the shock front with γ for weak shock

Magnetic case*					Non-magnetic case*				
γ	U/a_0	p/p_0	u/a_0	ρ/ρ_0	γ	U/a_0	p/p_0	u/a_0	ρ/ρ_0
1.2	3.2854	5.9863	4.1533	5.1533	1.2	1.2078	1.4335	0.3778	1.3778
1.4	3.4932	6.8194	4.1553	5.1533	1.4	1.1999	1.4666	0.3333	1.3333
1.66	3.7632	7.8978	4.1553	5.1533	1.66	1.1915	1.4983	0.2879	r.2879

*For all values of weak magnetic field

Table 3(b). Variation of shock velocity, pressure, particle velocity and density immediately behind the shock front for weak shock with strong magnetic field for $\gamma = 1.4$

β^2	U/a_0	p/p_0	u/a_0	ρ/ρ_0
1.1	2.2674	3.6519	1.9866	2.6942
1.2	2.3922	3.6319	2.0749	2.8942
1.3	2.6239	3.6519	2.2412	2.8942
2.0	3.2224	3.6519	2.6788	2.8942
2.5	3.6476	3.6519	2.9449	2.6942

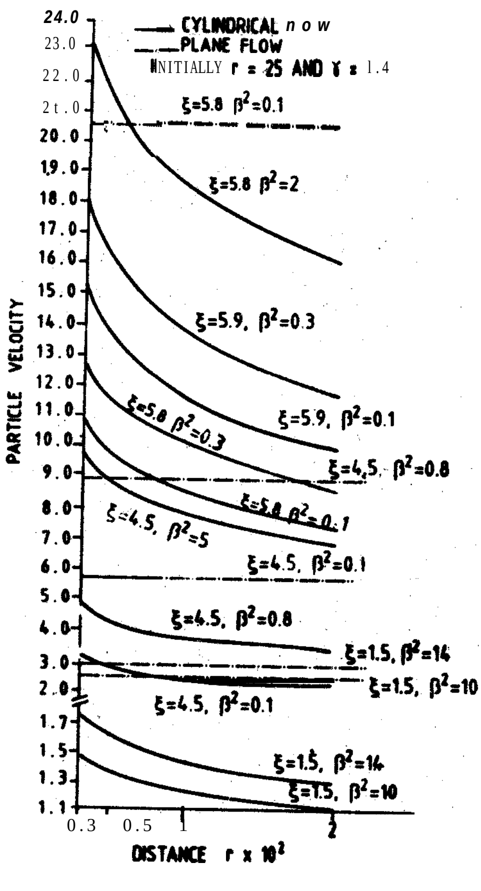

Figure 3. Variation of particle velocity immediately behind the shock front with propagation distance for strong shocks.

In the end, it is important to mention that CCW approximation is not affected by disturbances in the flow behind the shock. However, the effect of overtaking disturbances on the motion of a shock has been studied recently (Yousaf[8]) and found that when the strength of the overtaking disturbance is known, the CCW approximation may be modified to become an exact theory. It is, therefore, essential to modify the present analysis.

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