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Biaxial Flexural Strength and Estimation of Size on the Strength Properties of FRP Composites

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ABSTRACT

Fibre reinforced plastics (FRP) are widely used as structural materials. For designing structural components, a designer is provided with data based on unidirectional testing. But 'in real structural applications the component is subjected to multiaxial stress throughout the material. Hence a multiaxial test is a bettei gauge of the behaviour of FRP components in service. In the present paper a ring-on-ring method was adopted which produces biaxial flexural stress on the FRP specimen. Wubull's statistical weakest link theory was applied to standardize the complexity and to assess the reliability of the results.

1. INTRODUCTION

Increasing applications of fibre reinforced composites in aerospace and spacecraft industry necessitated the development of newer high performance composites. Their characterisation is all the more important when they are considered in applications primary structures. A large number of papers in recent years have appeared discussing the fabrication, testing and application of composites*'*.

Fibre reinforced plastics (FRP) have maximum strength and stiffness values parallel to, and minimum values transverse to, the fibre direction. However, in most applications the reinforcement is used in the form of woven fabrics, random mats and/or multi-layered laminates.

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Standard methods of tests in the tensile, compressive or flexural mode use narrow specimens usualiy cut parallel to the principal material axis. The designer is concerned with the prediction of the deformation and strength from such properties. In real structural applications the components are subjected to loading which produces multiaxial stressing throughout the material, usually of a sustained nature and with varying magnitude. Hence a multiaxial test is a better gauge of the behaviour of FRP components in service than a uniaxial test.

Because of the heterogeneous nature of the composites and the large number of processing parameters controlling the strength behaviour involved, a deterministic evaluation of the composites is often not possible and recourse to statistical evaluation has to be made. An inherent feature of the statistical nature of properties is the so-called size effect, a large volume of a material has a higher failure probability than a smaller volume"

With the advent of new applications more and more families of composites are being introduced every year. However basic data for these, for reliability-based design, is often unavailable. In this paper an attempt has been made to generate basic data for reliability-based design. Thus five types of composites have been studied for biaxial flexural strength.

Weibull' first formulated his statistical weakest link theory to characterize the mechanical behaviour of brittle materials. He later introduced the effect of size in his theory. Over the years, different workers like Babel and Sines[2], Batdorf and Sines[3], Radford aud Lange[4] refined the original Weibull theory and applied it to a large number of materials in different modes of testing. Trustrum and Jayatilaka[5] developed analytical solution for evaluating the combined effect of stress and size and failure modes. Shetty et *al.*[6,7] applied biaxial flexural strength testing to ceramics and glass-ceramics. In the last few years, Morona, Rice,[8-12] etc. have perfected the Weibull analysis and applied it to a number of composite materials.

In the original theory proposed by Weibull, the risk of rupture/failure probablity is given by

$$P = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{1}$$

where P is the failure probability at or below stress level, σ and σ_0, and m are Weibull parameters, called scale and shape parameters respectively.

As the volume of the specimen increases, the number of links in the model increases and hence failure probability increases. The net failure probability is

$$P = 1 - \exp - \int_V \left(\frac{\sigma}{\sigma_0}\right)^m dv \tag{2}$$

A closed form solution of the integral is often difficult to obtain. A close approximation can be obtained by dividing the specimen into finite elements and summing the failure probabilities of these individual elements.

$$\int_V \left(\frac{\sigma}{\sigma_0}\right)^m dv \approx \sum_{n=1}^{n=n} \left(\frac{\sigma_n}{\sigma_0}\right)^m \Delta V_n \tag{3}$$

We can equate the failure probability of the specimen volume under a variable stress with that for a smaller volume under maximum stress by the following equality

$$\sum_{n=1}^{n=n} \left(\frac{\sigma_n}{\sigma_0}\right)^n \cdot \Delta V_n = V_{eq} \tag{4}$$

where V_{eq} represents volume of a specimen under uniform tension/compression and having the same failure probability as the larger specimen with variable stress. Equation (4) enables us to apply the results obtained in one mode of testing to that in any other mode, if stress state is known.

To calculate equivalent volume, the specimen is divided into 5 elemental volumes as shown in Fig. 1. The innermost disc having a radius = $b = r_0$ and under a uniform stress and four annular rings with outer radii r_1, r_2, r_3, and r_4 respectively. The average stress in **these** is **taken** as the stress at the average radii $r_{0.5}$, $r_{1.5}$, $r_{2.5}$, $r_{3.5}$ respectively. Stress at any radius is given by

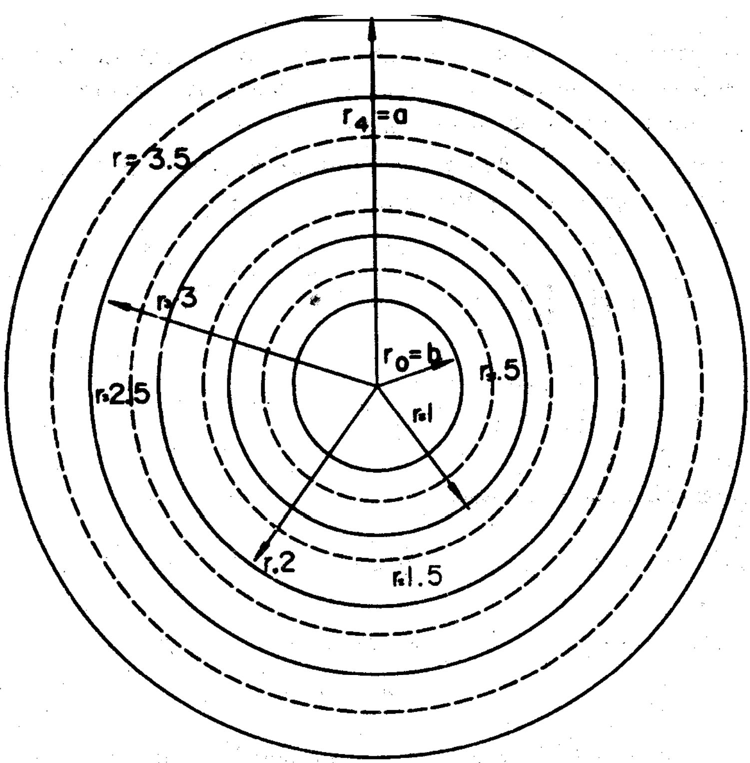

Figure 1. **Finite element analysis to calculate equivalent volume under maximum stress; $r_0 = b =$ load ring radius, r_b support ring radius; $A_{eq} = A_{act} (\sigma/\sigma_{max})^m$.**

$$\sigma_r = \sigma_{max}\left(\frac{\ln(a/r)}{\ln(a/b)}\right) \tag{5}$$

$$\frac{\sigma_r}{\sigma_{max}} = \frac{\ln(a/r)}{\ln(a/b)} \tag{6}$$

$$\left(\frac{\sigma_r}{\sigma_{max}}\right)^m = \left(\frac{\ln(a/r)}{\ln(a/b)}\right)^m \tag{7}$$

Volume of the first ring

$$= \pi \cdot t \cdot (r_1^2 - b^2) \tag{8}$$

where t is thickness and r_1 and b are outer and inner radii. Equivalent volume of the first ring =

$$2\pi \cdot t(r_1^2 - b^2) = \left(\frac{\ln(a/r)}{\ln(a/b)}\right)^m \tag{9}$$

(The factor 2 in Eqn. (9) has been used to account for the failure probablity in radial and tangential directions).

The equivalent volumes of other four rings can be calculated similarly. The equivalent volumes of four ring elements and volume of central disc under maximum stress is equal to the total equivalent volume.

For a given equivalent volume, the variation of failure probability is given by Eqn. (1). By rearranging the terms in Eqn. (1), we have

$$1 - P = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{10}$$

$$\frac{1}{1-P} = \exp\left[\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{11}$$

Taking logarithm twice, we have

$$\ln/\ln(1/1 - P) = m(\ln\sigma - \ln\sigma_0) \tag{12}$$

For $P = 0.632$, $\ln\ln(1/1 - P) \simeq 0$. Hence $\ln\sigma - \ln\sigma_0 = 0$

or $\sigma = \sigma_0 \quad a^2 = \gamma p/\rho$

Hence graph of In $\ln(1/1\text{-}P)$ versus $\ln\sigma$ is a straight line with slope m and with an intercept on abscissa equal to $\ln\sigma_0$.

$$\text{For } P = 0.5,\ \ln\ln(1/1 - P) = -0.3665 \tag{13}$$

Hence, the mean strength is given by the antilog of the intercept of the graph with the line $x = -0.3665$.

With the help of the foregoing analysis, we can calculate σ_0 and a,,, for various equivalent volumes for equal failure probabilities of 0.632. For two specimens with equivalent volumes V_1 and V_2, *we* have

$$\ln \ln (1/1 - P) = 0 = \ln V_1 + m \ln \sigma_{0,1} = \ln V_2 + m \ln \sigma_{0,2} \tag{14}$$

where $\sigma_{0,1}$ and $\sigma_{0,2}$ are the stresses at $P = 0.632$, for volumes V_1 and V_2 respectively. By rearranging terms in Eqn. (14) we have

$$\ln V_1 - \ln V_2 = m [\ln \sigma_{0,2} - \ln \sigma_{0,1}] \tag{15}$$

$$\ln (V_1/V_2) = m \ln (\sigma_{0,2}/\sigma_{0,1}) \tag{16}$$

or

$$\frac{V_1}{V_2} = \left(\frac{\sigma_{0,2}}{\sigma_{0,1}}\right)^m \tag{17}$$

$$\frac{\sigma_{0,2}}{\sigma_{0,1}} = \left(\frac{V_1}{V_2}\right)^{1/m} \tag{18}$$

Equation (18) shows that as the volume increases, the mean stress decreases. The plot of $\ln \sigma_{0,n}$ versus $\ln V$ is a straight line with a negative slope $= -1/m$. At the intercept on y-axis, we have $\ln V = 0$ (or $V = 1$ cc). Antilog of this intercept gives the statistical mean strength of a unit volume of the material. This value of $\sigma_{0,0}$ is a material property independent of volume.

If n number of samples are tested, then their failure strengths can be arranged **rankwise** in increasing order. The sample space is divided into n intervals. Each specimen is considered at the centroid of each interval. Though, there are complicated formulae for assigning probability in each interval, the following simplified formula suggested by **Lankford**[13] suffices.

$$p_r = \frac{r}{N+1} \tag{19}$$

where r is rank of specimen. Cumulative failure probability at the failure of the last specimen is given by

$$P = \frac{n}{n+1} \tag{20}$$

To calculate the mean strength at a unit volume of 1 cc and the size effect, we need to test a few series of specimens of different equivalent volumes V_1, V_2 stresses $\sigma_{0,1}$, $\sigma_{0,2}$, etc., and average m are evaluated. A plot of $\ln \sigma_{0,n} \rightarrow \ln V_n$ establishes the relation between the **strength and** volume.

The foregoing analysis enables a designer in reliability-based design where the, data obtained from small specimens tested in the laboratory is to be extrapolated to large structures under complex stress conditions.

The biaxial stress distribution in the ring-on-ring test, has been evaluated by Kristen and Wooley[14] using elementary plate theory. The maximum stress occurs in the central region inside the loading ring and is given by

$$\sigma_r = \sigma_t = \frac{3P(1+\nu)}{4\pi t^2}\left[2\ln\left(\frac{a}{b}\right) + \left(\frac{1-\nu}{1+\nu}\right)\left(\frac{a^2-b^2}{R^2}\right)\right] \tag{21}$$

where σ_r and σ_t are radial and tangential stresses, P is load, t is thickness, ν is Poisson's ratio, a and b are radii of support ring and load ring, respectively and R is the radius of the specimen. At any other radius σ_r and σ_t are slightly different.

$$\sigma_r = \frac{3P(1+\nu)}{4\pi t^2}\left[2\ln\left(\frac{a}{r}\right) + \left(\frac{1-\nu}{1+\nu}\right)\left(\frac{(a^2-r^2)b^2}{r^2R^2}\right)\right] \tag{22}$$

$$\sigma_t = \frac{3P(1+\nu)}{4\pi t^2}\left[2\ln\left(\frac{a}{r}\right) - \frac{(1-\nu)}{(1+\nu)}\left[\frac{b^2(a^2+r^2)-2a^2r^2}{r^2R^2}\right]\right] \tag{23}$$

Though variation in the two is not the same, we can consider the stress to be proportional to ln (a/r) as an approximation.

2. EXPERIMENTAL

2.1 Materials

The following materials were used in the experimental part : (a) phenolic resin supplied by Indian Plastics Ltd., Bombay; (b) glass fabric 1: 1 plain weave, aminosilane treated supplied by Unnati Corporation Ltd., Ahmedabad; (c) nylon fabric, wt 225 gm/m^2 procured from local market; (d) carbon fabric, TORAY-6141 procured from Toray, Japan; and (e) kevlar fabric, 1 : 1 plain weave procured from Fabric Development Corporation, USA.

2.2 Methodology

Composites were fabricated by first impregnating phenolic resin on the reinforcing fibre fabrics by brushing. The fabric treatment was repeated till the resin take up was 30 per cent w/w. The solvent was evaporated out, first at room temperature and then by keeping the impregnated fabrics in vacuum oven. The fabrics were then cut into specific sizes and were stacked in an open mould. The mould was placed on heated platens of hydraulic press and curing was carried out at 160°C for two hours under 70 kg/cm^2 pressure.

The following composite laminates were thus fabricated :

(i) Glass fabric only

(ii) Glass fabric 70 per cent w/w + nylon fabric 30 per cent w/w

(iii) Glass fabric 70 per cent w/w + kevlar fabric 30 per cent w/w

(iv) Glass fabric 70 per cent w/w + carbon fabric 30 per cent w/w

Glass fabric used in all the cases was of same quality, particularly in weave and surface treatment.

3. GLASS FABRIC REINFORCED PMR POLYIMIDE RESIN COMPOSITES

PMR polyimides are high temperatures resistant class of resins which retain their strength properties up to 300°C. These resins can be easily processed and curing is done *in situ*. The basic components of resin are taken in liquid form. The monomeric reactants react to produce amic acid which is further polymerised to polyimides. The end capping monomer reactant then cross links by reverse Diels-Alder reaction.

3.1 PMR Solution

PMR solution-2,2; 3,3;-benzophenone tetra carboxylic dianhydride purified by crystallization from boiling acetic anhydride, and 5-norbornene-2,3-dicarboxylic anhydride were weighed in two separate round bottom flasks fitted with reflux condensor. Enough methanol was added to provide a final 50 per cent w/w ester solution in each case. The mixtures were refluxed gently for two hours, cooked and then the two ester solutions were mixed. Diaminodiphenyl methane (DDM) was dissolved in methanol to prepare 50 per cent w/w solution. Ester solution and DDM solution were then mixed in required proportions.

Reinforced fabric was then treated with this solution ensuring complete wetting. Then the prepregs were dried at 70°C for one hour. The oven temperature was then raised to 220°C and the prepregs kept for one hour.

The prepregs as prepared above were stacked in semi-positive mould and moulded at 290°C and under 350 kPa pressure. The temperature was then raised to 320°C (in 10–15 minutes) and maintained for 2 hours at 700 kPa.

3.2 Biaxial Flexural Strength

The laminates were cut into square pieces of different dimensions. A ring-on-ring method was adopted to produce biaxial flexural stresses in the specimen. The coupen was supported on a support ring of radius *A* and loaded by a concentric ring of smaller radius *B* to failure, in a 16 tonne hydraulic press. Locating pins attached to the load ring with matching hole in the fixture of the support ring ensures concentricity. The failure in composites due to biaxial flexural stresses was found to be of complex nature with a combination of matrix failure, fibre pullout, shear failure at interfaces and fibre breakage. Hence to standardize the complexity and to assess the reliability, the result was subjected to Weibull analysis.

4. RESULTS AND CONCLUSIONS

A series of specimens of each of the materials were tested for a number of combinations of load ring and support ring radii *b* and a. The stress at failure for each specimen was calculated using Eqn. (21). The data was tabulated **rankwise** in the, order of increasing failure stress. Then a graph of lnln **(1/1-*P*)** versus In σ_t was plotted for each series of each composite. For the sake of brevity, only one table and the corresponding graph for the composite **RKM-3** (described in Table 1) are given (Fig. 2). From the graph, $\sigma_{0,V}$ at ***P*=0.632** and a,,,, at ***P*=0.5** for different equivalent volumes of each composite were determined. This data is presented in Table 2. **From Table** 2,

Table 1. Biaxial flexural strength test of composite RKM-3*

Rank	*P*	1/(1–*P*)	lnln(1/1–*P*)	σt (MPa)	$\ln\sigma_t$
1	0.1	1.111	-2.25	445.4	6.099
2	0.2	1.25	-1.5	485.7	6.186
3	0.3	1.429	-1.031	528.1	6.269
4	0.4	1.667	-0.6717	567.0	6.34
5	0.5	2.0	-0.3665	371.1	6.348
6	0.6	2.5	-0.0874	576.1	6.356
7	0.7	3.333	0.1855	577.8	6.359
8	0.8	5.0	0.4759	664.5	6.499
9	0.9	to.0	0.834	696.1	6.545

* V_{eq} under maximum stress = 0.6064 cc

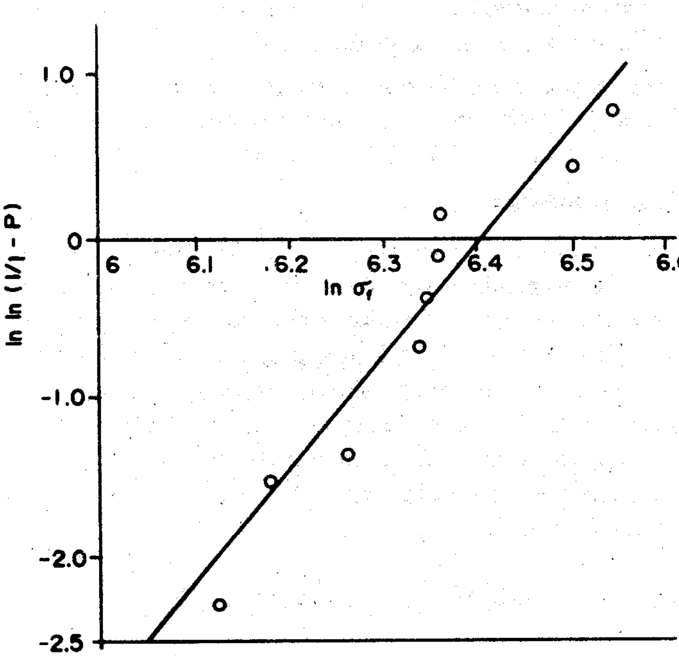

Figure 2. Graph **of lnln(1/1–*P*) versus ln** a,.

Table 2. **Mean biaxial flexural strengths, Weibull shape and scale parameters against equivalent volumes for composites** RKM-I, RKM-2, RKM-3, **RKM-4** and **glass/PMR**

Material	V_{eq} (cc)	$\ln V$	σ_0 (MPa)	σ_{mean} (MPa)	$\ln\sigma$	$\ln\sigma_{mean}$
RKM-1	2.386	0.87	635.9	620.2	6.455	6.43
	3.474	1.245	685.4	656.6	6.53	6.487
	3.862	1.351	669.8.	641.0	6.507	6.465
	12.944	2.561	539.2	521.7	6.29	6.257
RKM-2	13.202	2.58	553.2	553.2	6.316	6.316
	14.792	2.694	537.6	937.6	6.287	6.287
	21.66	3.075	534.9	530.6	6.282	6.274
RKM3	2.31	0.837	595.9	579.4	6.39	6.362
	3.474	1.245	690.2	675.5	6.537	6.51
	3.634	1.29	653.9	643.5	6.483	6.47
	12.944	2.561	540.2	514.4	6.292	6.243
	14.792	'2.694	555.6	510.8	6.32	6.256
RKM-4	0.6064	0.5	601.8	572.5.	6.4	6.35
	2.49	0.912	493.8	470.1	6.202	6.153
	0.476	-.742	642.3	620.2	6.465	6.43.
	2.413	0.881	559.5	539.2	6.327	6.29
Glass/PMR	0.89	-.1165	645.5	629.5	6.47	6.445
	4.252	1.447	623.3	621.7,	6.435	6.4325

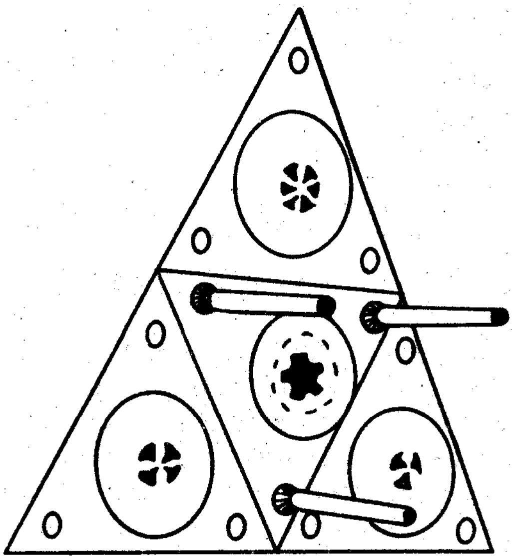

Figure 3. **Fixtures for ring-on-ring test.**

Table 3. **Values of mean flexural (for a volume of 1 cc) Weibull scale parameter and Weibull shape parameter for the composites RKM-1, RKM-2, RKM-3, RKM-4* and glass/PMR**

Name	Composition	$\sigma_{m,m}$ (MPa)	$\sigma_{0,0}$ (MPa)	m
RKM-1	65% E glass +35% phenolic resin	764.1	788.1	7.248
RKM-2	45.5% E glass + 19.5% nylon +35% phenolic resin	742.2	765.6	7.246
RKM-3	45.5% E glass +19.5% kevlar + 35% phenolic resin	511.9	561.2	7.14
RKM-4	45.5% E glass + 19.5% carbon fibres +35% phenolic resin	580	603.7	11.76
Glass/PMR	E glass + PMR resin	646.2	658.6	22.6

* Compositions RKM-2, RKM-3 and RKM-4 were arrived at by adjusting the plies of reinforcement during stacking for the laminate.

graphs of $\ln \sigma_0$ and $\ln$ a,. $\ln V_{eq}$ were plotted (Fig. 3). These graphs enabled the power law relation between σ and V to be established. Graphs for RKM3 and RKM-4 only are shown. $\sigma_{0,0}$ (mean strength for a specimen with 1 cc volume), Weibull scale parameter and m, Weibull shape parameter for the composites are presented in Table 3.

If we compare the systems RKM-1 and glass PMR, we find that RKM-1 has a larger strength, but more pronounced size effect. Let V be the volume at which both the systems have equal strengths. Then

$$\frac{764.1}{(V)(1/7.246)} = \frac{646.2}{(V)(1/22.6)} \tag{24}$$

Solving we obtain

$$V^{0.09375} = \frac{764.1}{646.2}$$

Therefore

$$V = 5.975 \text{ cc and strength } 597.1 \text{ MPa} \tag{25}$$

4. CONCLUSIONS

Amongst the composites tested during the study, glass fabric reinforced phenolic resin laminates were found to have highest biaxial flexural strength. Partial replacement of glass fabric by nylon, kevlar or carbon fibre did not improve the strength, where the maximum reduction was of the order of 20 per cent. Phenolic resin glass fabric laminates were found to have better biaxial flexural strength than PMR polyimide resin glass fabric laminates, thus indicating the brittle nature of PMR composites.

Systems RKM-1, RKM-2 and RKM3 have a fairly pronounced size effect whereas PMR glass composite has a very low size effect. For a specimen of size 1 cc, glass-phenolic and glass-PMR laminates have a strength value of 764.1 and 646.2. However, if calculated for equal strength, at a volume of 5.975 cc both systems will have equal strength of 597.1 MPa but for components larger than this volume. Glass-PMR composites will have higher strength than glass-phenolic laminates.

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